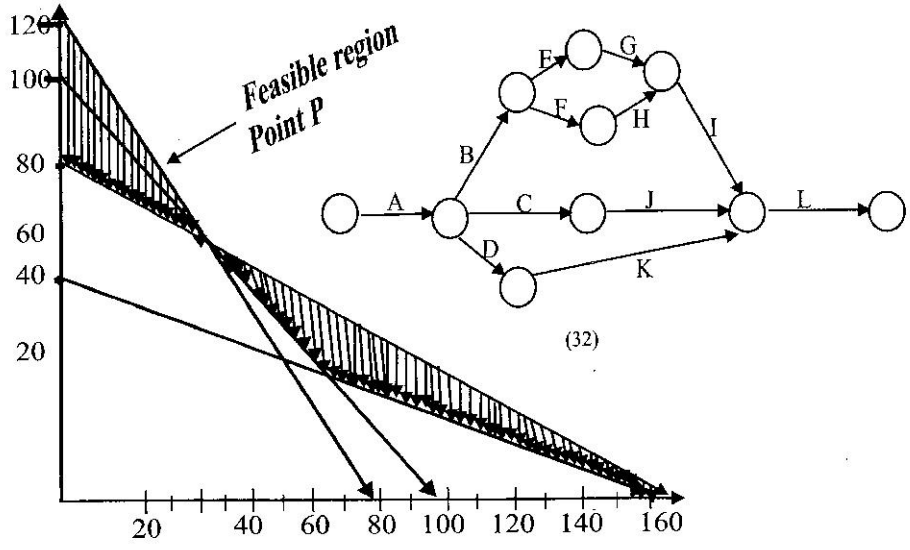
Questions and Solutions on Optimization Techniques



for

Post Graduate Students

(Planners and Managers)

and Under Graduate Students

Alamutu S. A.

Contents

Question 1.

Graph the linear inequality $4x + 8y < 16$ ----------(1)

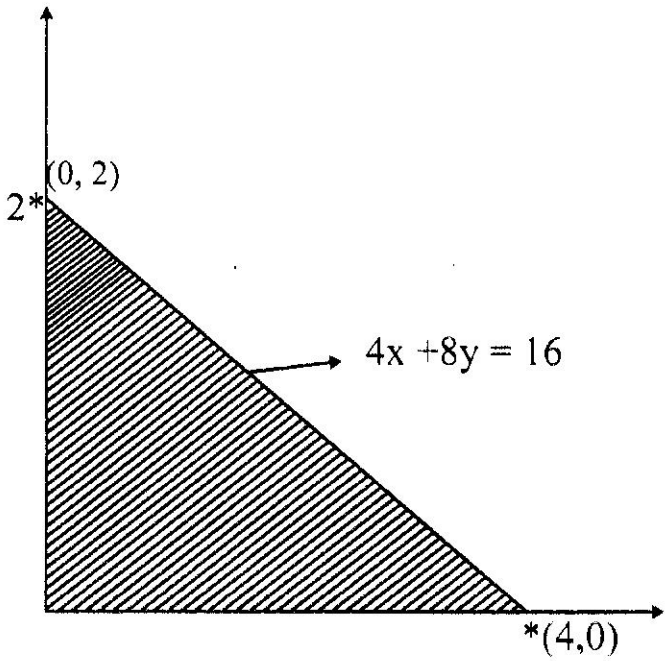
Solution:
First, make the inequality sign an equality sign.
Therefore, the required inequality from (1) becomes

$$4x + 8y = 16$$

Then, the graph of $4x + 8y = 16$ is the line whose slope is $-\frac{1}{2}$ and y intercept at $y = 2$

The graph is given below in fig. (1)

Fig (1)



Note: That the shaded part of the graph (fig. 1) denotes the set of poin whose coordinates satisfy.

$4x + 8y = 16$. This does not include the set of points on the line.

To indicate the set of points on the line and below it, we write;

$$4x + 8y < 16$$

And the set of points above the line is defined by the inequality

$$4x + 8y > 8.$$

Question 2.

Graph linear inequality $2x + y < 30$ and $x + 2y < 50$ on the same graph.

Solution:

Let $2x + y < 30$---------(1)

$x + 2y < 75$ ----------(2)

From (1) $2x + y = 30$ ----------(3)

And from (2) $x + 2y = 50$ --------(4)

Therefore, from (3) put $x = 0$, $y = 30$

and for line $2x + y = 30$, the coordinate is (0, 30)

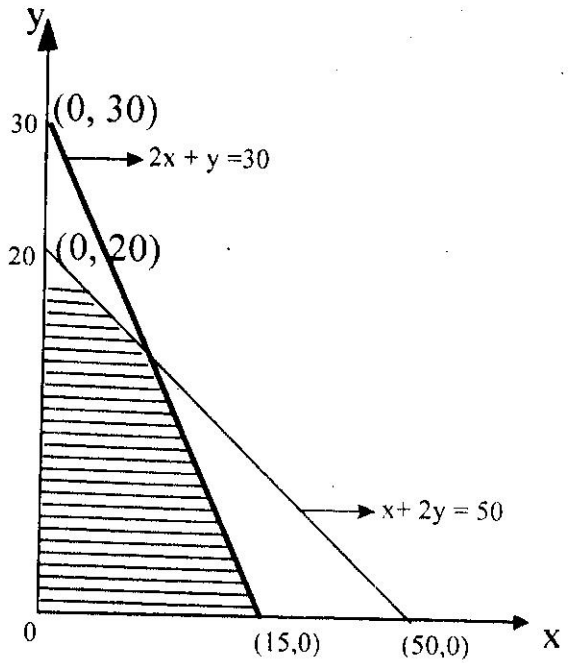
and when $y = 0$, $x = 15$ the coordinate is (15, 0).

Similarly, for line $x + 2y = 50$

Put $x = 0$, $y = 15$ the coordinate is (50,0).

Now if we plot the coordinates for the two graphs, we have

Fig. 2



(Please make sure that you chose correct scale 1 unit represent 10cm on y-axis and 1 unit represent 10cm also on x-axis).

Question 3:

An animal food is to be a mixture of two stuffs, each unit contains protein, fat and carbohydrate in the number of grams given in the table below;

Table 1.

	Food stuff	
	1	2
Protein	10	5
Fat	0.1	0.9
Carbohydrate	10	30

Each bag of the resulting mixture is to contain at least 40 grams of proteins, 1.8grams of fat, and 120grams of carbohydrate. Graph the system of linear inequalities, which shows the mixture that meets these requirements.

Solution:

Since each unit of food stuff 1 contain 10grams of protein and each units food stuff 2 contain 5grams of protein, and since each bag of the mixture must contain at least 40grams of protein, one inequality which must be satisfy is

$10x + 5y \geq 40$--------(1)

where x denotes number of units of food stuff 1 in the mixture and y the number of units in food stuff 2. Similarly, the other relevant inequality are;

for fat $0.1x + 0.9y \quad 1.8$----------(2)

for carbohydrate $10x + 30y \quad 120$----------(3)

We also have non-negativity constraints $x \geq 0, y \geq 0$.

: we now need to plot on y-axis and x-axis, equations (1), (2) and

(3). To do this, we have the following: from (1) $10x + 5y = 40$

Such that if $x = 0, y = 8$ the coordinate is (0, 8)

And when $y = 0, x = 4$ the coordinate is also (4, 0).

Similarly from (2), we have $0.1x + 0.9y = 1.8$

from this inequality, we have;

$\Rightarrow 0.1/10x + 0.9/10y = 1.8/10$

$\Rightarrow 1/10x + 0.9/10y = 18/10$

$\Rightarrow 10x + 90y = 180.$

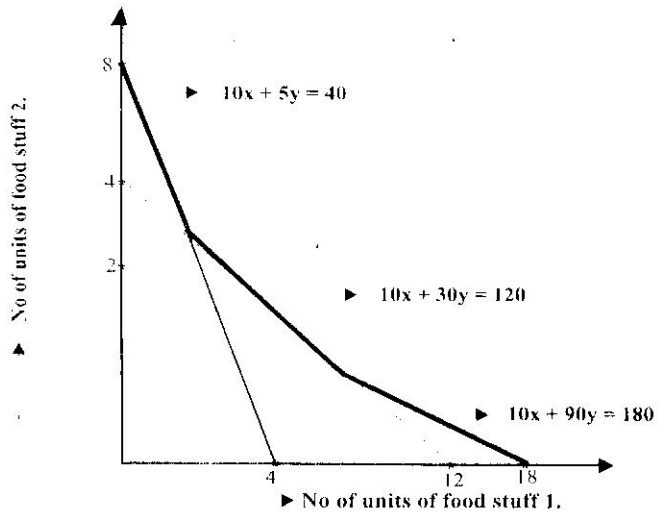
When $x = 0, y = 2$ the coordinate is (0, 2), and when $y = 0, x = 18$ the coordinate is (18, 0)

Also, from (3) we have: $10x + 30y = 120$

When $x = 0, y = 4$ the coordinate is (0, 4) and also when $y = 0, x = 12$ The coordinate is (12, 0).

What to do next is to plot the above coordinate on the same graph. Such that we have as follows;

Fig. 3



The portion of the graph, which lies above and to the right of the heavy line, indicates those mixtures, which meet these requirements.

Question 4.

The manufacturer of animal feed described in question 3 above seek to determine the combination of food stuff 1 and 2 will satisfy the specified nutrients required at the lowest possible cost. Assuming that each unit of food stuff 1 cost = # 40:00.

What is the optimum?

Solution:

Let x_1 denote the number of units of foodstuff 1 employed

and let x_2 denote the number of unit of foodstuff 2 employed.

$\therefore$ The objective function to be minimized is defined by

$C = 60x_1 + 40x_2$

The none negativity constraints are $x_j \geq 0$ where $j = 1, 2$.

The structurer constraints are;

$10x1 + 5x2 = 40$

$10x1 + 30x2 = 120$

$0.1x1 + 0.9x2 = 1.8$

$\therefore$ From your lecture, pick out the visible region and use the coordinates to determine the minimal cost.

Question 5.

A firm is planning production for the next week. It is making two products X and Y, each which requires certain foundry, machining and finishing capacities as shown below;

The following number of hours is available.

Table 2.

Product	Hours Per Unit		
	Foundry	Machining	Finishing
X	6	3	4
Y	6	6	2

In each area during the week planned

Foundry 110
Machining 150
Finishing 60

Graph the system of linear inequality, which shows the quantities of X and Y that can be produced.

Solution:

Since product X and Y each required 6hrs of foundry time for each unit produced, and since there are 110 foundries available, the total amount of the available foundry time, which is utilized, must satisfy the relation;

$6x + 6y \leq 110$----------(1)

Where x represent the number of the units of products X processed
And y represent the number of the units of product Y.

Similarly, the relations pertaining to machining and finishing capacity are respectively given as:

$3x + 6y \leq 150$----------(2)

and $4x + 2y \leq 60$----------(3)

Note that in addition to the production constraints indicated above, there are two additional conditions that any output combination must satisfy, i.e. the none negativity constraints $x \geq 0$, $y \geq 0$ (production cannot be negative).

Now, solve and plot this constraints or the three inequality on the same graph as follows;

From $6x + 6y = 110$

Put $x = 0$ $y = 18.5$, the coordinate is $(0, 18.5)$ and for $y = 0$

$X = 18.5$, then the

coordinate is $(18.5, 0)$

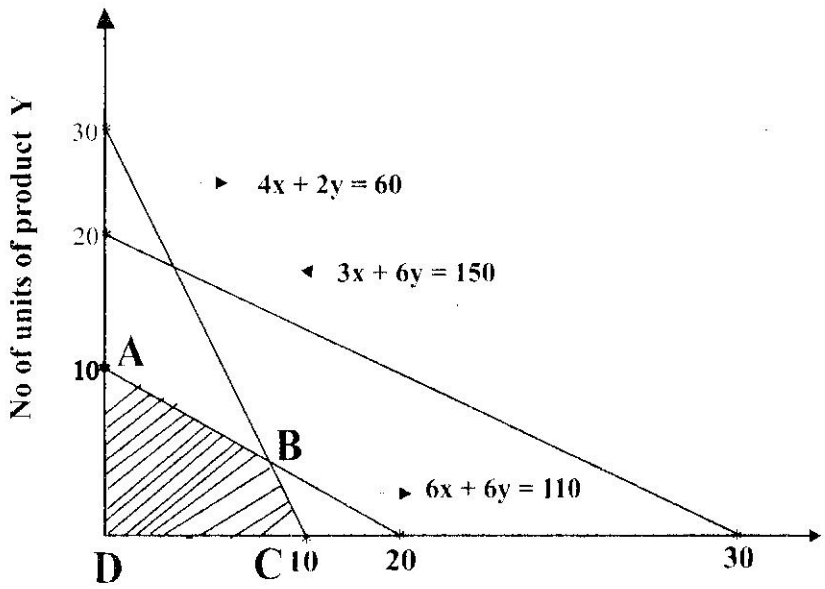
Also, for $3x + 6y = 150$

Put $x = 0$, $y = 25$, the coordinate is $(0, 25)$ and for $y = 0$, $x = 50$ coordinate is $(50,0)$.

And for $4x + 2y = 60$

put $x = 0$, $y = 30$ the coordinate is $(0,30)$ and for $y = 0$, $x = 15$ the coordinate is $(15, 0)$

Next plot the above coordinates on your graph Y-X-axis. We have;



No of unit of product X
(drawing not to scale)

The shaded portion of the above shows that output combination

Satisfying all restrictions. Note that in this problem, machining capacity is not a reality, a constraint at all; i.e. my output combination, which satisfies the other two constraints, will also satisfy machining capacity.

Question 6

Now assuming the above question (5) seeks to determine the combination of product X and Y, which is most profitable. If for instant each unit of X

produced result in a profit of #2 and each unit of Y result in a profit of #3 what is the optimum combination of products to produce?

Let x, represent unit of product X produced and

y, represent unit of product Y produced.

Therefore, the objective function to be maximized is defined by

$$P = 2x + 3y$$

The non-negativity constraints are:

$$x \quad 0, y \quad 0.$$

The structural constraints are:

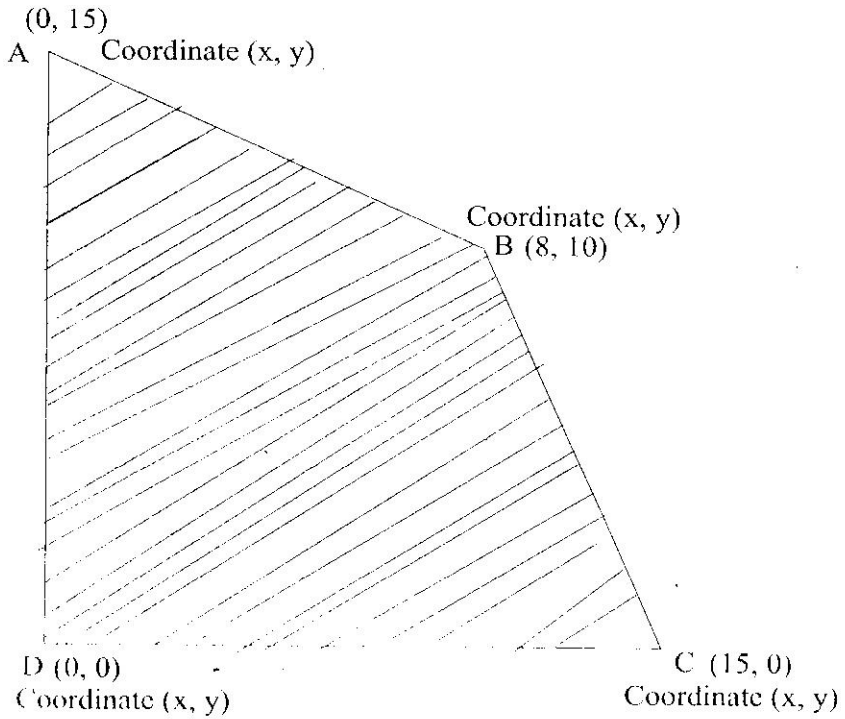
$$6x + 6y \leq 110$$

$$3x + 6y \leq 150$$

$$4x + 2y \quad 60$$

To determine the optimum combination of products, we must first determine which combinations of the two product x, y are capable of being produced i.e. which combinations satisfy the non-negativity and structural constraints. The shaded region, called technical feasibility polygon or feasible region i.e. from the graph, the visible region is given by Fig. 5.

Fig. 5



Check these coordinates from the graph use their value to determine the optimal combination for x and y. (Next question will give detail about how this can be worked out).

Question 8:

A manufacturer is producing two kinds of petrochemical product A and B. The profit per gallon of A is #2, and that of B is #3. Restricted supply of crude oil forces him to produces not more than 3 million gallons of petrochemical products in one production period. Limited processing plant shows that 1- gallon of A uses 1 unit and 1 gallon of B uses 3unit and there are more than 4 million units, available. Formulate the problem

inform of Linear Programming and obtain production schedule which maximizes the manufacturer profit.

Solution:

Let x be the quantity in million gallons of product A the manufacturer can produce and y be the quantity in million gallons of product B the manufacturer can produce.

Therefore

(i) The objective function is

$Z = 2x + 3y$ i.e.

$\text{Max } Z = 2x + 3y$

(ii) The limitation the manufacturer is facing

The 1st limitation, since he cannot obtain more than three million gallons i.e. he can't use more than this. Therefore the 1st constraint is the Crude Oil Limitation, what this is saying is

$x + y$ must not be more than 3 million gallons

$\Rightarrow \quad x + y \leq 3$----------(1)

(Crude Oil Constraint)

The 2nd limitation is the plant limitation, which is the 2nd constraint i.e. every gallon of A will use one unit of the processing plant and similarly for B, will also use 3 units of the processing plant, and that both cannot use more that 4 million units.

$X + 3y \leq 4$----------(2)

And that the maximum he can produce is 0.

$\Rightarrow$ $x \geq 0, y \geq 0$; it implies that he never can produce negative units.

Whence, the above equations are the model, which are to be solved we have as follows

max $Z = 2x + 3y$ subject to the following constraints

$X + y \leq 3$---------(1)

$x + y \leq 4$---------(2)

$x \geq 0, y \geq 0$

now use graphical method to solve the above

First plot your constraints with their respective coordinate

From (I)

i.e. $x + y = 3$

put $x = 0$, $y = 3$ gives $(0, 3)$ as coordinates

And if $y = 0$, $x = 3$ gives $(3, 0)$ as coordinates

And from (2) $x + 3y = 4$

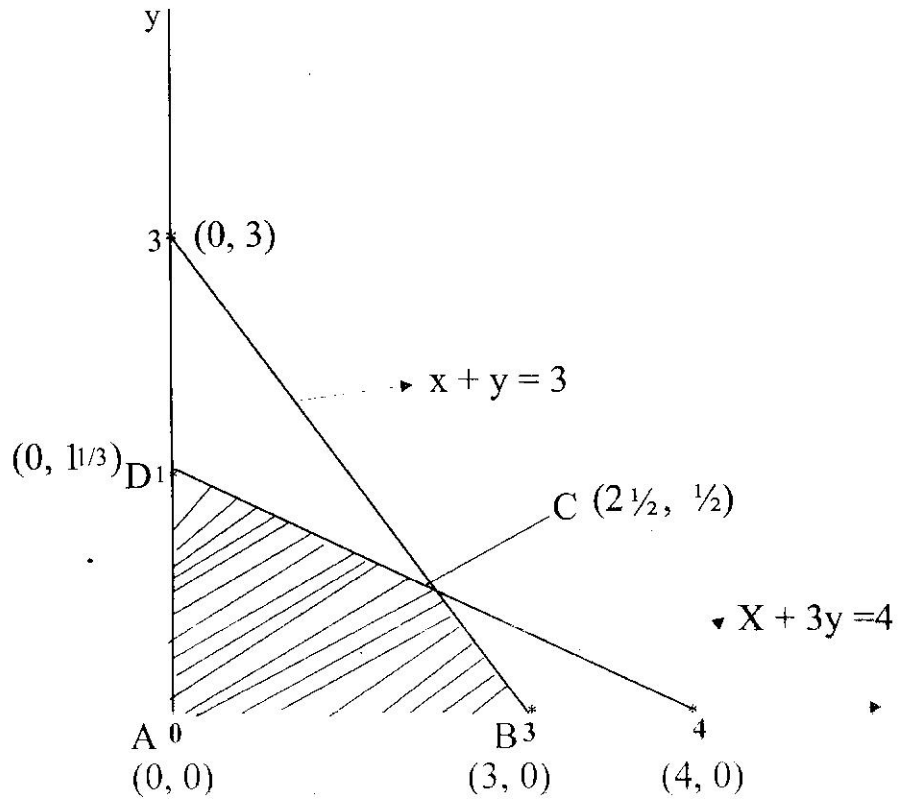
Put $x = 0$, $y = 4/3$ gives $(0, 4/3)$ as coordinates

And if $y = 0$, $x = 4$ gives $(4, 0)$ as coordinates

If we now plot all these coordinates on a graph,

We have; in fig 6 as follows;

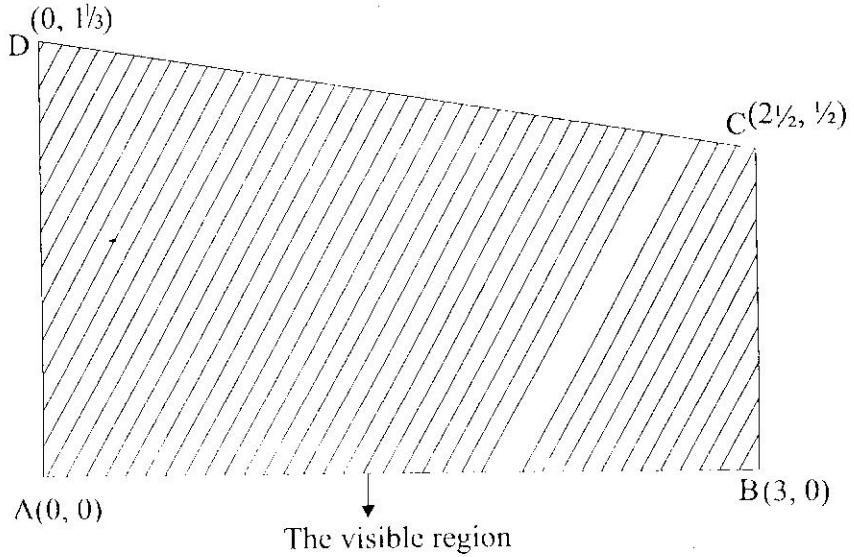
Fig. 6.



Now for line $x + y = 3$, and $x + 3y = 4$. choose a point below the line within the shaded area which must satisfy the inequalities (i.e. the two lines). Any point of such within this region is called the visible point and the region is called the visible region. Thus, we say that the visible region is those regions that satisfy any of the equations.

You can take the visible points out by drawing the polygon with their respective coordinates as follows.

Fig. 7



Now, what you do is to find the value of your objective function i.e. the profit (i.e. what will maximize the manufacturer profit).

Using your objective function $Z = 2x + 3y$

Therefore, for coordinate (0, 0). that the manufacturers is not producing anything i.e. No profit i.e.

$Z = 2(0) + 3(0) = \#00$

For $(0, {}^{4}/_{3})$ coordinates there are only ${}^{4}/_{3}{}^{of}$ B produced.

and $Z = 2(0) + 3({}^{4}/_{3})$

$= 4$

$Z = \#4$ million

For (3, 0) coordinates only 3 of A is produced

And $Z = 2(3) + 3(0)$

$= 6$

$Z = \#6$ million

For $({}^{5}/_{2}, {}^{1}/_{2})$ coordinates, ${}^{5}/_{2}$ of A and ${}^{1}/_{2}$ of B is produced. And $Z =$

$2({}^{5}/_{2}) + (3)({}^{1}/_{2}) = 6.5$ million

I.e. $Z = \#{}^{13}/_{2}$ million is realized.

Therefore, from our analysis, we shall advice the manufacturer that for maximum profit he should produce ${}^{5}/_{2}$ million gallon of A and ${}^{1}/_{2}$ million gallon of B.

Question 8:

A manufacturer is going out of business. However, he has two contractual agreements that must still be fulfilled. One agreement specifies that customer. A must be supplied, daily, at least six unit of product X and Y combined. The second agreement specifies that customer B must be supplied daily at least eight units of product X and Y combined. Graph the system of inequalities which shows, which output combinations, satisfy these requirements. Assuming, now that each unit of product X requires 1 man-hour and that there are only 20 man-hour available daily.

Show which output combinations satisfy the requirement. Finally, if a union rule prohibits the employment of less than 8 man-hours daily. Show the feasibility output combinations.

Solution.

(i) $x_A + y_A \geq 6$----------(1)

(ii) $x_B + y_B \geq 8$----------(2)

$X + y > 14$

$\Rightarrow$ $y \geq 14 - x$.....(3)

and $x + 2y \leq 20$

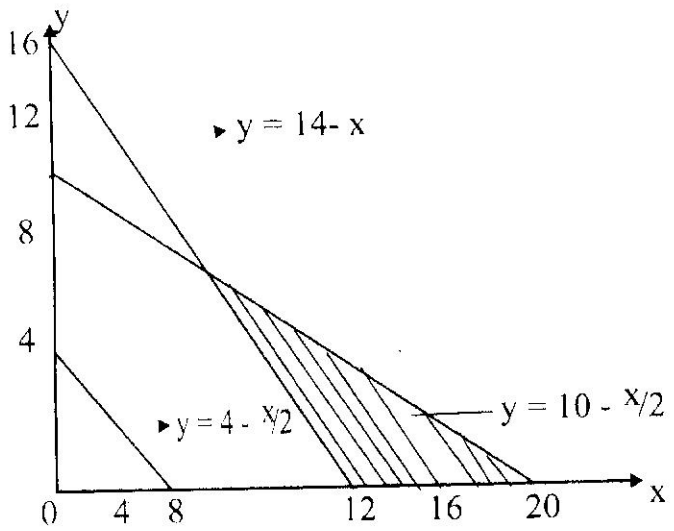
$\Rightarrow$ $y = 10 - x/2$.....(4)

and $x + 2y = 8$

$\Rightarrow$ $y = 4 - x/2$.....(5)

Work out the coordinates of (3), (4) and (5) plot them as follows:

Fig. 8.



DUALITY THEOREM:

Corresponding to any L-P problem, their exist L. P problem called the DUAL of the original problem.

The original problem is the PRIMAL. For instant if we have the primal i.e. original problem

max Z = cx (objective)

s.t Ax ≤ b

x ≥ 0 (constraint) non negative

Therefore, our DUAL will be

Min Z = yb

s.t yA ≥ c

y ≥ 0

Questions on Primal and Dual (9):

An engineering firm engaged by management of Nigerian port Authority called NACO. To produce five types of products X_1, X_2, X_3, X_4,.X_5, by using two products processes (grindings and drilling). After deducting raw material cost, each unit of product yield the following profit.

Table 3.

	x_1	x_2	x_3	x_4	x_5
	500	600	350	400	200
Grindings	12	20	--	25	15
Drilling	10	8	16	-	-

In addition, the fuel assembly uses 20 hours i.e. labour. The factory has 3 grinding machines, 2 drilling machines and works 6 days a week with 2 shifting of 8 hours each day. 8 men are employed in the assembly each working hours shift day. Now the problem is find how the factory will maximized profit and minimized cost using primal and dual respectively.

Solution : From above we shall see that the total number of hour available for grinding machine is

$3 \times 6 \times 2 \times 8 = 288$ hours

And for Drilling Machine, we have

$2 \times 6 \times 8(2) = 192$ hours

And the number of hours available since he employed 8 men each working for 8 hours for 6 days gives

$8 \times 8 \times 6 = 394$ hours or $4 \times 2 \times 8 \times 6 = 394$ hours

Therefore, our primal problem would be the objective function as follows;

$$\max Z = 550_{x1} + 600_{x2} + 350_{x3} + 400_{x4} + 200_{x5}$$

s.t $12_{x1} + 20_{x2} + - + 25_{x4} + 15_{x5} \leq 288$

$10_{x1} + 8_{x2} + 16_{x3} + - \leq 192$

$20_{x1} + 20_{x2} + 20_{x3} + 20_{x4} + 20_{x5} \leq 394$

$x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5 \geq 0$

and in matrix form we have

Max (550, 600, 350, 400, 200)

s.t. $\begin{bmatrix} 12 & 20 & - & 25 & 15 \\ 10 & 8 & 16 & - & - \\ 20 & 20 & 20 & 20 & 20 \end{bmatrix} \leq \begin{bmatrix} 288 \\ 192 \\ 394 \end{bmatrix}$

Now suppose an accountant wants to value each of the available resources (grinding, drilling and labour) this constraint was derived by finding profit for contributions of each of the factors for each unit produced.

Thus he will attach to each factor what we call shadow prices. (The Dual).

$$\text{Min} = 288y_1 + 192y_2 + 384y_3 \qquad (y_1 y_2 y_3 \text{ are cost})$$

s.t $12y_1 + 10y_2 + 20y_3 \geq 550$

$20y_1 + 8y_2 + 20y_3 \geq 600$

$16y_2 + 20y_3 \geq 350$

$25y_1 + 20y_3 \geq 400$

$15y_1 + 20y3 \geq 200$

And its matrix form is

$$\min \quad (288, 192, 384)$$

s.t $\begin{bmatrix} 12 & 10 & 80 \\ 20 & 8 & 20 \\ - & 16 & 20 \\ 25 & - & 20 \\ 15 & - & 20 \end{bmatrix} \geq \begin{bmatrix} 550 \\ 600 \\ 350 \\ 400 \\ 200 \end{bmatrix}$

(Remember that dual of the Dual is the primal).

Question 9:

Change the Primal problem to Dual

$\max Z = 2x_1 + 5x_2$

s.t $x_1 + 2x \leq 4$

$4x1 + 3x2 \leq 12$

$x_1 \geq 0, y_2 \geq 0.$

Solution:

The Dual is:

Minimize $Z = 4y_1 + 12y_2$

s.t $y_1 + 4y_2 \geq 2$

$2y_1 + 3y_2 \geq 3$

$x_1 \geq 0, y_2 \geq 0.$

Question 10:

Graph the following systems of linear inequalities:

(a) $x \leq 3$

$Y \geq 0$

(B) $x\ y < 2$

$2x\ y < 3$

$x\ 4y < 5$

(c). $x > 0$

$y \geq 0, 2x + 3y > 5$

(d) $x > 3$

$y < 4$

$x-y = -5$ (provide solutions to them).

Question 11:

If a person must have at least 900 units of vitamins and 1000 units of calories per day, express each condition as a linear inequality and determine what could constitute an acceptable diet. Graph the inequalities. (Hints: Let X be the number of units of vitamins needed and Y the number of units of calories needed).
(Provide solution.)

Question 12:

To meet a budgetary limitation an executive can hire not more than five secretaries and at least seven sales men. Write a system of inequality expressing the permissible combinations he can hire. If he must have at least one secretary, indicate the domain and range of the relation involved. Graph the system of inequalities.
(Provide solution.)

Question 13:

Determine directly from fig. 3 which of the following combination of food stuff meet all requirements. Indicate, for those combinations which do not meet these requirements, which of the constraints are being violent; if the figure is not sufficiently clear for this purpose refer to the relevant equations:

(a) $x = 2, y = 1$
(b) $x = 2, y = 3$
(c) $x = 2, y = 5$
(d) $x = 10, y = 1$
(e) $x = 1, y = 3$
(F) $x = 3, y = 1$
(g) $x = 14, y = 0.5$ (Provide solution).

Question 14

A firm produces different models of a product, model 1 and model 2. Each model must complete one unit of model 1, the drilling machine must work 2hrs and the lathe must work 4hrs; to complete 1 unit of model 2, the drilling machine and the lathe must work 4 and 2hrs respectively. A strong union enforces the rule that no machine may be operated more than 12 hours per day. Graph the system of linear inequalities which shows the combination of the 2 models; which can be produced daily. (Provide solution)

Question 15

A manufacturer chemically processes bauxite ore into product X and Y. Each unit of product X contains unit of ingredient A and 1 unit of ingredient B. a costumer desire at least 9 units of ingredient A and at least 18 unit of ingredient B, and he does not care how many unit of product X and Y he receive in order to obtain the specified amounts of 2 ingredients. In order to process the 2 products, the manufacturer must bake product X for 4hrs and product Y for 1 hr. he has only 24 furnace hour available each day, graph the system of inequalities which show the combination of X and Y which will satisfy the customer and still not exceed the available furnace capacity. (Provide solution).

Question 16

A big of fertilizer requires at least 6 units of ingredients A and at least 8 units of ingredient B. This fertilizer can be manufactured by mixing product X and product Y, former product contain 2 units of ingredient A and 1 and 3 of ingredient B, while the later contain 1 unit of ingredient A and 3 unit of ingredient B. Graph the relevant system of inequality and shade the portion of the resulting figure which represent feasible combinations of X and Y. (Provide solution).

Question 17

A firm has 2 plants, A and B. Plant A has the capacity to produce 80 units daily, and plant B is able to produce as many as 400 units daily. The firm has 3 warehouses, W_1, W_2, and W_3, which must receive respectively, 300, 400 and 500 units daily. The cost per unit of transporting from the 2 plants to the 3

Table A:

Plant	Warehouse		
A	1	2	3
	#4	#5	#8
B	#6	#3	#5

Warehouse is given in table A. How much should each plant ship to each warehouse for total transportation cost to be minimum? What is the minimum cost? (Provide Solution).

Question 18

Alamtex group manufacture 2 types of set, half-upholstered and full upholstered. The contribution per unit to profit is N80:00 for half-upholster N90:00 for full upholstered. The amount of materials needed per product and maximum available materials are given below;

Table:

Product	Unit of Materials		
Half-upholstery	2	2	5
Full-upholsterey	1	4	5
Maximum Available	12	24	36

Use graphical method to determine how many of each should be produced by Alamex group to maximize his profit and determine the maximum profit. (Provide Solution).

Question 19

A gambler with N2:00, makes a series of N1:00 bet. His probability of winning N1:00 is 2/5. He either wins N1:00 or loses N1:00, he decided to quit the game as soon as he gains N4:00 or loses N2:00. Let X_n be the amount he is having at time n. Write down the one step transition matrix for this process.

Solution

Let X_0 = N2:00

i.e. the initial take off. And the sample space = (0, 2, 3, 4)

i.e. the set of possible values of X_n (Result).

Where $P_i\, j + 1 = 25$----------(1)

$P_i\, j - 1 = 3/5$ i.e. $j \neq 0$.

$P_{ij} = 1$

$j = 0, 4$ or 0 (elsewhere)

The state space: ONE STEP TRANSITION MATRIX of the mark ov chain is given below;

Table 4:

i \ j	0	1	2	3	4
0	(0,0)	(0, 1)	(0, 2)	(0, 3)	(0, 4)
1	(1, 0)	(1, 1)	(1, 2)	(1, 3)	(1, 4)
2	(2, 0)	(2, 1)	(2, 2)	(2, 3)	(2, 4)
3	(3, 0)	(3, 1)	(3, 2)	(3, 3)	(3, 4)
4	(4, 0)	(4, 1)	(4, 2)	(4, 3)	(4, 4)

i \ j	0	1	2	3	4
0	1	0	0	0	0
1	3/5	0	2/5	0	0
2	0	3/5	0	2/5	0
3	0	0	3/5	0	1
0	0	0	0	0	1

Question 20

In the month of January Y2K the companies A, B and C share equally in a local market from month to month. A retained 60% of his costumer gain 10% of B's costumers and gains 30% of C's costumers. The company B retains 70% of its own costumers, gain 15% of A's costumers and gains 15% of C's costumers. The company retains 50% of his its own costumer gains 30% of A's customer's and 20% of B's costumers. You are required to;

(i) Write down the transition matrix of the brand switches among the companies.

(ii) Determine the market share in March 2001.

(iii) Determine the market share for A, B and C, at a steady state Condition.

Solution:
The transition matrix of the brand switches among the companies at the stable state is given by;

	A	B	C
A	0.60	0.10	0.30
B	0.15	0.70	0.15
C	0.30	0.20	0.50

(Ii) To determine the market share in March Y2K 1.
1st, the transition matrix is;

$$\begin{pmatrix} 0.16 & 0.1 & 0.3 \\ 0.15 & 0.7 & 0.15 \\ 0.3 & 0.2 & 0.50 \end{pmatrix}$$

Since the three companies enjoy equal market share, their probability are;

$$\begin{pmatrix} 33^1/_3\% \\ 33^1/_3\% \\ 33^1/_2\% \end{pmatrix}$$

∴ Market share in January is;

$$\begin{pmatrix} 0.6 & 0.1 & 0.3 \\ 0.15 & 0.7 & 0.15 \\ 0.3 & 0.2 & 0.5 \end{pmatrix}\begin{pmatrix} 0.33 \\ 0.33 \\ 0.33 \end{pmatrix}\begin{pmatrix} 0.6 \times 0.33 + 0.1 \times 0.33 + 0.3 \times 0.33 \\ 0.15 \times 0.33 + 0.7 \times 0.33 + 0.15 \times 0.33 \\ 0.3 \times 0.33 + 0.2 \times 0.33 + 0.5 \times 0.33 \end{pmatrix}$$

$$\begin{pmatrix} 0.198 & +0.033 & +0.099 \\ 0.050 & +0.231 & +0.050 \\ 0.099 & +0.066 & +0.165 \end{pmatrix} = \begin{pmatrix} 0.33 \\ 0.33 \\ 0.33 \end{pmatrix} = \mathbf{A}$$

And market share in February is;
0.6

$$\begin{pmatrix} 0.6 & 0.1 & 0.3 \\ 0.15 & 0.7 & 0.15 \\ 0.3 & 0.2 & 0.5 \end{pmatrix} = \begin{pmatrix} A \end{pmatrix}$$

$$\begin{pmatrix} 0.6 & 0.1 & 0.3 \\ 0.15 & 0.7 & 0.15 \\ 0.3 & 0.2 & 0.5 \end{pmatrix}\begin{pmatrix} 0.33 \\ 0.33 \\ 0.33 \end{pmatrix} = \begin{pmatrix} 0.33 \\ 0.33 \\ 0.33 \end{pmatrix} = B$$

And in March is,

$$\begin{pmatrix} 0.6 & 0.1 & 0.3 \\ 0.15 & 0.7 & 0.15 \\ 0.3 & 0.2 & 0.5 \end{pmatrix} = \begin{pmatrix} B \end{pmatrix}$$

$$\begin{pmatrix} 0.6 & 0.1 & 0.3 \\ 0.15 & 0.7 & 0.15 \\ 0.3 & 0.2 & 0.5 \end{pmatrix}\begin{pmatrix} 0.33 \\ 0.33 \\ 0.33 \end{pmatrix} = \begin{pmatrix} 0.33 \\ 0.33 \\ 0.33 \end{pmatrix}$$

(iii) To determine the market share for A, B and C at steady state condition is;

$$\begin{pmatrix} 0.6 & 0.1 & 0.3 \\ 0.15 & 0.7 & 0.15 \\ 0.3 & 0.2 & 0.5 \end{pmatrix}\begin{pmatrix} A \\ B \\ C \end{pmatrix} = \begin{pmatrix} A \\ B \\ C \end{pmatrix}$$

From here you need to form three equations as follows;

$$\begin{aligned} 0.6A + 0.1B + 0.3C &= A \text{----------(1)} \\ 0.15B + 0.7B + 0.1C &= B \text{----------(2)} \\ 0.3A + 0.2B + 0.5C &= C \text{----------(3)} \end{aligned}$$

And solve for A, B and C.

Question 21
Assuming that Alamtex group of company produced two types of car products K(Toyota car) and another H(Honda Accord). Currently, the market for these products has 60% and 40%, share each respectively. In

the process some switches takes place, of those who bought K the previous week, 70% buy it again, while 30% switched to H of those bought H the previous week,80% buy it again while 20% switched to K. determine the share of K and H after two weeks and after a long period.

Solution

First the stable state for the switching is;

Table:

	K	H
K	0.7	0.3
H	0.2	0.8

Secondly, you are to determine the transition matrix P.

$$\text{i.e. } P = \begin{bmatrix} 0.7 & 0.3 \\ 0.2 & 0.8 \end{bmatrix}$$

Thirdly, determine the market share;

$$\text{i.e. } M = \begin{bmatrix} 60\% \\ 40\% \end{bmatrix} = \begin{bmatrix} 0.6 \\ 0.4 \end{bmatrix}$$

Fourthly, market after 1st week = P. M.

$$P.M = \begin{bmatrix} 0.7 & 0.3 \\ 0.2 & 0.8 \end{bmatrix} \begin{bmatrix} 0.6 \\ 0.4 \end{bmatrix} = \begin{bmatrix} 0.7 \times 0.6 + 0.3 \times 0.4 \\ 0.2 \times 0.6 + 0.8 \times 0.4 \end{bmatrix} = \begin{bmatrix} 0.54 \\ 0.44 \end{bmatrix}$$

To determine the next week i.e. the 2nd week we have;

$$\begin{bmatrix} 0.7 & 0.3 \\ 0.2 & 0.8 \end{bmatrix} \begin{bmatrix} 0.54 \\ 0.44 \end{bmatrix} = \begin{bmatrix} 0.51 \\ 0.46 \end{bmatrix}$$

for long period, the market share would be;

$$\begin{bmatrix} 0.7 & 0.3 \\ 0.2 & 0.8 \end{bmatrix} \begin{bmatrix} G \\ A \end{bmatrix} = \begin{bmatrix} G \\ A \end{bmatrix}$$

$$0.7G + 0.3 = G \text{ ---------- (1)}$$
$$0.2G + 0.8A = A \text{ ---------- (2)}$$

$$\text{i.e. } 7G + 3A = 10G \text{ ---------- (3)}$$
$$2G + 8A = 10A \text{ ---------- (4)}$$

Now solve equations (3) and (4) simultaneously and get G terms of A or A in term G.

Question 22

Two local media station A and B, are in competition for Southwest audience. A is within a week able to retain 80% of its audience and captures 40% of its audience and captures 20% of the audience of A. what proportion of total audience will eventually be for A and B respectively.

Solution:

First, the probability matrix is given as follows (for the southwest).

P =

	A	B
A	0.8	0.4
B	0.2	0.6

The market share is 50% each.

$\Rightarrow$ A = 50%

and B = 50% because they have equal share of winning.

Now the Vector Matrix is;

$$\begin{bmatrix} .8 & .4 \\ .2 & .6 \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} A \\ B \end{bmatrix}$$

But, $A + B = 1$

$B = 1 - A$

$$\begin{bmatrix} .8 & .4 \\ .2 & .6 \end{bmatrix} \begin{bmatrix} A \\ 1 - A \end{bmatrix} \begin{bmatrix} A \\ 1 - A \end{bmatrix}$$

Therefore, for A, A = 2/3. Verify this.
i.e. from above

$$0.8\ A + 4(1 - A) = A$$

such that $A = {}^{2}/_{3}$

Similarly for B

Since $A + B = 1$

and $A = 1 - B$

we have

$$\begin{pmatrix} 0.8 & 0.4 \\ 0.2 & 0.6 \end{pmatrix} \begin{pmatrix} 1 - B \\ B \end{pmatrix} \quad \begin{pmatrix} 1 - B \\ B \end{pmatrix}$$

$$-\ 0.2\ (1 - B) + .6B = B$$

$$B = 1/3$$

Question 24 a

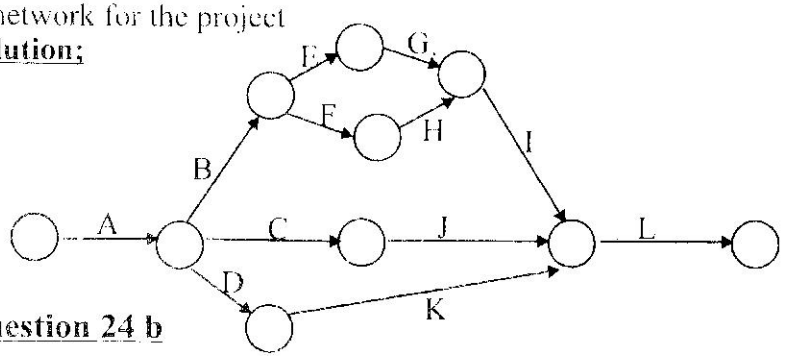
A project has the following characteristics

Activity	Preceding Activity
A	-
B, C, D	A
E and F	B
G	E
H	F
J	C
K	D
I	G and H
L	I, J and K

Draw the network for the project

Solution;

NI



Question 24 b

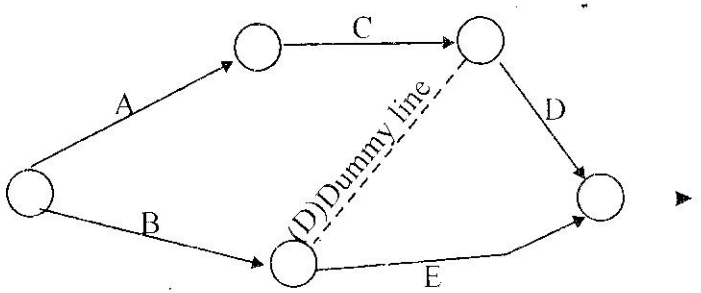
Consider the following activity for the excursion of a particular project.

Activity	Preceding Activity
A	-
B	-
C	A
D	B and C
E	B

Draw the network diagram for the project

Solution:

NII



Note:

The dummy activity is used solely to show that D must follow B as well because these activities should not be drawn between same events.

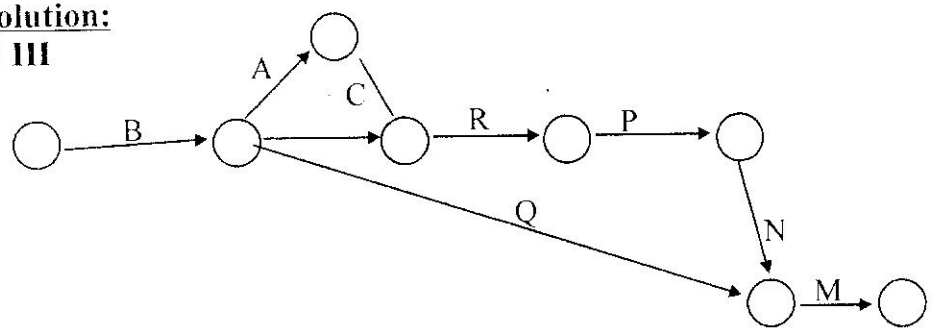
Question 25:

Draw the network diagram of the following project.

Activity	Preceding Activity
A	B
B	-
C	B
R	C,A
P	R
N	P
Q	B
M	Q,N

Solution:

N III



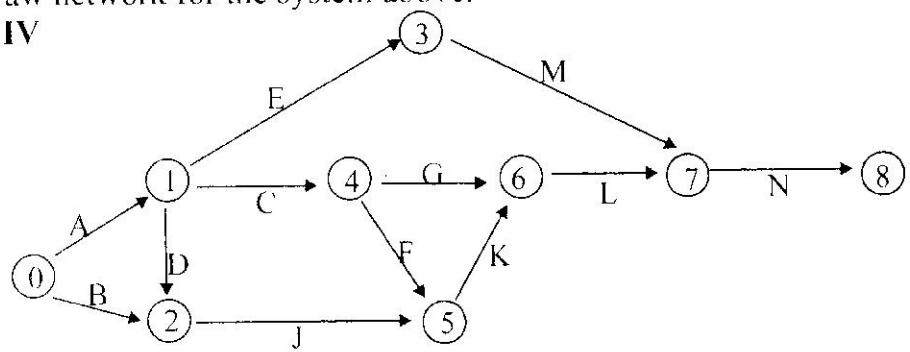
Question 26:

Project XYZ of Building a Boat is given as follows:

Activity	Preceding Activity	Activity Description
A	-	Design Hull
B	-	Prepare Boat Shed
C	A	Design mast and mount mast
D	A	Obtain Hull
E	A	Design Sails
F	C	Obtain mast mount
G	C	Obtain mast
H	C	Design Rigging
J	B, D	Prepare Hull
K	F, J	Fit mast mount to hull
L	E, H, G, K	Step mast
M	E H	Obtain Sails and Rigging
N	L, M	Fit Sails and Rigging

Draw network for the system above.

N IV



Question 27:

The management of a company has been asked to present a network diagram of the preparation of monthly budgetary control reports, in these report the actual cost of direct material and labour the mouth are compared with budgeted cost and an analysis of idle machine time during the month is also prepared. The individual activities required to prepared each set of monthly reports as follows.

Activity	Description of Activity	Duration
A	Complete the costing materials	3
B	Analysis material cost into direct and indirect cost	2
C	Complete the costing of labour	3
D	Analysis labour cost into direct and indirect cost	4
E	Prepare a list of machine stoppages	5
F	Analysis machine stoppages and breakdowns by cause	5
G	Establish the base for allocating direct wages and material cost of production	6
H	Divide the annual budget into monthly calendar periods	8
I	Prepare a summary of direct wages and material cost onto control sheets	4
J	Analysis Variances	2
K	Complete draft variance reports and idle time analysis Reports.	3
L	Type report	2
M	Photocopy and distribute report	1

You are required: to draw a network of the events and determine the routes through the network.

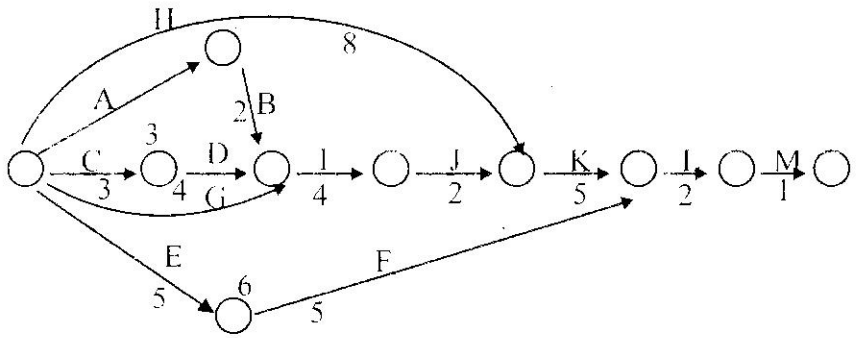
Solution:

In this case, activities A, C, E, G and H can all be started straight away because non is dependent on the completion of a previous activity. Below is the summary.

Activity	Preceding Activity
A	-
B	-
C	-
H	-
B	A
D	C
F	E
I	B, D & G
J	I and H
K	J and F
L	K
M	L

The Network Diagram is given below:

NV



Note:

(a) Duration time for each activity are shown against the activity (arrowed on the diagram).

(b) The length of each arrowed line does not in any represent the duration of the Activity.

(c) There are 5 routes through the network.

(i) A B E J K L M

(ii) C D I J K L M

(iii) G I J K L M

(iv) H J K L M

(v) E F K L M

Question 28:

Activity	Preceding Activity	Duration (weeks)
A	-	5
B	-	4
C	A	2
D	B	1
E	B	5
F	B	5
G	C, D	4
H	F	3

Require: What is the critical path and its duration? (Provide solution)

Question 29:

Calculate EST, LST, EFT, LFT and D where D = activity duration in a table Question 20. (Provide solution)

Question 31:

Below is a list of activities involved in the construction of a bridge in weeks

(i) Draw the network of the activities

(ii) Determine the critical path

(iii) Calculate the variance and the standard deviation

Activity	Duration	Intermediate predecessor
(1, 2)	4	- - -
(2, 4)	7	(1,2)
(2, 3)	8	(1, 2)
(2, 5)	6	(1, 2)
(2, 6)	15	(2, 3) (2, 4) (3, 4)
(3, 5)	9	(2,3) (2, 4)
(5, 6)	12	(2,3) (2, 5)
(6, 7)	8	(4,6) (5, 6)

(Provide solution)

Hint:

Use normal example and determine the EST, LST, EFT, LFT and the critical path.

Question 32:

Define the following term associated with network analysis

(i) Total float
(ii) free float and
(iii) Independent float

Solution:

(i) Total float for any activity is the amount of time by which its duration could be extended before it become critical. This is the same, of course, as the amount by which its duration could be extended without affecting the total project time. The effect on the time available for proceeding and subsequent activities is ignored, mathematically the total float is equal to the latest finished time minus the earliest starting time, minus the activities duration. That is total float TF = LFT EST D.

(ii) The free float of an activity is equal to the amount by which its duration can be increased without affecting either the total project time or the time available for subsequent activities. Preceding activities are ignored. Mathematically free float is equal to the earliest finishing time minus the earliest starting time minus the activities duration i.e. free float (FF) = EFT EST D.

(iii) The independent float is the amount the duration of activity could be extended without affecting the total project time, the time available for subsequent activities or the time available for preceding activities. The independent float is a sub-division of the free float. Mathematically, independent floats equal earliest finishing time minus latest starting time, minus activity of duration i.e. Independent float IF = EFT- LST - D.

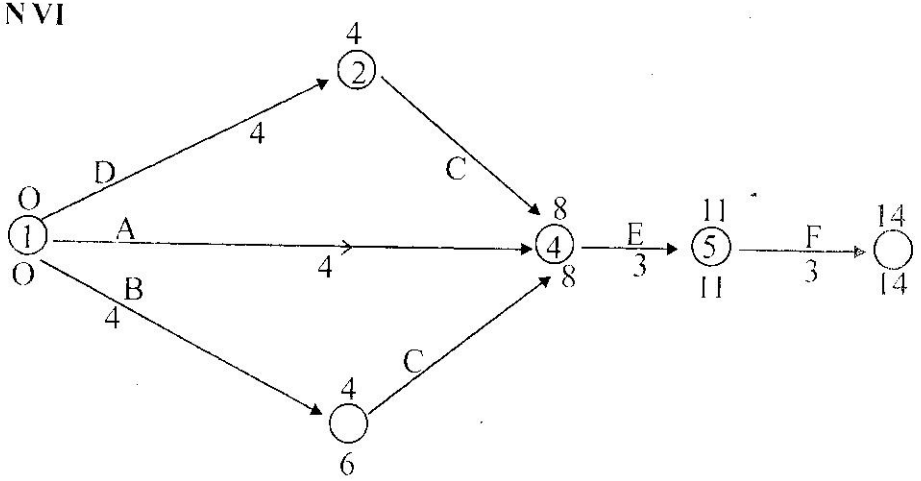
Question 33:

ALAMTEX GROUP is planning the annual shutdown of its furnace for maintenance purpose. The shutdown has 6 phases and the estimate normal time are as follows:

Phase	Preceding activity	Normal days taken
A	-	8
B	-	4
C	B	2
D	-	4
E	A, C, D	3
F	F	3

Required: Calculate the minimum time required for the shutdown and state the critical path. Calculate all the floats.

NVI



Solution:

The minimum time for shutdown is 14. There are 3 routes through the network They are

(i) D, Dummy, E, F
(ii) A, E, F
(iii) B, C, E F of these the critical route is AEF. The floats could have been calculated as follows:

Activities (i)	(ii)	EST (iii)	LST (iv)	EFT (v)	LFT (iv - I - v)	D	Total Float (iii) - (i) - (v))	Free float
A	0	0	8	8	0	0*	-	
B	0	0	4	6	4	2	0	-
C	4	6	8	8	2	2	2	
D	0	0	8+	8	4	4	4	
E	8	8	11	11	3	0*	-	
F	11	11	14	14	3	0*	-	

*Critical activity

+ for activity D; the subsequent dummy activity should be treated as an extension of activity and the EFT of D day 8, not day 4.

Question 34

A project consist of the following activities and time estimate. Time estimate (weeks).

Predecessor Events	Successor Events	Time to	Estimate tm	tp	Weeks
1	2	4	8		
12	3	8	11		
12					
1	4	8	14		
24					
2	5	5	9		
10					
3	+	2	6		
8					
3	5	2	4		
8					
4	5	6	10		
14					
5	6	1	3		

6. You are required to:

(1) Construct the operational network.
(2) Locate the critical path
(3) Explain the importance of critical path analysis
(4) Calculate the mean and S. D for the critical path
(5) From the distribution what is the probabilities of completing the project in more than 3 weeks (31 days).

(Provide solution).

Note that important of PERT is its ability to handle uncertain i.e. events that are probability in nature.

Question 35 a

Explain the following in detail with example.

(i) Transition matrix
(ii) Stochastic process
(iii) State space

(iv) Independent Increment (both stationary and homogeneous).
(v) Weiner process
(vi) Markorv process
(vii) Markorv chain

(Provide solution)

Question 35 b

A developer is to develop and market a new fertilizer, which is to be mixture of two ingredients X and Y: The properties of the two ingredients are

	CALCIUM	NITROGEN	LIME	PHOSPATE	COST /Kg
Ingredient X	20%	30%	40%	10%	#12
Ingredient Y	40%	10%	45%	5%	#8

Has been decided that:

(i) The fertilizer will be sold in bags containing a minimum of 100kg
(ii) It must contain at least 15% nitrogen
(iii) It must contain at least 8% phosphates
(iv) It must contain at least 25% calcium.

You are require to determine the amount of ingredient X and Y the manufacturer should produced in order to meet the above requirement at the minimum cost.

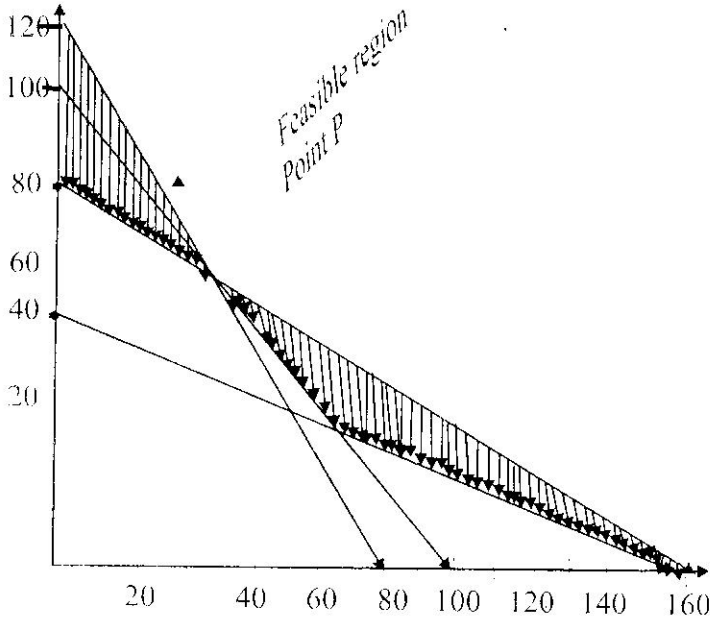
Solution

Let X_1 = Kg of ingredient X
X_2 = kg of ingredient Y
Thus the LP model
The objective is to minimize

$12X_1 + 8x_2$ (cost)

Subject to the following constraints

$X_1 + x_2 \geq 100$ (Weight Constraints)
$3x_1 + 0.1x_2 \geq 15$ (Nitrogen Constraints)
$0.1x_1 + 0.54x_2 \geq 8$ (Phosphate Constraints)
$0.2x_1 + 0.4x_2 \geq 25$ (Calcium Constraints)
$x_1, x_2 \geq 0$ (Non Negativity)



Question 36: ABC Company installed new equipment at an average rate of three per hour.
Customers needing this service arrived at the company at the average of two per hour, the company studies the queuing models and feels that all condition for single channel mode are met. Calculate the numerical values of the operating characteristics.

SOLUTION:

Let = 2, Cars arriving per hour
μ = 3, Cars served per hour
P = /μ = 2/3 (Probability that the server is busy)

(1) p_0 = 1 p 1- /, = 1/3 = 0.33 (probability that are 0 cares in the system)
(2) p / 1 - p = $^2/_3 / _1 - ^2/_3 / ^1/_3$ = 2 (carsine the system on average)
(3) $^1/(1 -)\mu$ = $^1/(1\ 2/3)$ = 1hr (hours that an average can spent in the system)

4 $\frac{p}{(1-p)u} = \frac{^2/_3}{(1 - ^2/_3)^3} = {}^2/_3 = 40$ mins

5 $\frac{p}{1-p} = \frac{(^2/_3)^2}{1 - ^2/_3} = \frac{^4/_9}{^1/_3} = \frac{4}{3} = 1.33$

(Car waiting in line on average).

6. $\frac{1}{1-p} = \frac{1}{1 - ^2/_3} = \frac{1}{^1/_3} = 3$

(Car waiting in line when there is queue).

Question 37:
At a tool service center the arrival is two hour and the service potential is three per hour. The hourly wage paid to the attendant at the service center is #150 per hour and the hourly cost of a machinist away from work is #400. you are required to:

(a) Calculate the average number of machinist append waiting or waiting to be served at any given time.
(b) Calculate the average time a machinist being served for service.
(c) Calculate the cost of the system if there were two attendants working together as a team each paid #150:00 per hour to serve an average of two customer per hour.

Solution:

(a) Arrival rate is = 2. since rate is μ = 3 then = /μ = $^2/_3$
Average number of machine in the system in the system is
$\frac{p}{(1-p)u} = \frac{^2/_3}{1-1\ ^2/_3} = \frac{^2/_3}{^1/_3} = 2$

$\frac{p}{(1-p)u} = \frac{p}{1-p} = 1$

(b) average time spent in the queue is
= 2 x $^1/_3$ = $^2/_3$h

(c) Attendant wage for an 8 hours day is #(8 x 250) = #1,200. on average there are 2 machinist in the system through out the day so, 2 x 8 = total cost of waste of time is 400 x 16 = # 6,400. hence that total cost of

operating this for an 8 hours day is =N=1,200 + #6,400 = #7,600.

(d) There is a single Channel system since the attendant work as a team. The Effective service rate is 4 per hr.

Thus; $\lambda = 2$ $\mu = 4$

$\Longrightarrow$

$p = \lambda / \mu = 2/4 = \frac{1}{2}$

$$p = \frac{\frac{1}{2}}{1 - \frac{1}{2}} = \frac{\frac{1}{2}}{\frac{1}{2}} = 1$$

The average Number in the system is $= \frac{1-p}{}$

Now wages for an 8 hours day are =N= (2*8*150) = #2400. each cost #400 so the value of lost production = =N= 400* 8= #3,200. so total cost of operating the system is #2,400+#3,200 = #5,600.

Question 38:

AT ALAMTEX SOAP FACTORY, material arrives on the goods in games section of the factory at an average rate of five loads per hour. The material is handled by a folk lift which has an average load of seven loads per hour. Management requires to know:

(a) The average number of loads at the section waiting to be moved when there is a queue.

(b) The average length of times an arriving load spent waiting for service.

(c) What is the average service rate must be in order to reduce the expected Waiting time to 20 minutes for a load.

(a) $\lambda = 5$ $\mu = 7$ $\Longrightarrow$ $p = \lambda/\mu = 5/7$

Solution:

Average number in the queue when there is a queue is

$$\frac{1}{1-p} = \frac{1}{1 - \frac{5}{9}} = \frac{1}{\frac{2}{7}} = 3.5$$

That is, the average number of loads to moved when there is a queue = 3.4

(a) Average time in queue

$$\frac{p}{(1-p)\mu} = \frac{\frac{5}{7}}{(1 - \frac{5}{7})\,7} = \frac{\frac{5}{14}}{14} = 21.4$$

That is, the average time an arriving loads waiting for service is 21.4

minutes

(c) Let the average service rate needed in order to reduce the waiting time To 20 minutes (i.e.1/3) be x. then we have; $\frac{p}{(1-p)\mu} = \frac{\frac{5}{3}}{(1 - \frac{5}{3})x}$

$$x(x - 5) = \frac{5}{\frac{1}{3}}$$

i.e. $x^2 - 5x\ 15 = 0$

so $x = \frac{5 \pm \sqrt{25 + 60}}{2} = 5 \pm 9.2195$

The negative value is meaningless in this context so x = 14. 2195/2 = 7.11 That is, the service rate must be increased to 7.11 per hour.

Question 39:

In three channel system, the rate of service at each channel is five costumers per hour, and customers must arrive at rate of 12per hour. What is the probability that;

i. There are no customers in the system at a given time?

Ii. That there two costumer in the system.

Solution

C = 3, $\lambda = 12$, $\mu = 5$

$P = \lambda/\mu c = 12/5 \times 3 = 4/5 = 0.8$

$Po = 3!\,(1 - 0.8) / (0.8.\ X\ 3)^3 \quad +3!\,(1 - 0.8)x$

$$\text{Where } X = \left[\sum_{n=0}^{c=1} 1/n!\,(pc)^n\right] = \sum_{n=0}^{2} 1/n!\,(pc)^n$$

$= 1/0!\,(0.8 \times 3)^0 + 1/1!\,(0.8 \times 3)^2$

$= 2 + 2.4 + 22.88 = 6.28$

$po = 6(0.2) / (2.4)^3 + 6(0.2)(6.28) = 0.056$

Multi-channel Queue Formulae

(a) Probability of n costumers in the system is if $n < -1$

$$P_n = p^n C^{\infty} \qquad P_0 \text{ if } n > C$$
$$C!$$

(b) Average number of customers in the system is

$$p(pc)^c \qquad P^0 + pc$$
$$C!\,(1-p)^2$$

(c) Average number of customer in the queue is $PUEx/C!\,(i-p)^2 \times P$

(Including time when there is no queue)

(d) Average number of customers in the queue is

1/ (1-p) (average only overtimes when there is a queue).

(e) Average times a customers spends in the system is

$$\frac{P(PC)}{C!\,(1-p)\ c.\mu} \quad P_0$$

(f) Average times a costumer spend in the system is

$$\frac{P\,(pc)}{C!\,(1-P)\ c.\mu} \quad P_0$$

Question 40:

In the three-channel system, given in the question 40, the rate of service at each channel is 5.

Customers per hour, and customers arrive at the of 12 per hour. You are required to calculate the item (a) - (f) given above.

Solution:

(A) $C = 3$, $\lambda = 12$, $\mu = 5$

$\therefore P = \lambda / C\mu = 12/3 \times 5 = 4/5 = 0.8$

= Probability that no customer is the system

$$Po = \frac{3!(1-0.8)}{(0.8 \times 3)3 + 3!(1-0.8)y}$$

Where $y = 1/0!\,(0.5 \times 3) + 1/1!\,(0.8 \times 3) + \frac{1}{2}!\,(0.8 \times 3)2 = 6.28$

$= 1 + 2.4 + 2.88 = 6.28$

$Po = 6.28 \times 0.2 = 0.056.$

(b)(i) Probability of two customers in the system is

$$P2 = \frac{(pc\ p0/n!)^2}{(0.8 \times 3)} \qquad \text{Since } 2 = C - 2 = 3 - 1$$
$$P2 = \frac{}{2!}$$

(ii) Probability of 5 customer in the system is

$$P5 = \frac{(pc)^2 po}{n^{5}} \text{ Since } 5 > C = 3 \text{ (because maximum is 3. } \therefore \text{ for use 3!}$$

$$P5 = \frac{(0.8)5 \times 3}{3!} = 0.056 \text{ (complete the solution)}$$

(c) Average number of customer in the system is

$$\left(\frac{P(p)C}{C!\,(1-0.8)}{}^{3} Po\right) + pc$$

$$\frac{3!\,(0.8 \times 3)^2}{3!(1-0.8)} = 0.056 \times 0.8(3)$$

(complete the solution)

(d) Average number of customer in the queue (including times when here is no queue) is

$$P(pe)C_2^3 \qquad Pc = \frac{0.8\,(0.8 \times 3)^2}{3!(1-0.8)} = 0.056 \text{(complete the solution)}$$
$$C!\,(1-p)$$

(e) Average time a customer spends in the queue (only over times when there is a queue) is

$$1/(1-p) = 1/(1-0.8) = 5$$

(f) Average time a customer spends in the system is

$$\frac{P(pc)^2}{C!(1-p)\ c.u_3} Po + 1/u$$

$$\frac{(0.8 \times 3)^2}{3!(1-0.8)} = 0.056 + 1/5$$
$$(3)(5)$$

(g) Average time a customer spend in the queue is

$$\frac{(Pc)C}{Po + 1/u}$$

$$C!(1-p)^2 \, c.u^{3}$$

$$= \frac{(0.8 \times 3)^{2}}{3!(1-0.8)} = 0.056$$

$$\frac{}{(3)(5)}$$

(complete the solution)

Question 41:

(a) Describe with the aid of realistic example the basic features of a queuing problem. Your answer should indicate what information we need for analyzing (i) The impute process (ii) The service mechanism (iii).The queue discipline. You should also comment on the major assumption implicit in the queuing mode.

(b) Discuss in general the major limitation of queuing theory and indicate with ways in which these limitations might be overcome. (Provide solution)

Question 42:

The proprietor of shop "n" shop supermarket with one cashier is desirous of minimizing delays to customer and efficiently utilizing the service facilities. You have ascertained that customer arrive randomly in time at supermarket at a mean rate of 51per half hour that the service time of the cashier is exponentially distributed with a mean rate of 60 per hour. You are required to determine the following.

(a) Mean queue length excluding customer being served

(b) Mean queue time

(c) Probability that the whole queue length exceed the capacity of say eight people

(d)Mean waiting time queue time + service time.

(Provide solution).

Question 43:

At a quarry a crane is used to load tippers. It has been reliable ascertain than the tipper arrive at random for loading at an average rate of live tippers per quarter of an hour and that tippers are loaded per quarter of an hour. You are required to calculate:

(I) The crane utilization

(ii). Mean waiting time per tipper

(iii) Mean number of tipper waiting to be loaded

(iv) Mean queue length including the tipper loaded

(v) Mean queuing time.

(Provide solution)

Question 44:

Average arrival rate 5 per hour while service at the SOKOTI bank is 7 per hour. Assuming a random distribution for arrival times and service time calculate;

(i). Mean queue length excluding customer being served

(ii) Mean queuing time

(iii) Mean waiting time (queuing time + service time)

(Provide solution)

Question 45:

The woman's department of large stores employees one tailor for customer fittings. The number of customers requiring fittings appears to follow a position distribution with mean arrival rate 24 per hour Customer are fitted on a first-come first served basis, and they are always willing to wait for free. The time it takes to free a customer appear to be exponentially distributed with a mean two minutes.

(A) What is the average time should a customer expect to spent in the fitting room?

(b) How much time should a customer expect to spent in the fitting room?

(c) What percentage of time is the tailor idle?

(d) What is the probability that a customer will wait more that ten minutes for the tailor service?

(e) Determine the average wait for the tailor's service experienced by these customers who have wait at all.

(Provide solution)

Question 46:

A fast food restaurant is operated by one person, the owner. The arrival pattern of customer on Monday appears to follow a Poison distribution, with a mean arrival of people per hour customers are served on a FIFO basis, and because of the reputation of the restaurant. They are willing to wait for service once they arrive. The time it takes to serve customer is estimated to be exponentially distributed, with an average service time of 4 minutes. You are required to determine:

(a) The probability that there is a queue
(b) The average size of the queue
(c) The expected time that a customer must wait in the queue, and
(d) The probability that a customer will spent less than 12 minute in the restaurant. (Provide solution)

Question 47:

KAT hire Ltd is a haulage Company that owns 100 trucks. It is estimated that on average of 8 trucks break down everyday and that this follows normal distribution. Everyday that a truck is out of action, by being under repairs or waiting in the queue to be repaired, the company loses #80 in earring.

The company owns workshop for servicing of it vehicles. The workshop/shop car handle on average 10 vehicle per day and this follows an exponential distribution. The cost of operating the workshop is #150 per day. The alternative is for the company to close the workshop. This saving #150 daily cost and contract out the repairs to an outside Mechanic Rash international. The outside mechanic will charge #220 per day and has capacity to repairs 12 trucks and this also follows and exponential distribution. Should KAT HIRE Ltd continue to run its workshop or should it contract out its trailers?
Estimate the dialog saving of the alternative selected. (Provide solution)

Question 48

A filling station has unfortunately been reduced to one petrol pump owing to a mechanical breakdown. Normally 40 cars an hour stop petrol and on average drivers serve themselves in one minute. The owner is afraid that people will not wait and ii considering employing a temporary attendant who would reduce average service time to 45 seconds. The station owner obtains and average profit contribution of #10 per car. You are required to:
(a) State the usual assumption of simple queue
(b) State the implied value of driver's time if the cost employing the attendant is #50 per day. (Provide solution)

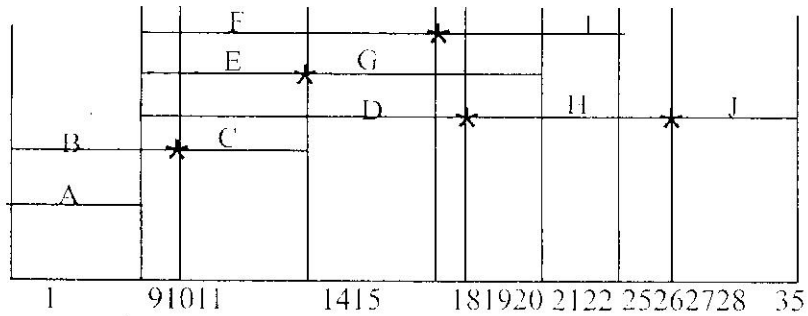
Question 49

A project to erect an exhibition stand at the Kaduna International trade fair required the following activities and men.

Activity	Preceding Activity	Duration	Men Required
A		9	7
B	-	10	3
C	B	4	6
D	A	10	5
E	A	5	7
F	A	9	6
G	E	7	6
H	D	8	5
I	C, F	7	7
J	G, H, I	8	8

Determine the minimum number of men required to complete the network in the shortest possible overall time? Use a bar chat in your solution

Solution The figure Grant Chart showing the combination of activities in progress throughout

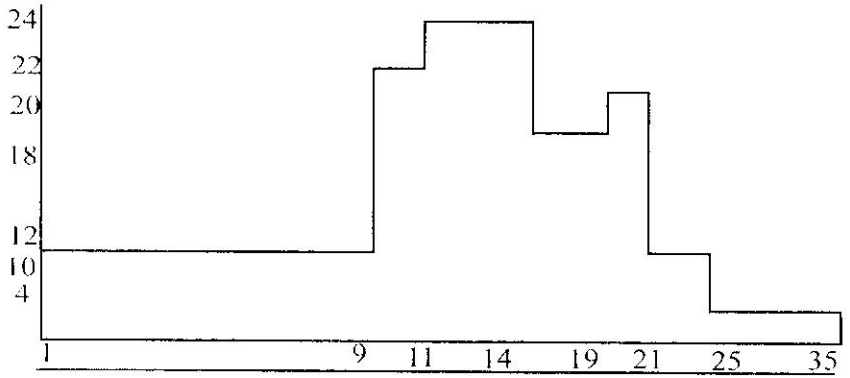


Using the bar chart and the information given on manpower needs of each activity, we can draw up a table of total manpower needs on each day of the 35 days of the project.

Days	Activity in progress	No of men needed
1 9	A, B	1
10	B, D, E, F	2
11 - 14	C, D, E, F	24
15 - 18	D, F, G	17

19	D, I, G	18
20 21	G, H, I	18
22 25	H, I	12
26 27	H	5
28 35	J	8

The bar chart showing manpower requirement appears below



We see that maximum requirement is for 24 men on day 11 14. By examination of the bar chart above we see that this peak could be reduced if the requirement that every activity start as early as possible is relaxed.

Question 50

1. Briefly describe four objective which critical path analysis scheduling may possible achieve
2. A project has been following characteristics

Activity	Preceding Activity	Duration normal	Week (crash)
A	-	2	
B	-	8	5
C	-	6	5
D	A	8	7
E	B	6	3
F	B	4	3
G	C	3	
H	D & G	12	8
I	F & G	8	

(a) Construct the network for the project under
  (i) Normal time
  (ii) Crash time
(b) What is the normal time required for the project?
(c) Can the project be complete in18 days?

3. The following activity dependency table concerns the overhaul of a pies of equipment located in one of the station in the swamp area of the country.

Activity	Description	Preceding Activity	Duration of work in month
P	Move tool & material to side	-	4
Q	Erect scaffolding	P	3
R	Remove piping, valving and hoses	Q	
S	Fabricate piping	P	5
T	Check bearing	R	2
U	Fit cylinder wall liner	R	5
V	Fit hoses	R	4
W	Replace piping	R, S	8
X	Replace worm bearing	T	1
	Replace valuing	U, X	2
Z	Weld and insulate	V,W, Y	4

Form the above, draw a carefully annotated network diagram and calculate the critical path.

(Provide solution)

Question 51:

The "PARAPO" community association is preparing the program for its 10th year anniversary and has identified the following activities:

	Activity No	Prior Activity	Duration Days
Determine celebrating day	A	-	12
Agree with speakers	B	A	36
Secure Advertising material	C	A	33
Send out notice	D	-	60
Produce list of financial Member	E	D	18

Print celebration programme	F	B, C, E	21
Print membership list	G	F	30
Obtain member's addresses	H	E	15
Dispatch programme	I	G, H	12

(i) By drawing a critical path diagram establish the critical event
(ii) Prepare a bar chart and establish the total float for each event
(iii) How many staff will be required if a staff is not expected to work on more than one activity at the same time. (Provide solution).

Question 52:
You are give the following information concerning a project which consist of eight activities 1 to 8

Activity	Preceding	Duration (Days)
1	-	4
2	-	5
3	1	2
4	1	3
5	2, 3	3
6	2, 3	4
7	4, 5	5
8	6	2

Required:
(a) Draw network for the project and determine the activities that lie on the critical path and also its length
(b) If activity 6 has to precede activity 7, will the critical path change? If it does change, draw your new network and compute the length of the new critical path.
(c) Assume that it has been found that certain activities in the original network can be shortened by hiring extra resources at the following costs.

Activity	2	4	5	6	7	8
Shortened duration (days)	3	2	2	3	3	1
Extra cost (#)	200	150	50	200	250	300

On the assumption that the extra cost are linear determine the possible expedited times and their association cost.

Question 53:
Consider the L. P model below
maximize $9x + 2y$ (Contribution)

Subject to

$4x + 2y \leq 80$ (Machine hours)
$2x + 6y \leq 120$ (Drilling hours)
$x, y \geq 0$ (Non - negativity)

where x = number of unity of casting x
y = number of unity of casting y

Therefore you are required to use simplex method to determine how many of each x and y should be made to maximum contribution.

Solution:
The first step in the simple method is to make the constraints into equation by introducing into each one a slack variable. The represent the amount of the resource left unused. For example we have

$4x + 2y + 3_1 = 80$
$2x + 5y + s_2 = 120$
Maximize $C = 9x + 12y = 0$
The first initial table

	Solution Mixture		Slack Variable		
Basic	X	Y	S1	S2	Solution
S1	4	2	1	0	80
S2	2	5	0	1	120
P	-9	-12	0	0	0

1. Select the highest negative number in the P row (i.e. the row are presenting the objective function) i.e. 12 under y.
2. Divide the positive number in the y-column A into the solution quantity column
   i.e. 8/2 = 40
   120/5 = 24
3. Select the row that gives the lowest answer (in this case the row

indemnified s_2). Ring the element, which appears in both the identified column (y)and the identified row (s_2) this element is know as the pivot element, thus, initial tableau reproduced as follows;

Row Number	Solution Variable	Products X	Y	Slacks Variable S1	S2	Solution Quantity
R1	S1	4	2	1	0	80
R2	S2	2	5	0	1	120
R3	P	9	12	0	0	0

Row Number	Solution Variable	Products X	Y	Slack Variable S1	S2	Solution Quantity
R4 -R1-4R5	S1	16/5	0	1	-2/5	32
R5 1/5R3	Y	2/5	1	0	1/5	24
R6 R3+ 12R5s	P	-21/5	0	0	12/5	238

The only negative is in column X.
32/16/5 10 and 24/2/5 60
Select the row minimum of these value i.e Row S_1
Hence, the pivot element 16/5
Row Number

Row Number	Solution Variable	Product X	Y	Slacks Variable S1	S2	Solution Quantity
R7–5/10R4	X	1	0	5/16	-1/8	10
R8 R5 -2/3R7	Y	0	1	-1/8	¼	20
R9=R6+ 21/5R7	P	0	0	21/6	15/8	280

The contribution will be maximize at #280:00 if 10 units of X and 20 unity of Y.

Question 54:
A company manufacture three products, tanks, trays, and tubes, each of which pass through three process, X, Y and T, Give below, the time required each products in each process and, for a certain production period the total process time available. The contribution to profit of each product are #2:00, #3:00 and #4:00 per unity respectively.
Process hour per unity:

Process	Tanks	Trays	Tube	Total Process Hour Available
X	5	2	4	12,000
Y	4	5	6	24 ,000
Z	3	5	4	18,000

You are required to calculate:
(a) How many of each product should be produced to make maximum profit, and state the profit figure?
(b) State how much slack time, if any is available in the process;
(c) Interpret the shadow prices.

Solution:
Let X, be the number of units of tanks produced.

X_2 be the number of units of trays produced.

X_3 be the number of units of tube produced.

The objectives is to maximum $2x_1 + 3x_2 = 4x_3$ (contribution) Subject to the following constraints;

$5x_1 + 2x_2 + 4x_3 \leq 12{,}000$ (process x hours)
$4x_1 + 5x_2 + 6x_3 \leq 24{,}000$ (process y hours)
$3x_1 + 5x_2 + 4x_3 \leq 18{,}000$ (process z hours)
$x_1, x_2, x_3 \geq 0$

We shall introduce slack variable s1, s2 and s3 to represent number of unused nourishing in processing x, y and z respectively, so that

$Z = 2x_1\ 3x_2\ 4x_3 = 0$
$5x_1 + 2x_2 + 4x_3 + S_1 = 12{,}000$
$4x_1 + 5x_2 + 6x_3 + S_2 = 24{,}000$

$3x_1 + 5x_2 + 4x_3 + S_3 = 18{,}000$

Our initial tableau is

Row No	Variable Solution	X1	X2	X3	S1	S2	S3	Solution
R1	S1	5	2	4	1	0	0	12,000
R2	S2	4	5	6	0	1	0	24,000
R3	S3	3	5	4	0	0	1	18,000
R4	Z	-2	-3	-4	0	0	0	0

Row Operation

R5= 1/44R1	X5	21/16	½	1	¼	0	0	3000
R6= R2 - 6R3	S2		2	0		1	0	5000
R7= R3 - 4R5	S3	-2	3	0	-1	0	1	6000
R3= R4+ 4R5	Z	3	-1	0	1	0	0	12000

Row No	Variable in solution	X1	X2	X3	S1	S2	S3	Solutio
R9= R5- ½ R11	X1	10	0	1	s	0	-1/6	2,000
R10= R6 -2R11	S2	-16/6	0	0	-5/6	0	-2/3	2,000
R11= ½ R1	X2	-2/3	1	0	-1/3	0	1/3	2,000
R12= R8+11	Z	7/3	0	0	2/3	0	1/3	14,000

Since every shadow price in the solution row is positive, an optimal solutio has been found.

(a) Contribution is maximized at #14,000 by making 2,000 units of tube #2,000 units of trays.

(b) There will be 2000 slacks hours in process y. process X and process Y will be fully utilized.

(c) (i) The shadow price of process X time is #1/3 per hour, which means that for every excess hour of process X time made available contribution would be increased by $\#^1/_3$.

(i) The shadow price of time is $\#^7/_3$ per hour. This may be interpreted.

(ii) The shadow price of tanks is #7/3. This means that for every unit of tanks made, contribution would fall by #7/3.

This means that for every units of tanks made contribution would fall by #7/3

Question 55:

In relation to linear programme, explain the implication of the following assumption of the model

(i) Linear of the objective functions and constraints
(ii) Continuous variables
(iii) Certainty.

(b) Sak producer limited, manufactures two types of paints-sak emulsion and sak-gloss. "Emulsion" value at #295.25 per gallon needs 5kg of raw material and nine hours of machine time. Gloss valued at #325.05 per gallon needs 6kg of the same materials and 12 hours of machine time. Establish the maximum value of each of the product that can be made from 400 hours of machine time and 500kg of raw materials.

Question 56:

A-Z oil company manufacture tow brands of lubricants namely X and Y. X valued at #150 needs 15kg of raw materials and nine hours of machine time. Lubricant Y also valued at #150:00 needs 10kg of the same raw materials and 12 hours of machine time. Establish the maximum valued of products that can be made from 360 hours of machine time and 375kg of raw materials and the respective quantity of lubricant X and Y.

Question 57:

A caterer has 1600gm, 1100gm and 1500gm of yams, fish and maize respectively she require 100gms of fish, 110gms of meat and 200gms of yams to prepare a plate of pounded yam. To prepare a plate of porridge costs #30:00 and prepare a plate of pounded yam for #50:00. how many plates of each she would prepare to maximize her sales?

Question 58:

T. farms limited can buy two types of fertilizer which contains the following percentage of chemicals.

	Nitrites	Phosphates	Potash
TYPE A	18	5	2
TYPE B	3	2	5

For a certain crop the following minimum quantities kg are required:
Nitrates 100, Phosphate 50, Potash 40.
Type A cost #100:00 per kg and Type and wishes to maximized its expenditure on fertilizers. You are require to:

(a) Write down the objective and the constraints for T-farm limited
(b) Draw a graph to illustrate all the constraints (equations/ inequalities) shading the feasible regions
(c) Recommend the quantity of each types of fertilizer which should be bought and the cost of these amount
(d) Find the saving T-farm limited can make by switching from its current policy to your recommendation
(e) State briefly any limitations of using this approach to problem solving in practice.

Work Sheet

TRANSPORTATION PROBLEM

This is a special case of linear programming (L.P.) problem. An homogenous product is available in known question each of in origins (source). There are n destinations each requiring a known quantity of this product. There is a cost of transporting one unit of product from an origin to a destination and this is known. We wish to determine the transportation schedule, which minimize the total transportation cost of shipment.

Let a_i be the quantity of the product available
And b_j be the quantity of the product available
a_{ij} be the cost of transporting a unit of the product from origin i to destination j.
if x_{ij} is the amount of quantity transported from of origin to destination j

As an LP, $\text{Min } Z = \sum\sum c_{ij} x_{ij}$
Such that $\sum x_{ij} = a_i$ where $i = 1,2,\ldots m\ a_i > 0$ origin)
$\sum x_{ij} = b_j$ where $j = 1, 2 \ldots . n\ b_j > 0$ destination) (1)

Equation 1 gives us m + n equation in mn unknowns. We will assume that

$\sum a_i = \sum b_j$ Feasible solution.

If more are available that is required $\sum a_i > \sum b_j$ we invert an (n + 1) destination to receive the supplies ($\sum a_i - \sum b_j$) at zero cost $C_i m + 1 = 0$ for all I.

If the amount required is more than the amount available ($\sum a_i < \sum b_i$) must inver a dimming $(m + 1)^{th}$ origin of it Serial out the Shack.

$\sum_{bi} - \sum_{ai}$ at zero cost $C_{m+1}, j = 0$, for all I

Let us look at a case of 2 origin and the definition

$\text{Min } Z = \sum\sum C_{ij} x_{ij}$
$\sum x_{ij} = a_i \quad i = 1, 2 \quad \sum x_{ij} = b_j \quad j = 1234$
$\text{Min } Z\ c_{11}x_{11} + c_{12}x_{12} + c_{13}x_{13} + c_{14}x_{14} + c_{21}x_{21} + c_{22}x_{22} + c_{23}x_{23} + c_{24}x_{24}$
$S.T = x_{11} + x_{12} + x_{13} + x_{14} = a_1$
$x_{11} +$
$x_{21} +$

UNIMODULAR L.P

The a matrix has a simple form. Each column has only two none zero at entries a one from the origin row and one from the destination row.

A is a **unimodular** if the determinant of any square submatrix of A taken one of the 3 value 0, +1, -1.
Each column of A can be written as $P_{ij} = e_i + e_m j$ where e_i is the i^{th} unit vector.

Let $S^i = i^{th}$ row of origin
$d^i = j^{th}$ row of destination

$\sum S^i - \sum d^i = 0$, i.e. every row can be express as a binary combination of the remaining m+n-1 row.

$\Rightarrow$ m+n rows are linear dependant and r(A) = m+n-1

REMARK : Unimodular property: Every minor of A matrix TP can only take the value of $^{\pm}$ 0.

PROOF: Let Ak be a i^{th} order Submatrix formed from any K different rows and k different columns of A. we are to show that
$A = 0$. thus we have 3 cases:

Case I There is at least a column or a row containing all zero $|A_k| = 0$

Case II when each column contains z ones
$\sum Si - \sum di = 0$ linear dependence: $|Ak| = 0$

Case III if at least a column of A_k with only the element (e_i)

Expand $|A_k|$ by that column $= {}^{\pm}|A|$. then the A_{k-1} is a minor
having property I or II and III. Continue to solve until you reduce $|A_k| = 0$ or a x matrix which is 1, $|A_k| = 0$ or -1 or +1

EXERCISE Check with 2x4 IP matrix

N/B:
(1) A necessary and sufficient condition for (1) to have a solution

is for (2) to hold

(2) if a_i and b_j are integers, any optimal solution of T. P will be integer

(3) A directed graph is a set of value vertices and area, which are directed from one vertex of the other

Origin	Destruction
(1)	(i)
	(ii)
	(iii)
(2)	(iv)

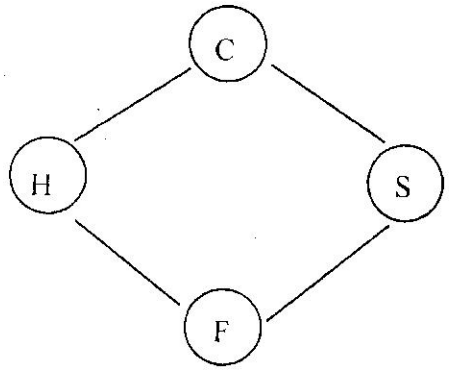
Transportation problem is a special case of directed graph called BIPARTITE graph since all vertices fall into 2 groups and this exist no are bet members of some groups.

(4) r(A) = m + n -1 (an important result in transportation problem), i.e. Not more than m+n-1. x_{ij} are different from zero.

(5) TP is represented by adjacent matrix

Destination/ Origin	D_1	D_2	D_3
01	x_{11}	x_{12}	c_{11}
02	c_{21}	c_{22}	c_{29}
	X_{21}	x_{22}	x_{2n}
0m	x_{m1}	x_{m2}	x_{mn}

(6)Tree Graph: A tree graph is one which consist of vertices and not n-1 arces such that E a unique chain of acres connecting every pair of vertices.



(7) In TP, a basic solution correspondent to a tree

Example:

Available icons		20	15	18	38	25	21
I=1	32	6	8	6	4	6	4
=2	27	11	13	15	8	12	1
=3	48	3	7	4	2	3	3
=4	30	8	9	8	5	9	6

Now suppose you are given a proposed solution as

Dest		(5)	(6)	(7)	(8)	(9)	(10)
Origin	20	15	18	38	25	21	$=Eb_j$
(1) 32			(15)			(17)	
(2) 27				(27)			
(3) 48	(20)	(3)			(25)		
(4) 30		(15)		(11)	(4)		

$\Sigma a_i = \Sigma b_j = 137$

m = 4? the no of origin

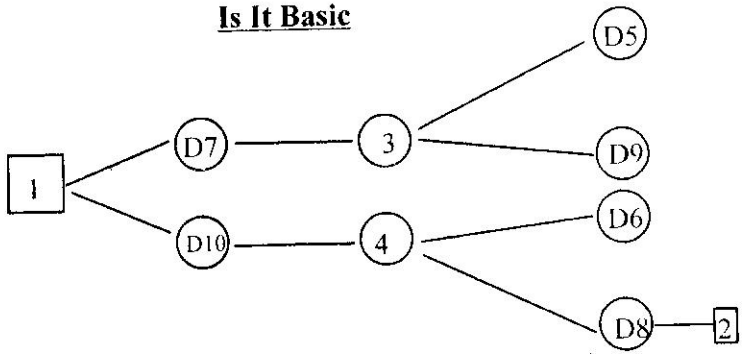
n= 6? no of destination

(1) so for **feasibility** we have

$\Sigma x_i = a_i$ $\Sigma x_i = b_i$ $\Sigma a_i = \Sigma b_i$

Thus, if any of this condition is not okay, then it implies that problem is not feasible, then you will introduce Domin.

(2) **Is It Basic**



It is tree because it completed the no of vertices, i.e.

M+n = vertices 4 + 6 = 10 vertices

And the number of arc = m+n-1 = 10- 19 arc so it is basic feasible solution.

(3) **Check for optimality**

We know that in one L.P problem we have

Min Cx Dual = maxyb

S.L Ax = b S.t Ya ≤c

x≥ 0 y =0

For optimality YA-c ≤ 0

$Y_i\ y_i\ c_{ij} = 0$

$Ilij = y_i\ _y i\ c_{ij} = 0$ for used rout

And ≤ 0 for unused rout

And to obtain our dual values, the following procedure is followed:

A. Choose and arbitrary origin to have dual value 0

B. To get the remaining values:

(a) When moving from the origin to the destination and the original values for the cost of transportation on used values a column dual value.

This gives y_i

(a) Subtract the cost of the routs when going from column to a row.

This gives Y_i

(b) Compute y_i. y_i. c_{ij} for all i and j

Now using the clear result, we can now compute our dual and obtained the above tree result

yi / yi	10	12	11	8	10	9
5	-1	-1	(6)	-1		(4)
0	-1	-1	-4	(8)		
7	(3)		(4)		(3)	
3		(9)		(5)		(6)

Now we can obtain our y_i and y_i as follow since we have agreed to choose origin 2. now to check for optimality

10-5-6 = -1

12-5-8 = -1

8-5-4 = -1

All the values is either 0 or -ve whence our solution is optimal.

Now the next is what is the cost

i.e $Z = cx = c^p x^p$

for each of the basic roots. Find the cost and multiply by the quantity

$= 6(15) + 4(17) + 8(27) + 3(20) + 4(3) + 3(25) + 9(15) + 5(11) + 6(4)$
$= 723$ units of money

Or cost = yb

		20	15	18	38	25	21
		10	12	11	8	12	9
32		5					
27	0						
48	7						
30	3						

yb = (20x10) + 15 x12 ... 21x9 (32x 5)x 27 x14 ... 3(30) = 735

3. DIFFERENT METHOD FOR GETTING FEASIBLE SOLD OR SAY GETTING A 1ST FEASIBLE SOLUTION

In T.P, a feasible is easily available and have easy inverse. We consider 3 Methods

1. NORTH WEST CORNER RULE

STEP1: begin with cell (1,1)
Set x11 min $(a_1 b_1)$

Either the origin or the destination requirement is the satisfied. For the satisfied row or column, label as non basic, the remaining cell but not both.

STEPII: if $a_1 > b_1$ set $x_{12} = \min(a_1\ b_1, b_2)$

Oral $< b1$, set $x_{21} = \min(b_1 a_1, a_2)$

$a_1 = b_1$, Set $x_{12} = 0$ evidence of degeneration.

The process continues at the i^{th} step on origin or destination requirement is satisfied. Ultimately, we will obtain an initial basic feasible solution if there exist m+n-1 basic solution. Now suppose we have the following T.P solution

D	30	50	20	40	30	11
0						
50						
40						
60						
31						

X_{12} =min (20,50)
Sine 20 has least value
$x_{12} = 20$

for x_{12}

	30	50	20	40	30	11
50	X11	X12	X13	X14	X15	X16
	(30)	(20)	x	x	x	x
40	X21	X22	X23	X24	X25	X26
	x	(30)	(10)	x	x	x
60	x	x	(10)	40	(10)	x
31	x	x	x	x	(20)	(11)

X_{11} = min (30, 50
30 units is sent to destination I
Sine 30 is < 50
x11 = 30
But since it remaining 20

we put 20 -1 = x_{12}
for x_{22} the remaining of
50 = 30, we put 30 into x_{12}

Now to find out whether they are correct, we have m+n-1 =9
Meaning the no basic feasible solution 4+6-1 = 9, hence the above is basic.

CHEAPEST METHOD (Min cost method)
Find the cheapest rout, i.e. min (c_{ij}). Assign the max feasible x_{ij} for the routs. Go to the next cheapest route and so on

	30	50	20	40	30	11
50	2	1	3	3	2	5
40	3	2	2	4	3	4
60	3	5	4	2	4	1
31	4	2	2	1	2	2

	30	50	20	40	30	11
50	x	50	x	x	x	x
40	20	0	20	x	x	x
60	10	x	x	9	30	11
31	x	x	x	31	x	x

You choose either column or row and assuming non basic, i.e. Matrix.

Now check for basic m+n-1 = 9, it is basic and hence it is an example of degenerate solution.

VOGEL'S METHOD OR RULE

This is a method of choosing the cell that has the greatest immediate alternative cost. Compare the difference between the lowest cost c_{ij} and the next lowest cost cit for each row (c_{ij} c_{kj}) for all I.

1. Compute the difference between the lowest c_{ij} and the next lowest c_{kj} for each column (c_{ij} c_{kj}) for all I.

2. Choose the cell for which excess cost route or columnwise is grater. Assign the biggest possible value to this cell.

3. If there is a tie, resolve the tie by choosing the cell where grater flow can be assigned.

D	8	17	11	Row diff
0				
12	24	10	28	8
19	18	10	20	8
15	5	1	8	6
Column diff	13	9	12	

Now consider the highest value here, it is 13, Now assign the max flow here, i.e. the quality in the Destination.

	8	17	11				8	17	11	
12	x	16	28	12		12	x	12	x	
9	x	10	20	10	=	9	x	10	20	10
15	8	1	8	7		15	8	1	8	7
	x	9	12				x	9	12	

	8	17	11
12	x	12	x
9	x	5	4
15	8	x	7

Whence basic because m+n-1 = 3+3 = 5

Whence we have <m+n-1 positive basic variable generating.

Getting initial basic solution:

Now the next thing is to find our whether it is optimal, we have

BASIC CHANGE IN ORIGIN/DESTINATION MATRIX

Stepping Stone Algorithm

Suppose you are given this problem

Di / Oi		20	15	18	38	25	21
32	6	8	6	4	6	4	
27	11	13	15	81	2	10	
48	3	7	4	2	3	3	
30	8	9	8	5			
63	3	1	2	2	3	1	

Here we have 4 origin available each of them having various quantity to send to different destination and you are to advice the manifestation. The quantity to send to obtain max solution. Assuming Vogle's Rule: by this you look from the cost difference as we have above

	20	15	18	38	25	21	
32	x	8	6	4	x	4	0
27	x	13	15	8	x	10	2
48	20	7	4	7	25	3	1
30	x	9	8	5	x	6	1
	3	1	2	2	x	1	

		20	15	18	38	25	21
32	x	8	6	4	x	4	0
27	x	x	x	27	x	x	x
48	20	7	4	2	25	3	1
30		9	8	5	x	6	1
	3	1	2	2	x	1	

	20	15	18	38	25	21	
32	x	8	6	4	x	4	0
27	x	x	x	27	x	x	2
48	20	x	x	3	25	x	x
30	x	9	8	5	x	6	1
	3	1	2	2	x	1	

S

	20	15	18	38	25	21
32	x	8	6	4	x	x
27	x	x	x	27	x	x
48	20	x	x	3	20	x
30	x	9	8	5	x	x

20	15	18	38	25	21
32	8	6	x	x	21
27	x	x	x	27	x
48	20	x	x	3	25
30	x	9	8	5	x

	20	15	18	38	25	21
32	x	x	11	x	x	21
27	x	x	x	27	x	x
48	20	x	x	3	25	x
30	x	9	8	5	x	x

Now our dual is given as

	9	12	11	8+0	9	9
5	-2	-1	6	-1	-2	4
0	-2	-1	-4	8	-3	-1
6	3	-1	1	2	3	0
3	-2	9	8	5	-3	0

Now for optimal, we expect our Π_{ij} to be = 0 for all ij is either ve or 0 from the table we see that the optimality is not satisfied because of the +1 in one of the rows and this that we are losing for not sending anything through that route because it would have been cheaper to send through that route. In essence we say that there is only one non-basic route that shows the profit

x_{33}

Now what we shall do is to choose 0 = min (3, 7) min will be 3 for table 10

∴ from table K we have as follow

	20	15	18	38	25	21
32		11			21	
27				27		
48	20			3	25	
30		15	7	18		

Now we can construct our dual as follows:

	10	12	11	8	10	9
5	-1	-1	6	-1	-1	4
0	-1	-1	-4	8	-2	-1
7	3	-2	4	-1	3	-1
3	-1	9	8	5	-2	0

Now we can check for optimality $\Pi_{ij} \leq 0$

∵ from the table, we see that all the table basic are either ve or zero, i.e. those without cord.

We have obtained our optimal.

Now we can check for optimality $\Pi_{ij} \leq 0$

from the table, we see that all the table basic are either ve or zero, i.e. those without cord,

∵ We can now obtained our optimal .

Now we can compute our Z = C

= 11 x 6 + 4(21) + 27(8) + 3(20) + 4(3) + 3(25) + 9(15) + 8(4) + 5(11)

= 735

Another example

	15	16	11	13
12	8	9	6	3
20	6	11	5	10
13	3	8	7	9

And that we are given an initial basic feasible solution as

	15	16	11	13
17	15	2		
20		14	6	
13			5	13

Now the question is for the optimal result

∴ Since we have been given the basic feasible solution, you don't need to use any of the method mentioned earlier, but you just need to itrate (now what you will do is to write the corresponding cost, i.e. fine dual)

Now our dual is given by

	8	9	3	5
0	8	9	-3	2
-2		11	5	-3
-4	9	5	7	9

Since we have +2, +9, +4

We choose to move from 9 which is greater, hence we can find the least value of θ (also note that because of the values got in (a), the optimal is not yet attained)

For the value of (consider all her, $-\theta \Longrightarrow$ (514 15)

∴ θ with min value is $5 \Longrightarrow \theta = 5$

Hence our Dual now would be

	8	9	3	14
0	8	9	-3	11
-2	4	11	5	6
-4	7	-4	-9	9

Thus because of the values again got from our dual, we have to compute our values again from table 11, 6, 4 $\Longrightarrow$ our θ value will start from position 11 because it is the greatest.

	15	16	11	13
17	10	7		+0
20		9	11	
18	5			-0

$\therefore \theta = \min(10, 13)$

$\theta = 10$

	0	7	10
	9	11	
15		3	

	3	11	5	5
2		9		3
0		11	5	
-4	7			9

The final answer of the above is given by

	15	16	11	13
17		4		13
20		9	11	
18	15	3		

	6	11	5	5
2	9		3	
0	11	5		
3	3	8	8	

$\therefore Z = 298$

Question 59:

a. Define the following:
   i. Strategy
   ii. Two person Zero sum game
   iii. Pay off (iv) Dominants

b. The table shown below display the pay off between player A and player B negative pay off shows the total amount in thousands that A losses to B as result of the strategy they played simultaneously. Define if the following situations occur.
   i. A dominant
   ii. Saddle point
   iii. Pure strategy
   iv. Mixed strategy

	B1	B2	B3	B4
A1	-1	-2	4	5
A2	0	-3	1	2
A3	-2	-3	3	1
A4	-1	4	-3	-2

SOLUTION 2

a. Strategy is the technique that a game player adopts or decide to adopt concerning his move in order to maximize his gain

ii. Two person zero sum game. It is a game where every naira that one player wins comes out from the other player's pocket, so the two players have totally conflicting interests. Thus cooperation between the two players would not occur. If player A wins ₦X. This means that the other player B would have lost ₦X and so, the total sum of the winning from the game is ₦X + (-X) = ₦0

iii. Pay off: It is a table or matrix that analyse various alternative course of actions to be performed or that are possible in a scenario.

iv. Deminants: G is the most outstanding event among other series of events.

	B1	B2	B3	B4	Minimum (least element)	
A1	-1	-2	4	5	-2	*
A2	0	-3	1	2	-3	
A3	-2	-3	3	1	-3	
A4	-1	4	-3	-2	-3	
Maximum	0	4	4	5		

Latest
Element *

i. Dominant loss is 3 to B player and gain to A Player
ii. Saddle points occur if there is an entry in the pay off matrix which is simultaneously the minimum element in its row and the maximum element in its columns. Here there is no saddle point as -2 is not in the same row or column with O.
ii. Pure strategy - This is a game that its situation determines the table
iv. A mixed strategy involves more than one strategy such that if one strategy fails. The reserved strategy will be adopted.

Question 60

Nigerian Breweries Plc is a company that has production operations in Lagos, Ibadan and
Benin. The company has four distributions centres located in Sagamu, Warri, Kaduna and
Port Harcourt. The production capacity, the demand of the distribution centres as well as
the transport cost from the distribution centres are show belows.
Advise the company management on the most economic deliveries

	Sagamu	Warri	Port - Harcourt	Kaduna	Supply Capacity
Lagos	3	2	7	6	5,000
Ibadan	7	5	2	3	6,000
Benin	2	5	4	5	2,500
Demand Requirement	6,000	4,000	2,000	1,500	

SOLUTION
USING VOGELS APPROXIMATION METHOD

	S	W	PH	K	SC						
L	3 1000	2 4000	7	6	5000	3-2 1	6-3 3	X	X	X	X
I	7 2500	5	2 2000	3 1500	6000	3-2 1	3-2 1	3-2 1	3-2 1	3-2 1	2
B	2 2500	5	4	5	2500	4-2 2	4-2 2	4-2 2	X	X	X
DD	6000	4000	2000	1500	13500	—	—	—	—	—	—
	3-2=1	X	4-2=2	5-3=2	—						
	3-2=1	X	4-2=2	5-3=2	—						
	7-2=5	X	4-2=2	5-3=2	—						
	7	X	2	3	—						
	X	X	2	3	—						
	X	X	2	X	—						

The result is exactly the same as that of least cost method. Hence total Minimum cost is N42,000

Question 61
ai. What is a project
ii. Define the following terms:
Critical paths and slack

Solution:
a. Project: This is an activity or event or contract that requires a process or stages of completion. The
activities can be overlapping concurrent, dependant or progressive.
ii. Critical Paths:- This is the longest path that it takes to complete the critical activities. It is the longest
day it takes to complete a given project.
Slacks: This the amount of time we can delay the start of the activity beyond its earliest startime time without delaying project completion beyond EFT

Question 62

a. What is production management?

b. Differentiate between production management and project management.

C. Fit a trend line to the following data by the semi-average method and hence predict
the likely sales of the company by 2005.

YEARS	1980	1981	1982	1983	1984	1985	1986
SALES	102	105	114	110	108	116	112

Solution:

a. Production Management

This is the process of managing the resources of an organization which are devoted to the
transformation of inputs into goods and services

b. Production Management Vs Project Management. Production management are series of checks
and balances put in place to ensure that raw materials put into a production process generate
expected output. Project Management are series of monitorings ensure that a contract is
concluded as planned and meet up to requires standard and specifications.

c.

Year	1980	1981	1982	1984	1984	1985	1986	sum
Sale (y)	102	105	114	110	108	116	112	
T	0	1	2	3	4	5	6	

$$b = \frac{n \times \sum yt - (\sum y) \times (\sum t)}{n\sum t^2 - (\sum t)^2}$$

Y	t	Yt	t^2
102	0	0	0
105	1	105	1
104	2	228	4
110	3	330	9
108	4	432	16
116	5	580	25
112	6	672	36
767	21	2347	91

$$b = \frac{7(2347) - (767)(21)}{7(91) - (21)^2} = \frac{16,429 - 16,107}{637 - 441} = \frac{322}{196} = 1.64$$

$a = \bar{y} - b\bar{t}$

$$a = \frac{\sum Y}{n} - b\frac{(\sum t)}{n} = \frac{767}{7} - 1.64286\ \frac{21}{7}$$

$= 109.57 - 4.92858$

$= N104.64$

$Y = a + bt$

$Y = 104.64 + 1.64286t$

For Sales in 2005: note that t = 5 for 1985 and t = 6 for 1986

Year	1987	1988	1989	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000
T	Y	8	9	10	11	12	13	14	15	16	17	18	19	20

Year	2001	2002	2003	2004	2005
T	21	22	23	24	25

t = 25

$Y = 104.64 + 1.6428t$

$= 104.64 + 1.6428(25)$

$Y = 104.64 + 41.07$

$Y = N145.71$

Question 63:

4. ABC of planning a new operation which will involve the following activities and time duration.

ACTIVITY	DESCRIPTION	DURATION			PREDENT ACTIVITY
		a	m	b	
I	Design Logistic	4	2	6	
II	Solution Refine logistic	3	4	5	I
III	Solution Prefare planning	5	3	7	I
IV	application System requirement	3	2	1	III
V	Suppliers briefing/Negotiation	2	2	6	I
VI	Obtain planning Content	7	5	9	III
VII	Mechanical/Handling Equipment	2	3	4	III, II
VIII	Building contrinction	1	4	1	V
IX	Tractor procurement curement	20	12	40	VI, VII
X	Trailer procurement	2	4	6	VIII, IX

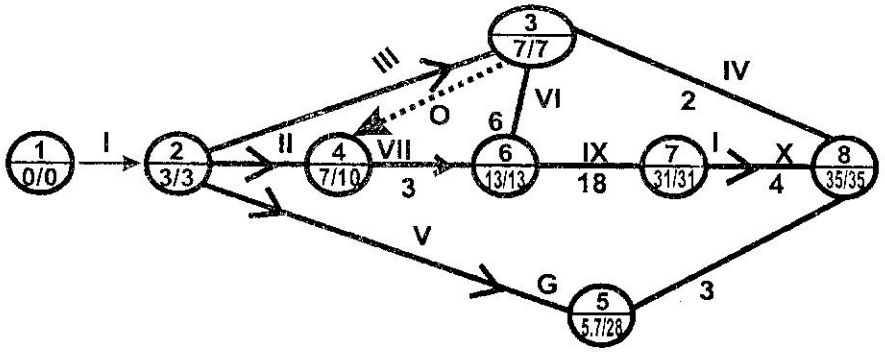
Required:

Draw the network for these tasks.

Solution

4.

Activity A	Preceeding IPA	Duration te = a + 4m + 6/6
i	-	te = 4+4(2)+6 / 6 = 3
ii	I	te = 3+4(4)+5 / 6 = 6
iii	I	te = 5+4(3)+7 / 6 = 4
iv	iii	te = 3+4(2)+1 / 6 = 2
v	I	te = 2+4(2)+6 / 6 = 2.7
Vi	iii	
Vii	iii,ii	te = 2+4(4)+1 / 6 = 3
Viii	v	te = 1+4(4)+1 / 6 = 3
Ix	vi,vii	te = 20+4(12)+40 / 6 = 18
X	viii, ix	te = 2+4(4)+6 / 6 = 4



i.e THE NETWORK FOR THE ABOVE

Question 64.

a What do you understand by forecasting? Are there any differences in meaning between the concept of forecasting and predicting? State the relevance of forecasting to production

b. Fit a straight line trend of the Y=a+bx by method of least squares to the following time series data. Compute the trend value and hence predict the value of production by 1976, 2000.

YEAR	1960	1961	1962	1963	1964	1965	1966	1967	1968
PRO(000)	11	13	15	14	15	16	16	17	18

Solution:

a. Forcasting is a term which relates to anticipation of likely conflow movement a near future. It usually relates to most part of an organisation actvies e.g. Sales forcast, production forcast, purchase budget etc

Predicting or estimation is a term which relates to obtaining what an aspect of cost will be after a given period. This is usually carried out using

Regression analaysis using least square $Y = a + b\,x$

b

Year	Production Y'	t	Yt	t^2
1960	11000	0	0	0
1961	13000	1	13000	1
1962	15000	2	30000	4
1963	14000	3	42000	9
1964	15000	4	60000	16
1965	16000	5	80000	29
1966	1600	6	96000	36
1967	17000	7	11900049	
1968	18000	8	144000	64
	135000	36	584,000	204

$$b = \frac{n \times \sum yt - (\sum y)(\sum t)}{n\sum t^2 - (\sum t)^2}$$

$$= \frac{9(584,000) - 135000)(36)}{9(204) - (36)}$$

$$= \frac{5256,000 - 4860,000}{1836 - 1296} = \frac{396,000}{540}$$

$$= ₦733$$

Where $a = \frac{\sum y}{n} - b\frac{\sum t}{n}$

$$a = \frac{135,000}{9} - 733\frac{(36)}{9}$$

$$= 15,000 - 2,932$$

$$= ₦12,068$$

$\therefore Y = a + bt$

$Y = 12,068 + 733t$ for 1968, $t = 8$

Then $t = 9$ for 1969, $t = 10$ for 1970, $t = 11$ for 197/ and $t = 16$ for 1976

$Y = 12,068 + 733(16) = N23,796$

For 2000 year, try and obtain "t" yourself and substitute as usual and provide solution.

Question 7:

1a. Define the following terms in relation to game theory

I Moves

Ii Game

Iii Decision maker

Iv. Conflict

V. Decision

Vi. Strategy

vii. Perfect information

Viii. Pay-offs

bi State the maximum criterion

Ii. Use the criterion to find the optimal strategies for players X and Y and the value of the play-offs of the following

Player Y's strategy

		Y_1	Y_2	Y_3
	X_1	8	-3	7
	X_2	6	-4	5
	X_3	2	2	3

Solution:

(ii). Game: Game theory is a mathematical strategy which assign analyzing decision environment containing conflict

It is useful making decision incases where two or more decision environment containing conflict. It is useful for making decisions in cases where two or more decision makers have conflicting interests

(iii). Decision Maker: is the person who will be finally committed and responsibility consequence of the decision made

(iv). Conflicts: Where two or more decision makers disagrees

(v). Decision: is the final course of action committed to. It the choice made out of a number of different alternative

(vi). Strategy is defined as a complete predetermined plan for selecting a course of action, under every possible circumstance

(vii). Perfect information: This is a complete facts or knowledge

(viii). Pay-offs:-

(i). Moves: This is the Strategies that a game player adopt in order to maximize his gain or reward.

bi. The maximum criterion = 8

ii.

	Y1	Y2	Y3	Min	Max
Xi	8	-3	7	-3	-
X2	6	-4	5	-4	-
X3	2	2	3	2	2
Max	8	2	7	-	-
Min	-	2	-	-	-

Optimal Strategy - Pure strategy
Pay - Off = 2

Question 65:

a. Define the following terms in relation to:
(I) Queuing theory (i) Arrival (ii) Service
(Ii) List the various assumption in Queuing theory

b. Idowu-Foba and Co., a shoe manufacturing and repair company employs 6 operators who on the average work on 10 pairs of shoes per hour. The average number of customers to be attended to are 20 per hour while loss on any person left unattended to is N3
Required:
(i) Computer the traffic intensify
(ii) The loss per hour
(Iii)The total of people on queue.

Solution:

2a (i) Queuing Theory: This is a systematic way of circumstances which involve trend of people or item that are available for services

ii. Arrival: Implies the term at which items/people come for service and this can be random approach or regular approach

iii. Queue - a waiting line formed whenever customers arrived and facility is busy

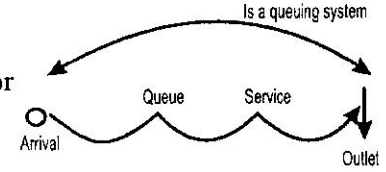
iv. Service: This is the act of serving customers. The service is provided by a service facility

vi. Assumption in Queuing theory

* Arrival must be poison in nature
* Services must be Poisson in nature
* There is only one queue & service point
* customers are served on FIFO basis

(b) Traffic intensify = $\frac{\lambda}{N}$ - Arrival / Service

or



Arrival rate = λ = 10 pair of shoe per hour

Service rate = N = 20 customers are expected to bring 20 pair of shoes for repair per hour

(i). Traffic intensify = $P = \frac{\lambda}{N} = \frac{10}{20} = \frac{1}{2} = 0.5$

(ii). Time on queue (Tq) = $\frac{1}{N} \frac{\lambda}{N-}$

$= \frac{10}{20} \frac{1}{20-10}$ $= \frac{1}{20}$ hrs or $\frac{1}{20} \times 60 = 3$mins

Lost per hr delay = $\frac{1}{20}$ hrs x ~~N~~3

$= \frac{~~N~~3}{20}$ $= (N3/20)$

(ii). Total people on queue = (Nq) = $\frac{\quad}{N\ (N-\ )}$

$Nq = \frac{10}{20} \frac{10}{20-10} = \frac{10}{20} \frac{(10)}{10} = 0.5$

Question 66:

a. List at least 4 areas of applications where network analysis has proved effective. Trace the origin of critical path method and project evaluation review Technique

b. The schedule below represent activities required to overhaul a piece of equipment

Activity	Proceeding activity	Duration (in days)
A	A	4
B	B	1
C	C	3
D	C	5
E	C	2
F	C,D	5
G	F	4
I	F	8
J	F,I	1
K	G,H,I	4

From the above schedule
(i) draw the network and explain why a dummy has to be introduced
(ii) Compute the EST, EFT, LFT, Float
(Iii) Compute the project completion time

Solution:

a. Four areas of application of network

i. Construction e.,g. Houses

ii. Product change over

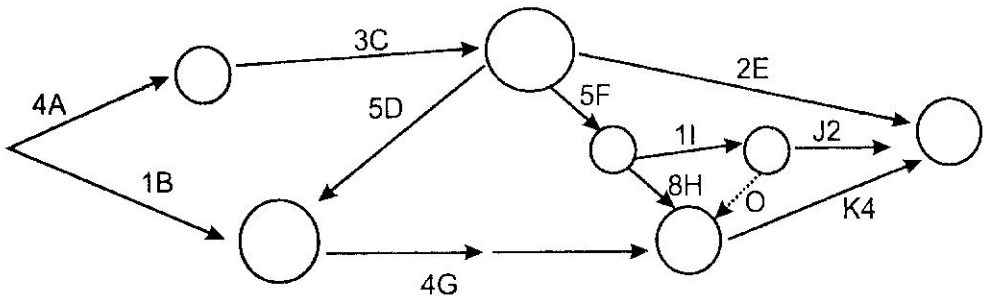
iii. Town Planning

vi. Civil Engineering eg. Bridges

Note that the critical parth method assist in calculating the float for each activity and the PERT for planning, controlling and evaluating the project.

3b

Activity	IPA	Duration	ES	EF	LS	LF	Float/ Slack	Critical Park
A	–	4	0	4	0	4	0	* A
B	–	1	0	1	15	16	15	
C	A	3	4	7	4	7	0	* - C
D	C	5	7	12	11	16	4	
E	C	2	7	9	22	24	15	* - F
F	C	5	7	12	7	12	0	
G	C,D	4	12	16	16	20	4	
H	F	8	12	20	12	20	0	* - H
I	F	1	12	13	19	20	7	
J	F,I	2	13	15	22	24	9	
K	G,H,I	4	20	24	20	24	0	* - J



Event I and H are Con Current starting from event F hence a dummy is required to connent I to H,G to make G,H,I required for activity K to start.

Question 67:

CFAO Plc a manufacturing company manufacturers hand set type called Nokia A and Motorola - B brand. Each of the product relevant data are stated below

Product	Nokia A	Motorola B
Cutting hr per (unit)	4	2
Finishing hrs per unit	6	6
Unit cost (N)	56	50
Selling price	68	58
Maximum sales (per week)	400	

Maximum available cutting hours is 880 and number of finishing hours available is 1720, other resources are abundantly available

(i) Calculate the number of units that should be produced of each set brand profit .

(ii) Calculate the maximum profit

(iii) Calculate the shadow price from the above

Solution:

Profit/Contribution Calculation

	Nokia A ₦	Motorolla B ₦
Selling Price	68	58
Unit Cost	(56)	(50)
Profit Per Unit	12	8

	Nokia	Motorolla	Hrs (Max)
Cutting hrs	4	2	880
Finishing hrs	6	6	1720
Profit	12	8	

Constraint

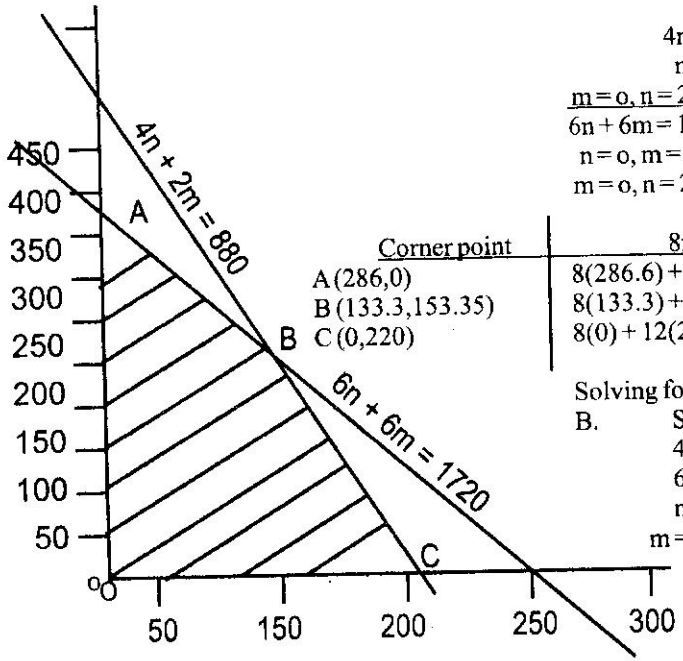
Maximiz $Z = 12n + 8m$

S.T. $4n + 2m \quad 880$

$6n + 6m \quad 1720$

$n, m \quad o$

The 400 maximum sales is to ensure that what ever recommended sales you make must not excised 400.



$4n + 2m = 880$

$n = o, m = 440$

$m = o, n = 220$ $\therefore$ A(220,440)

$6n + 6m = 1720$

$n = o, m = 286.6$

$m = o, n = 286.6$ $\therefore$ B(286.6,286.6)

Corner point	$8m + 12n = Z$
A(286,0)	$8(286.6) + 12(0) = 2292.8$
B(133.3,153.35)	$8(133.3) + 12(153.35) = 2906.6$
C(0,220)	$8(0) + 12(220) = 2640$

Solving for corner point B. Simultaneously using

$4n + 2m = 880$ and

$6n + 6m = 1720$

$n = 153.35$

$m = 133.3$

Sincere cornerpoint B produce the highest profit result, then it recommended that 33 3 of m and 153.35 on N be produced. The maximum profit being = 2906.6 ie $8(133.3) + 2(153.35)$

(iii) Concept of shadow cost

If one extra hour of cutting hour is available

Then $4n + 2m = 881$ -> cutting

$6n + 6m = 1720$ -> finishing

Solving it simultaneously

$-12n + 6m = 2643$ (1 multiply e.g. (1) by 3)

$6n + 6m = 1720$

$6n = 923$

$n = 153.84$ and solving for m

$4n + 2m = 881$

$(153.83) + 2m = 881$

$615.33 + 2m = 881$

$2m = 881 - 615.33$

$2m = 265.66$

$m = \frac{215.66}{2} = 132.83$

Hence $n = 153.83$ and $m = 132.83$

New profit Z $= 12n + 8m$

$= 12(153.83) + 8(132.83)$

$= 2908.6$

But the old Profit is N2906.6

Hence there will be an increase in profit by 2908.6 = N2 if one hour of cutting time is available on the other hand, if one hour of finishing time is available then

$4n + 2m = 880$---------1

$6n + 6m = 1721$--------2

Multiply equation (1) by 3

$12n + 6m = 2640$

$- \underline{6n + 6m = 1721}$

$6n = 919$

$n = \frac{919}{6} = 153.16$

$6n + 6m = 1721$

$6(153.16) + 6m = 1721$

$919 + 6m = 1721$

$6m = 1721 - 919$

$6m = 802$

$m = \frac{802}{6} = 133.67$

New Profit = Z $= 12n + 8m$

$= 12(153.16) + 8(133.67)$

= N2907.28)

Less existing profit (2906.6)

0.68 increase

Hence, if one hour of finishing time is available only 68 kobo increase will be made. It is therefore more profitable to utilize more cutting hour than finishing hour.

Question 68:

a. Itemise and briefly discuss the major assumptions that must be satisfied before practical problem can be forumated as a linear program

b. Two products P1 and P2 require resources R1, R2, and R3 to be produced. A unit P1 requires 2 units of R1, 1-5 of R2 and 1-5 of R3; while a unit P2 requires a unit R1, 4 of R2 and 2 of R2 to be producedR1, R2 and R3 are available at the rate of 500 units, 720 and 450 per day respectively P1 and P2

are sold at average profit rate of N50 and N45 per unit respectively. You are required to determine the optimal production rate per day of the products.

Required:

i. To formulate the problem in form of an allocation table
ii. Systematically formulate the problem as a linear program
iii. Solve completely using the graphical approach

Solution:

Assumption of LP

a (i) Objectivity Assumption
(ii) Linearity Assumption
(iii) Divisibility
(iv) Non negativity Assumption

- Objectivity Assumption: means two objective cannot obtained at the same time, one can either minimize or maximize
- Linearity Assumption: This states that there must be relationship between cost & revenue
- Divisibility Assumption: This states that fractional part of a product must be producable
- Non-negativity Assumption: States that an organisation may produce or not but cannot produce negative production.

The problem is use by presented inform of a table

b (i)

	Products		Available
	p1	p2	
R1	1.2	1	500
R2	1.5	4	720
R3	1.3	2	450
	50	45	

(ii) Let P1 represent x
P2 represent y

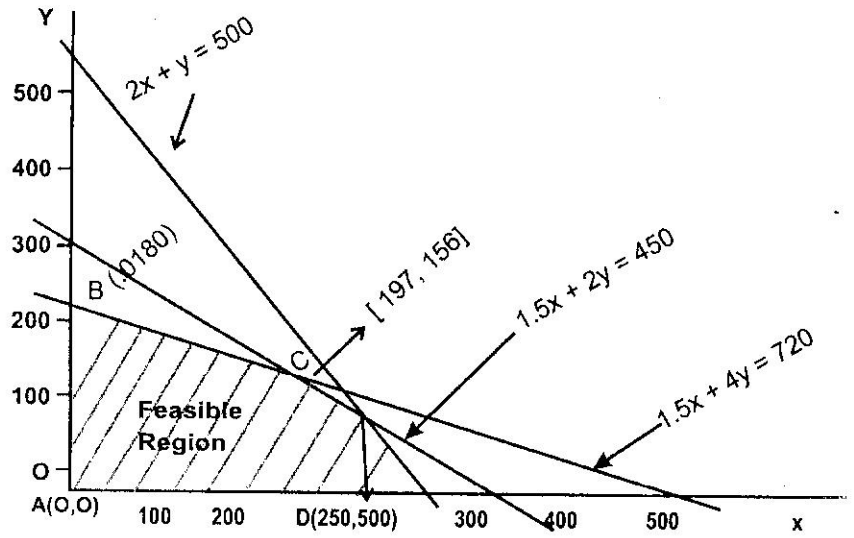
Max C: 50x + 45y (objective function)
Subject to: $2x + y < 500$ (R1 Constraint)
$1.5x + 4y \leq 720$ (R2 Constraint)
$1.5x + 2y \leq 450$ (R3 Constraint)

Non-Negativity $X, y > 0$

$2x + y = 500$ $\qquad$ $1.5x + 2y = 450$
When $x = O, y = 500$ $\qquad$ when $x = 0, y = 225$

$X = 250 \quad y = 0$ $\qquad$ $x = 300 \quad y = 0$

$1.5x + 4y = 720$
When $x = 0, y = 180$
$X = 480 \quad y = O$



To get point C

$2x + y = 500 \times 4$
$1.5x \ 4y = 720 \times 1$
$8x + 4y = 200$
$1.5x + 4y - 720$
$6.5x = 1280$
$X = 197$

Solve for x in
$2x + y = 500$
$2(197) + y = 500$
$394 + y = 500$
$y = 500 - 394$
$y = 106$

Points	product		Objective	Value
	x	y	50x + 45y	
A	o	o	50(0) + 45 (0)	0
B	o	180	30(0) + 45 (180)	8,100
C	197	106	50(197) + 45 (106)	14,620
D	250	500	50(250) + 45 (500)	35,000

Decision: The company should produce 250 on product x and 500 on y to make N35,000

Question 69:

A. Define and explain the following terms as they are used in games theory

i. Zero sum game
ii. Non-zero sum game
iii. The N-persons game
iv. Pure strategy
v. Saddle point

b. Two nations negotiated on the tonnage price of rice. The joint strategies with relatives selling prices (seller's perspective) are as follows

BUYER

Seller		B1	B2	B4	B5	B6
	S1	6	9	18	-12	15
	S2	12	15	9	15	12
	S3	21	12	24	18	18
	S4	9	12	21	3	15

i. Determine the optional strategy for each player
ii. What is the value of the game?

Solution:

a(i). Zero sum Game: In a zero - sum game, the winner receive the entire amount of the payoff that is contributed by the loss sum of a game is always strictly competitive

(ii). Non-Zero sum game: In a non-zero sum game, the gain of one player differs from the losses of the other. It is not strictly competitive

(ii). The N-persons game

(iv). Pure strategy: This is a game that its situation determines the table i.e. Maximum of the row is equal to minimum of the column

(v). Saddle Point: This occurs when the entry in the puremitrix is simultaneously the minimum element in its now & maximum element in its column

b.

	B1	B2	B3	B4	B5	B6	Min	Max
S1	6	9	18	6	-12	15	-12	
S2	12	15	9	-6	15	12	-6	
S3	21	12	24	9	18	18	9	9
S4	9	12	21	6	3	15	3	
Max	21	15	24	9	18	18		
Min				9				

ii. The optimal strategy is pure strategy since the row is equal to the column
iii. The value of the game = 9

Question 70:

Explain the following concepts

(a) Characteristics of competitive game
(b) PERT
(c) CPM
(d) What is optimal strategy in game theory?
(e) Queuing Theory

Solution:

a Characteristic of competitive game - we have two types off competitive gains, the business game and the military game, both are competitive games. There characters teach us that in business game, this provides the us with an opportunity to test out business skills and decision making ability in a competitive environment while in the military game, this is used all over the world to test offensive and dependence strategies and to examine the effectiveness of equipment and aims.

b PERT: (Programme Evaluation and Review Techniques)
- Pert is used when we have uncertain activities or a project task, and when this happens, there is need to apply a statistical analy to estimate the duration for each activity concerning the projects. Here, we have three types of estimates which are normally employed in the computation i.e the most optimistic the most pessimistic and the most likely time.

c CPM (Critical Path Method) - This is used precisely known time of which activities will take place, is usually given but may not be reliable in practice or in real situations and uses only two time estimate.

(d) we could says that The CPM is a techniques used in project mgt and is a deterministic technique that assumes certainly, the duration of every activity, it is assumed to be constant and known. While PERT is a stochastic in which uncertainty in the duration of the activities is allowed and it is measured by 3 times estimates namely, most optimistic, most likely or probabilitic time estimate and most pessimistic.

d Optimal Strategy in game theory in is: Game theory is a body of knowledge, which deals with the modeling and analysis of competitive situations. This is the expected wing of the game if the game is played for a large no of times it is the average pay off expected by each player who sticks to his best strategy all the time over an infinite number of plays of the game. It is also offal to the numerical value

of the saddle pt in a pure strategy game.

c. Queuing Theory: This studies situations in which customers arrive for service and must wait if the service facility is occupied or in used. Or we say that queuing theory is a system describing the waiting time and the service rate in supermarket, as in a movie as well as product given in a production plant.
There are a number of queuing model that deals with different queuing systen
The most common types of the system are
(i) Single services waiting the system
(ii) Multiple server waiting the system

Question 71:

a. Contrast "output" from manufacturing versus service delivery systems
b. Identify some major advantages of using models in production management

c. What do you understand by the concept production management
d. What are the features of production management?

Solution:

1(a) Manufacturing are those things produced and in turns those in charge producing the goods while service delivery systems are the people that takes th goods from the manufacturer to their respecter destination i.e the marketer.

(b) Some major advantages of using models in production management includ the following - for quality, for reliability, for delivery per Cheadle, for costin for flow of organizations. It could also be on policy formulation, like policy work in progress and lower material stock, also on policy on high pla utilization and for policy on industrial flexibility.

(c) Production Management is the process of designing, Operating and Controlli a Production System Capable of transforming physical resources and hum talent into needed goods and services. OR WE COULD SAY THAT Producti Management is the application of management functions such as planing a scheduling to the Conversion of inputs (both tangible and intangible) into goo and services with the budgeted cost.

(d) The feature of production management are as follows i.e Product Desig Production Planning, Production Scheduling, Production Control, Purchasi and Material Management. Also for Inventory Control, for Work Layout, Forecasting and Quality Control.

Question 72:

a. State the assumptions made in the use of linear programming
b. Toyota construction company constructing houses at two locations, required for minimum supply of 10 lorry loads of granite, 25 lorry loads of sand and 20 lorry loads of gravel to the locations. The table below show the quantity supplied to each site.

Lorry loads of Sand	Site		Total available
	A	B	
Lorry loads of Granite	2	3	10
Sand	10	17	25
Gravel	5	4	20

Each supply to site A costs the Company N60,000 while that supplied to site B costs N75,000

(i) Formulate a linear programming model for the problem
(ii) Find the quantity of supply to A and B that will minimize the total cost

Solution:

(a) Assumptions made in the use of linear programing is as follows:

(i) Additivity - The total contribution to the objective function and constraint equation is obtained by adding up the individual constraint of the individual activities.

(ii) Liniarity - This implies that constraint returns to scale. the decision variables Xj are raised to power I. i.e (must be linear a equation.

(iii) Proportionality - If the decision variables isdoubled, tripled or halved, its contribution to the objective function equation, and constraints equation will also be doubled, tripled or halved.

(iv) Divisibility - This implies that the decision variable Xj can have fractional values.

(v) Deterministic - this implies certainty i.e the values of all the parameter gj bi aij0 are known for certain.

(vi) Non - Negativity - This implies that the decision variable must be zero i.e Xj $\geq$ 0.

(vii) Finite - ness - This implies that all the activities, parameters and variable are finites Identifiable.

B. Maz Z = 60,000x + 75,000y.

S.t $2x+3y \leq 10$ -----(1)

$10x+17y \leq 25$ -----(2)

$5x+4y \leq 20$ -----(3)

FOR $2x+3y=10$

$x=0$ ---->$3y=10$ ---->$y=1073$ $= 3.3$

$y=0$ ---->$2y=10$ ---->$y=1072$ $= 5$

(5, 3.3)

FOR $10x+17y \leq 25$

for $x=0$ $17y=25$ $y=25/17$ $=1.5$

for $y=0$ $10x=25$ ----> $x=25/0$ $=2.5$

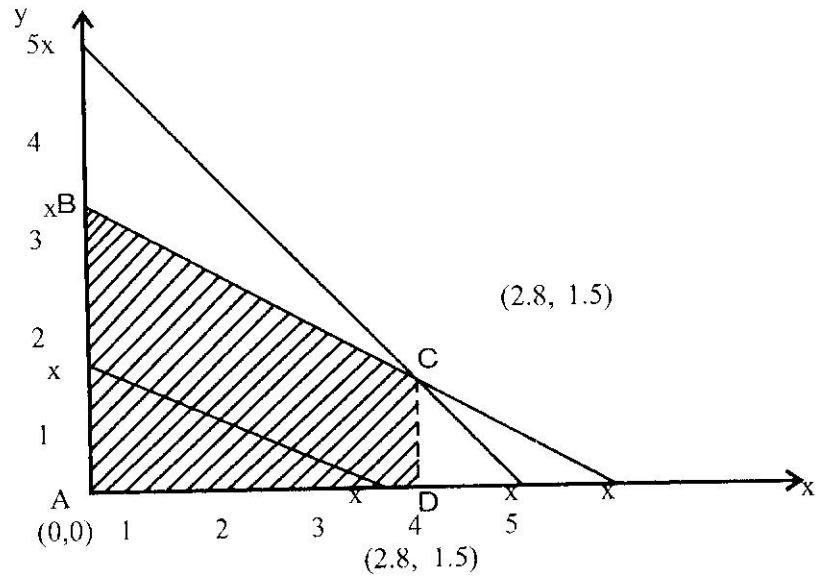
(2.5, 1.5)

FOR $5x+4y \leq 20$

----> for $x=0$ $4y=20$ ----> $y=5$

for $y=0$ $5x=20x=20/5$ ----> $x=4$

(4, 5)



Question 73

Given the data in the table below

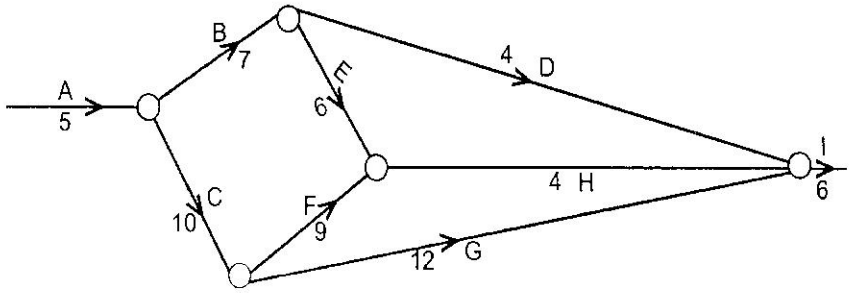
Activities	A	B	C	D	E	F	G	H	I
Immediate Predecessor	-	A	A	B	B	C	C	E,F	D,H,G
Time	5	7	10	4	6	9	12	4	6

(i) Construct a network for the data

(ii) Identify the critical activities

(iii) Determine the project completion time.

(i) Network for the data is as follows



(ii) The critical activities are

(a) A B D I

(b) A B E H I

(c) A C F H I

(d) A C G I

(iii) The project completion time.

EF = ES f t = For A = EF = 0 + 5 = 5

for B = 5 + 7 = 12

for C = 5 + 10 = 15

for D = 12 + 4 = 16

for E = 12 + 6 = 18

for F = 15 i 25 = 25

for G = 15 + 12 = 27

for H = we have ES as 18 and 24

∴ for H = 24 + 4 = 28

for I = 28 + 6 = 34

Question 74:

a. What is job design and what approach is taken to the design of jobs
b. What types of systems do organizations use to set employee compensation?
c. Critical activities
d. Slack

Solution:

(a) Job design is the general arrangement of different part of a body that is made, such as a building, hoah boat or build up of a studio , marchines or design of Cars, Aircraft, Automobile engine etc. (building the site to be used, location, cost e.t.c.). And the approach that is taken to design of jobs like a product layout which is a special configuration of machines as the support facilities arranged in accordance with the sequential manufacturing requirement of the product. the unique characteristic of the job (product, layout) design is the work center and equipment follow a specialized sequential order as required by the nature of the product being manufactured. In essence of works and equipment such layout is common where a standard product is being produced in large volume. Car assemble are a typical example of product - oriented layout and condition favoring these type of design i.e product layout design is most appropriate, where the following condition are present

(a) Very limited product variety
(b) High product standardize for
(c) Mass production

(b) Types of systems that organization used to set employee are as follows, There should exist Human resources Division of the organization that would set machinery in motion for all involvement of staff to get the organization moving i.e The staff or the employee that would work in the organization with various incentives.

(c) Critical Activity is an activity that has no room for schedule slip page. OR It is an activity with zero slack.

(d) Slack is the amount of time that an activity or group of activities can slip without causing delay in the completion of the job or project.

Question 18:

A. List steps involved in Operation Research/Production management.
b. Onifade Taiwo Produces both interior and exterior with the raw materials M1&M2.
The following table provided

	Exterior	Interior	Maximum Avaliability
Raw material	6	4	24
Raw material	1	2	6

A market Survey indicate that the daily demand for interior paint cannot exceed that of the exterior paint by more than one.
Also the Maximum daily demanded of interior paint is 2 tons.

i. Onifade Taiwo want to determine the optimum or best product mix of interior and exterior paint that maximizes the total profit.
ii Use any LPP method to solve the problem.

The wholesale prices per tons is n2000 for exterior and n3000 for interior paints. How much interior and exterior paints should the company produce maximize gross income.How much interior and exterior respectively and the requirement at I, II, III, IV. Are 9, 5, 4 and 2 units respectively.

Solution:

a. Steps involved in or / Production mgt are
i Use of all possible formulation of the problem in the objective function in state objection function
ii. The construction of the model, been it additive, linear, proportional, divisibility, deterministive or non-negativity
iii. Derivation of the solution
iv Test and evaluate the model
v. Interpretation and maintenance of the solution

b. Maz Z =2000x + 3000y
i S.t $6x + 4y \leq 24$--------1
and $x + xy \leq 6$ ----------2
when $x, y \geq o$.

ii Solve 1 and 2 as following
1st, $6x + 4y = 24$
$6(0) + 4y = 24$ $-4y - 24$
For pt A (0,6)
Also for $y = 0$ -> $6x + 4(0) = 24$
-> $6x = 24, x = 4$
From pt B (4, 0)

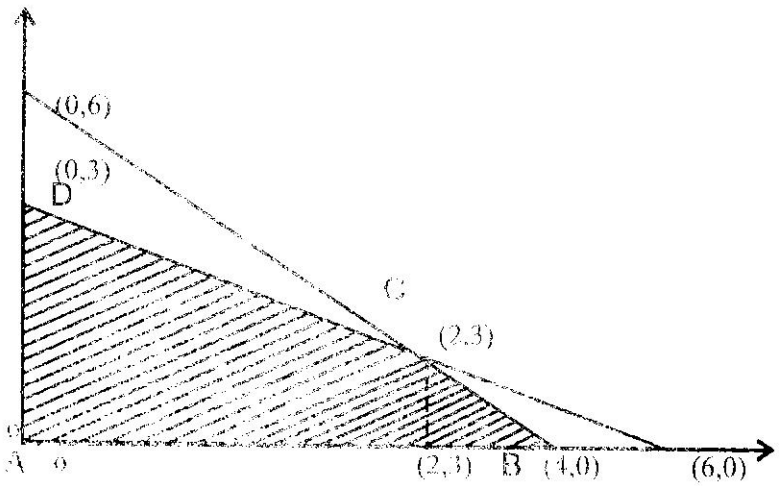
For the 2nd line x + 2y = 6

From x=0 -> 2y = 6 -> y = 3

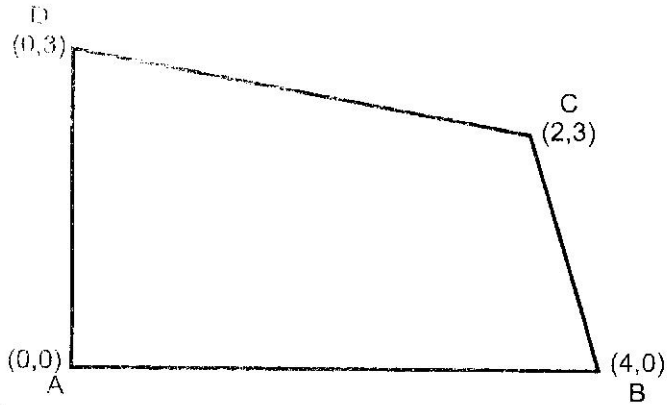
A(0,3)

Also for y = 0 -> 2(0) + x = 6 ->x=6

B(6, 0)



To determine the optimal production, we have the following, from the graph above



at Pt A (0,0)

Z = N0,000

at Pt (0,3)

Z = 2000(0) + 3000(3)

= N9000

at Pt (2,3)

Z = 2000(2) + 3000(3)

= 4000 + 9000

= N 13,000

The interior and exterior parts to produce the max profit, we have 2 for the interior and 3 for the exterior part

Question 75

a. Obtain the optimal plan of transporting Onifade Cement from four depots A, B, C and D to four destination I, II, III, IV. The availabilities of A,B,C and D are 3, 4, 5 and 8 units respectively and the requirement at I, II, III, IV, are 9,5,4 and 2 units respectively.

The cost of transportation for the cement between depots and the destination are given below:

		Destination			
		I	II	III	IV
	A	8	5	2	1
	B	5	8	6	2
	C	6	9	5	3
D	2	4	1	5	

i. Use the above cost to determine the maximum item to carry through different route using least cost method.

ii. Determine the initial cost allocation.

b. What is Dummy row and Dummy Column?

Dept \ Best	I	II	III	IV	Supply
A	1 [8]	2 [5]	- [2]	- [1]	3
B	- [5]	3 [8]	- [6]	- [2]	4
C	- [6]	- [9]	3 [5]	2 [3]	5
D	8 [2]	- [4]	- [1]	- [5]	8
Demand	9	5	4	2	20 \ 20

i. The initial cost allocation as given above is shown on the table and the cost is

= 1(8) + 2(5) + 3(8)

= 1(6) + 3(5) + 2(3) + 8(2) = 85

b. Dumy Row and Dummy Column - occurs where the special transportation condition is violated, that is when the total availability is not equal to the total requirement and vice versa and when such things occurs, we introduce the dummy row and column so as to be able to balance the transportal problem and solve the problem i.e case (i) When {ai<{ri here we introduces dummy row and similarly where {r;<{ai we introduce the dummy column.

OR

	I	II	III	IV	Availability
A	- [8]	1 [5]	- [2]	2 [1]	3
B	4 [5]	- [8]	- [6]	- [2]	-4
C	1 [6]	4 [9]	- [5]	- [3]	5
D	4 [2]	- [4]	4 [1]	- [5]	8
Demand	9	5	4	2	20

The total cost (c) using the least cost method we law
TC = 4 (1) + 2 (1) + 4 (2) + 4 (5) + 1 (5) + 1 (6) + 4 (9) = 81

ii. The initial cost allocation are the cost in the bracket above or cell above (8,5,2,1) (5,8,6,2) (6,9,5,3) and (2 ,4 ,2,5)

Question 76:

a. What are the assumption of a transportation model.

b. Obtain an optimal solution for the transportation plan below:

Solution

Assumptions of a transportation model is given as follows.

(i). The objective function is

Min Z = m n ci; xi;

i=i j=i

s.t. the following

(ii). The supply constraint

i.e n xi; = aj

i=i

(iii). The Demand constraint

i.e m xi; = r;

i=i

(iv). The non-negatively constraint must be obayed i.e xi; ≥ o

(v). The special transportation condition must be adhead to i.e m n

ai = rj

and the number of occupied cell must be = m t n-1 i=i j i

b.

	A	B	C	D	Supply
1	80 [4]	- [7]	20 [7]	- [1]	100
2	- [12]	- [3]	100 [8]	100 [8]	200
3	- [8]	90 [10]	- [16]	60 [5]	150
	80	90	120	160	450 \ 450

The optimal solution is given as follows: From the table, since the number of occupied cells is m + n - 1 = 3 + 4 - 1 = 6

--> The optimal solution can be given as follow

--> 80 (40) + 20 (7) + 70 (3) + 130 (8) + 120 (16) + 30 (5)

= 320 + 140 + 210 + 1040 + 1920 + 150 =3780

Or

Destination / Factory	A	B	C	D	Supply
1	80 [4]	- [7]	20 [7]	- [1]	100
2	- [12]	- [3]	100 [8]	100 [8]	200
3	- [8]	90 [10]	- [16]	60 [5]	150
Demand	80	90	120	160	450 \ 450

-------(B)

B is more appropriate ∴ it is the correct solution

∴ The optimal solution = 80 (4) + 20 (7) + 100 (8) + 100 (8) + 90 (10) + 60 (5)

= 320 + 140 + 800 + 800 + 900 + 300 = 3260

Question 771:

A. Consider the following project with the optimistic most likely (m) and pessimistic (b) estimation of duration in months is given below

Task	Preceded by	duration		Estimation
		a	m	b
A	-	5	8	11
B	-	5	10	1
C	-	15	18	33
D	A	25	43	33
E	B	2	5	8
F	C	6	1	44
G	C	18	22	47
H	A	17	25	31
I	D,E,F	12	12	29
J	G	10	O	O
K	H,I,J,			

i. Draw PERT Network diagram for the project.

ii. Determine the critical part of the project and hence the expected

completion tone of the Project.

b. Define the following:

I. Earliest event time

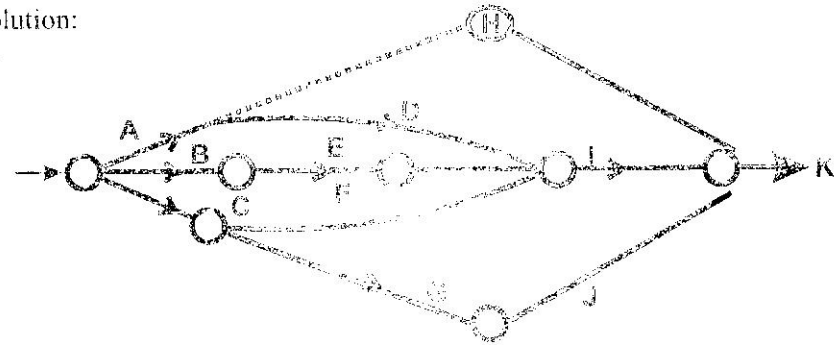
ii. Latest event time

iii. Slack time

iv. Total float.

Solution:

ai.



(Ii) The Critical path of the project is as follow

(a) A H K

(b) A D I K

(c) A E I K

(d) E F I K

(e) C G J K

And the expected completion time of the projects is given below using $t = \frac{(a+4b+c)}{6}$ make a = a, m = b, b = c

Note: The estimated time is incorrect.

(bi.) Earliest event time - This is the earliest this is an activity can start infringing on immediate predecessor requirement

ii. Latest event time - This is the latest time an activity can begin without delaying the whole project.

iii. Slack time - Is the longest time path through the network.

iv. Total float - is the amount of time by which its duration could be extended before you become critical.

And it is also the same as the amount by which its duration could be extended without affecting the total project time.

Question 78:

a. Name two important monitoring review and evaluation technique and state their uses.

b. Explain the concept of 'queuing'

c. State the 'duality theory' in operations Research.

d. What is traffic intensity in queue theory?

e. Define linear programming model.

Solution;

(a) The two important monitoring review and evaluation technique is the CPM (Critical Path Method) and the PERT (Programm Evaluation and Review Tech). There uses are as follows - For planning, Moniting, Scheduling and Controlling of the project or programm, also used for the following:

(i) To known when the entire programme or project will be completed.

(ii) To known what the critical activities or tasks in the programme are.

(iii) To know what the None - Critical activities are.

(iv) To know the duration for the completion of the projects intermay probability measured.

(v) To know whether the funds spent by any given date is equal to, more than or less than the budgeted amount.

(vi) To know whether there are enough resources to complete the project on time.

(b) The concept of "queuing" is noticeable in a queue system which can be divided into four elements as follows i.e the Arrival -Queue-Service-Outlet Query System.

Where:

Arrival is the element concerned with how items arrive in the system (the items include, people, component, cars, ships e.t.c).

Queue is the element concerned with what happens between the arrival of an item requing service and time when service is carried out.

Service is the element concerned with the time taken to service a customer which may be constant or varied. While outlet is the exit.

(c) Duality theorem: Corresponding to any L-P problem, their exit L.P problem called the DUAL of the original problem. The original problem is the PRIMAL. For instance, if we have the primal i.e original problem

$$\text{Max } Z = CX \text{ (objective)}$$
$$\text{S.t. } Ax \leq y$$
$$x \geq o \quad \text{constant}$$

Our DUAL will be

$$\text{Min } Z = y\,b$$
$$\text{S.t. } y\,A \geq C$$
$$Y \geq o$$

(d) Traffic Intensity in queue theory is the most important measure of a simple queue and is define as the ratio of the average arrival rate ( ), to the average service rate(N)i.e=p=≥and also called the probability of queuing on arrival. m

(e) Linear Programming (LP) model can be simply defined as a business decision Technique or mathematical technique which seeks to maximize an objective or goal used in providing solution to problems i.e profit or minimize an objective or goal e.g cost. It is an aspect of Mathematical Programming, it is called linear because all the decision variables (Xj) involved are raised to power one (X^1). It is called programming because it involves scheduling (mapping out) the activities.

Question 79a:

a. Customers arrive at one window drive in a bank according to Poisson distribution with mean 10 per hour. Service time per customer is exponential with mean 5 minutes. the space in found of the window including that for the serviced car can accommodate a maximum of 3 cars, others can wait outside this space.
   i. What is the probability that an arriving customer can drive directly to the space in front of the Window?.
   ii. What is the probability that an arriving customer will have to wait outside the indicated space.
   iii. How long is an arriving customer expected to wait before starting service.

b. Discuss the criteria for assessing the effectiveness of a production system.

Question 79b

a. What do you understand by forecasting? Are there any differences in meaning between the concept of forecasting and predicting? State the relevance of forecasting to production

b. Fit a straight line trend of the Y=a+bx by method of least squares to the following time series data. Compute the trend value and hence predict the value of production by 1976,2000.

YEAR	1960	1961	1962	1963	1964	1965	1966	1967	1968
PRO(000)	11	13	15	14	15	16	16	17	18

Provide Solution

Solution to Question 23a

a. Λ = 10

N = 5

C = 3

(I) § = $\frac{\Lambda}{C.N} = \frac{10}{5X3} = \frac{2}{3} = 0.66$

(ii) I - § = 1-0.66 = 0.333

(iii) Expected time of Arrival =Provide Solution

(b) Criteria for assessing the effectiveness of a production system include the following

- Design the product
- Planning and schedule
- Controll of the product
- Marketing the product
- Quality controll test e.t.c

Question 80:

(a) What are the requirements of a linear programming problem?

(b) Solve the following linear programming problem by graphical method.

Minimize Z = 20x + 10y

Subject to: $x + 2y \leq 40$

$3x + y \geq 30$

$4x + 3y \geq 60$

$x, y, \geq o$ (Provide Solution)

Question 81

(a) What are the requirement of a transportation model?

(b) State the assumption of a transportation model.

(c) The Production Manager of MOSIMI depot has five jobs A, B, C, D and E and five teams I, II, III, IV and V and has to assign one team to a job. The Production Manager assesses the number of hours each team would take to perform each job as follows:

	TEAM				
Job	I	II	III	IV	V
A	30	40	80	50	100
B	50	70	120	50	100
C	100	150	200	80	150
D	150	180	200	100	250
E	80	80	120	60	100

Use the Hunganan method to assign the job so as to minimize total number of hours of performing the Jobs.

(Provide Solution)

Question 82

(a) Write short notes on the following:
- (i) critical path
- (ii) project crashing
- (iii) critical activities
- (iv) slack time

(Provide Solution)

Question 83

(a) What are the assumption of games theory?

(b) Define the following terms of a competitive environment
- (i) Saddle Point
- (ii) Dominance Rule
- (iii) Zero Sum Game

(c) Two nation negotiate on the tonnage price of rice. The joint strategies with relative selling prices are as follows:

		BUYER				
	B_1	B_2	B_3	B_4	B_5	B_6
SELLER S_1	6	9	18	6	-12	15
S_2	12	15	9	-6	15	12
S_3	21	12	24	9	18	18
S_4	9	12	21	6	3	15

(I) Determine the optimal strategy for each player.

(ii) What is the value of the game? (Provide Solution)

Question 84

(a) Explain the following terms in Network Analysis.
- (i) Critical Path Method (CPM)
- (ii) Program Evaluation Review Technique (PERT)

(b) Using the table given below, determine:
- (i) the critical path and the shortest duration.
- (ii) the total float of all non-critical activity.

Activity	Preceeding Activity	Duration
A	-	6
B	-	4
C	A	3
D	A	6
E	A	4
F	C	3
G	D	5
H	B, E	7
I	H	3
J	F, G, I	7

(Provide Solution)

Question 85

A small project consists of seven activities whose three time estimate are given below:

Activity	Activity's Name	Time Estimate (days) t_o	t_i	t_p
1 - 2	A	3	3	12
1 - 3	B	3	6	9
1 - 4	C	6	6	15
2 - 5	D	3	3	3
3 - 5	E	6	15	24
4 - 6	F	6	15	21
5 - 6	G	9	15	27

Where t_o is the shortest time
t_i is the best estimate
t_p is the longest possible time

Determine:

(a) The expected duration and variance for each activity.

(b) Draw a Network diagram

(c) The expected project length (duration)

(d) The variance and standard deviation of the project length

(Provide Solution)

Question 86
If 5% of the products of a Manufacturing Company is known to be defective. Compute the probability that 50 randomly selected items from the company's warehouse will contain:

(a) Exactly two defectives
(b) At least two defectives
(c) Exactly three defectives
(d) At most three defectives.

(Provide Solution)

Question 87:
(a) Under what condition is Assignment model applicable?
(b) Suppose four professors are each capable of teaching any of the four courses operations research (OR), Information Management System (IMS), Law and Management (MGT). However, the average weekly class reparation time in hours for any course varies from professor to professor. Since the professors' department is highly research oriented, the dean would like to assign each professor one and only one course to minimize the total of course preparation times. Preparation time for each course by each professor is given below:

Courses

Professors	**OR**	**IMS**	**LAW**	**MGT**
Dajuma	2	10	9	7
Bello	15	4	14	8
Chike	13	14	16	11
Atanda	4	15	13	9

(i) Interpret the value **'16'** in the above time matrix.
(ii) Formulate the Dean's problem as an Assignment problem, and provide the solution using the Hungarian method. (Provide solution)

Question 88:
(a) State the assumptions of the basic EOQ model and discuss why it is not a realistic model.
(6) A company recruits 1000 graduate trainees who leave at a steady rate of 200 per year. The graduate trainee are recruited in groups and put through a training course. Each course costs N150,000 to run. Meanwhile, if the graduate trainees are not needed immediately on the job, this costs the company N5,000 per graduate trainee.

(I) How many graduate trainee should be engaged onto each training course?
(ii) How many courses per year?
(ii) What is the minimum total cost of training the graduate trainee?

(Provide solution)

Question 33:
(a) Explain what is meant by **'feasible regions'** in graphical solution method of linear programming.
(b) Apaper mill produces two grades of paper namely X and Y. Because of raw material restrictions, it cannot produce more than 400 tons of grade X and 300 tons of grade Y in a week. There are 160 production hours in a week. It requires 0.2 and 0.4 hours to produce a ton of products X and Y respectively with corresponding profits of Rs.200 and Rs. 500 per ton. Formulate the above as a linear programming problem and solve graphically.

(Provide solution)

Question 89:
Compare and contrast

(i) A critical path and a slack part
(ii) An activity duration and an activity float
Consider a project having seven activities and the following precedence relationship:

Activity	Predecessor	Duration (weeks)
A	-	6
B	a	3
C	a	4
D	a	6
E	c	4
F	b	2
G	d,e,f	2

(I) Draw a network of the project and identify the critical path for the project. Hence the duration of the project.
(ii) Calculate the floats for all the relevant activities in the network.

(Provide solution)

Question 90:
The Management of a company has asked the head of its research unit to advise on the best decision to make based on the given payoff table below:

	State of Nature	
Decision	**O1**	Q2
Recruit Professionals (Q 1)	600	300
Train me Existing Staff (Q2)	8000	400
Hire Consultants (Q3)	2000	800

Required:
Advise the management on the best decision using:
(A) Maximin criteon
(c) Minimax criterion
(d) Minimax reget criterion
(e) Expected monetary values for each action using probability of 6. and probability of 9, as 0.4 and 0.6 respectively. (Provide solution)

Question 91:
(a) What do you understand by proactive inventory system?
(b) Outline the factors affecting the adoption of proactive inventory system in Nigerian business environment.
(c) A company has a demand for a product at a steady rate of 1,500 units per week. it is observed that the cost of placing an order is N8 and 15k to hold a unit for a year. Calculate the:
(i) batch size to minimize inventory cost.
(ii) number of orders to be placed per year.
(iii) total cost of the firm's stock policy.
(Provide solution)

Question 92:
A company embarked on a project with an initial cost of N250,000 which generates the cash flow of N80,000, N75,000, N60,000, N45,000 and N40,000, respectively for the five-year life span of the project.

(a) Determine the Net Present Value(NPV) of the project if the annual rate of return of 10 per cent is allowed.
(b) Advise (with reasons) the company on the profitability of the project.
(c) Calculate the internal rate of Returns of the project by interpolation.
(Provide solution)

Question 93:
a. Customers arrive at one window drive in a bank according to Poisson distribution with mean 10 per hour. Service time per customer is exponential with mean 5 minutes. The space in found of the window including that for the serviced car can accommodate a maximum of 3 cars, others can wait outside this space.
i. What is the probability that an arriving customer can drive directly to the space in front of the window?.
ii. What is the probability that an arriving customer will have to wait outside the indicated space.
iii. How long is an arriving customer expected to wait before starting service.
b. Discuss the criteria for assessing the effectiveness of a production system.
(Provide solution)

Question 94:
ai Define an inventory and state its model
(ii) When does an inventory problem exists
(iii) If the demand for raw materials in a manufacturing industry is 10,000 tons per year
the carrying cost is =N= 0.75 per unit and the ordering cost is "N" 50 per unit cost
Determine the following:
(i) the economic order quantity
(ii) The no of order per year
(iii) Time between order within a year
(iv) The total cost

b. A contractor had to supply 10,000 bearing per day by an Automobile manufacturer. He found that when. he starts production he can produce 25,000 tons per day, the cost of hiding bearing stock is 2 kobo per annum and the set up cost of production run is =N= 18.00k. How frequently should the production value be made? (Provide solution)

Question 95:
a (i) What is a queue theory (ii) What is the difference between a queue process and queue situation (5 marks)
(b) In a large maint nce department, filters drawn frm the part stores which is at present staffed by one store man. The maintenance foreman is concerned about the time spent by filters getting parts and want to know if the employment of a store labourer would be worth while. On investigation it was found that
(i) a simple queuee exists
(ii) Filter cost =N=6.5 per hr
(iii) The store man cost ^N=5 per hr and can deal, on average, with 10 staff per
(iv) A store labourer could be employed at =N'=3 per hour and would increase the capacity of the store to 12 per hr (10 marks)
(c.) In a tree channel system, the rate of service at each channel is five customers per hr, and customers arrive at a rate of 12 per hr.
(i) What is the probability that there exist five customers in the system?
(ii) What is the average number of customers in the system?
(iii) What is the average number of customers in the queue including time when, there is no queue

(Provide solution)

Question 96:

a, Define simplex method in solving linear programming problem and when do we apply this method

(ii) List all the procedure involve in using simplex method for solving linear programming problem.

(iii) Consider the linear programming model below:

Maximize $9x + 12y$ (contribution) ,

Subject to $4x + 2y \leq 80$ (machine hours)

$2x+5y \leq 120$ (drilling hours)

$x, y, \geq o$

You are required to use simplex method to determine how many of each of x and y should be made to maximize contribution

(b) ALAMTEX group of company manufactures three (3) products, tanks, trays and tubes each of which pass through three processes x, y and z. Given below, the time required for each product in each process and for a certain production period, the total process time available. The contribution profit of each product are =N=2.00, ==N=3.00 and =N=. 00 per unit respectively

Process hour per unit

Process	Tanks Tray	*Tubes*	Total process	Hours available
X	5	2	4	12,000
Y	4	5	6	24,000
Z	3	5	4	18,000

You are required to find

(I) How of each product should be produced to make maximum profit and state the profit figure.

(ii) State how much slack time, if any is available in the processes

(iii) Interpret the shadow price

(Provide solution)

Question 97:

i. Define transportation model and state the constrains that attached to the model

Ii State when do we have degenerating problem in transportation model with examples

b. A ladies fashion shop wishes to purchase the following quantities of summ dresses.

Dress size	I	II	III	IV
Quantity	100	200	450	150

Three manufacturers are willing to supply dresses. The quantities given below are the maximum they are able to supply of any combination of orders for dresses

Manufacturer	A	B	C
Total Quantity	150	450	250

The shop expects the profit per dress to vary with the manufacturer as given below

Manufacturer	I	II	III	IV
A	*2.5*	*4.0*	*5.0*	*2.0*
B	*3.2*	*3.5*	*5.5*	*1.5*
C	*2.0*	*4.5*	*4.5*	*2.5*

You are require to

i Use the transportation technique to solve the problem of how the orders should be placed on the manufacturers by the fashion shop in order to maximize profit.

ii. Explain how you know that there is no further improvement possible, showing your workings.

(Provide solution)

Question 98:

i (i) Describe the component of time series and give detail analysis of each one of the components

ii) List and explain in detail three ways by which a trend can*be* estimated in a time series

ii. The table below shows the number of murders (in millions) in Nigeria for the year 1985 to 1995 construct a .4 yrs central moving averages for the data and use the method of semi average, obtain the trend values for the data by taken the average as the median.

Yrs	1985	1986	1987	1988	1989	1990
Murderers in millions	19.0	20.6	20.1	20.7	21.5	23.4
Yrs	1991	1992	1993	1994	1995	
murderers in millions	24.7	23.8	24.5	23.3	21.6	

(Provide solution)

Question 99:

1a. Write short notes on the following:

(I) Assumption of linear programming models.

(ii) Steps for formulating linear programming models.

(b). A Lagos based company makes two types of ornaments (type A and B) using brass and copper. Types A contain 100 grammes of copper and 600 grammes of brass and it sold for ^100 per unit. Type B contains 500 grammes of copper and 200 grammes of brass and the selling price is 140 per unit. The company has 15 kilogrammes of copper and 12kilogrammes of brass in stock.

(i). Formulate this problem as a linear program.

(ii) Solve your model in b(i) above using simplex method.

(Provide solution)

Question 100:

a. You have been asked by the chairman of vision 2020 to deliver a paper: "Decision Theory and its importance to the private sector in 2020". Discuss and highlight the functional areas of management where knowledge of decision theory can be used.

b. A client asks an estate agent to sell three properties A, B and C for him and agrees to pay him 5% commission on each sale. He specifies certain conditions. The agent must sell property A first, within 60 days, before he can receives his 5% commission on that sale. He can either backout at this stage or nominate and try to sell one of the remaining two properties within 60 days. The agent only has the opportunity to sell the third property if his sells the second one within 60 days. The table below gives the prices, selling costs incurred by the estate agent whenever a sale is made.

Property	Prices of property	Selling costs	Probability of sale
A	₦12,000.00	₦400.00	0.7
B-	₦25,000.00	₦225.00	0.6
C	₦50,000.00	₦N 450.00	0.5

(1) draw up an appropriate decision tree for the estate agent

(ii) What is the estate agents best strategy under EMV approach.

(Provide solution)

Question 101:

a. List at least six assumptions that are usually made when dealing with sequencing problems?

b. Six jobs A, B, C, D, E and F have arrived at one time to be processed on a single machine. Assuming that no new jobs arrive thereafter

Job	A	B	C	D	E	F
Processing time (minutes)	7	6	8	4	3	5

Determine the:

(i) Optimal sequence using SPT rule

(Ii) Completion time of the jobs

(iv) Average in-process inventory. (Provide solution)

Question 102:

a. Define transportation problem

b. As the human resources manager of a consultancy firm and faced with problem of allocation your client at different sites within the country. The firm has three network branches and three clients. The first branches (A) is ready to supply 10 staff; branch (B), 20 staff and branch (C), 30 staff; while client X needs 15 staff, client Y, 20 staff and client Z, 25 staff. You are provided with the unit transport costs as shown below:

Clients

Branches	X	Y	Z
A	100	N80	N 110
B	N70	N 120	N80
C	N90	N60	N50

(i) Generatel the transportation tableau for this problem

(ii) Optimize on your solution obtained in b(i) above, if possible. (Provide solution)

Question 48:

a. List and explain the principal cost associated with inventory.

b. Given the following variables in inventory model

D	=	Annual demand
Q	=	Quantity ordered
P	=	Cost of placing an order
H	=	cost holding one unit in inventory
C	=	unit cost of items in inventory
I	=	Annual inventory carrying charge

(i) Write the equation of the total inventory cost and hence derive the economic ordered quantity from the relation

(ii) Under what condition(s) is the EOQ model applicable?

(Provide solution)

Question 103:

a. Compare and contrast, critical path method (CPM) and performance Evaluation Review Techniques (PERT).

b. A project schedule has the following characteristics.

Activity	1-2	1-3	2-4	3-4	3-5	4-9	5-6	5-7	6-8	7-8	8-10	9-10
Time (Days)	4	1	1	1	6	5	4	8	1	2	5	7

(I) Construct the network diagram for this problem

(ii) Determine the critical activities and the project completion time. (Provide solution)

Question 104:

1. Suppose we have data on the quarterly sales of LASU SPTS Departmental stores for 4 years as follows:

YEAR	QUARTERS (N '000)			
	1	2	3	4
2002	70	41	53	83
2003	78	44	48	85
2004	83	54	51	96
2005	85	49	54	89

Apply the ratio-to-moving averages method to eliminate the irregular variations and adjust the quarterly averages to obtain your seasonal indexes. Determine the effects of seasonality of sales in 2004.

(Provide solution)

Question 105:

a. Distinguish between one-way and two-way classification under the analysis of variance technique.

b. A sample of size n is selected from different k populations with the assumptions that the k populations are independently and normally distributed with means M1 M2 ..., Mk and a common

Variance. Let aij == the jth observation from the ith population, Sij = the deviation of the jth observation of the ith sample from the corresponding population mean and p: == effect of the ith population.

Show that aij = p + p; + £ij where u, = mean of all the u; Six different machines are being considered for use by the manufacturing firm. The machines are being compared with respect of tensile strength of the product. A random sample of 4 seals

from each machine is used to determine whether or not the mean tensile strength varies from machine to machine. This is illustrated below:

A	B	C	D	E	F
17	15	21	13	15	19
16	20	16	17	21	15
14	18	18	21	15	18
19	14	17	17	20	19

(ANOVA)

Perform the analysis of variance at the 5% level of significance and indicate whether or not the treatment means differ significantly. (Provide solution)

Question 106:

a. State six basic steps in hypothesis Testing.

b. Define the uniformly most powerful Test in Hypothesis Testing.

c. Given that

Yi = a + bxi + ci

And the understated data.

Xi	Yi
1	7
1	8
2	6
4	3

(i) Obtain the regression equation for the data.

(ii) Calculate the goodness of fit for the equation obtained in c(I) (Provide solution)

4a. Define the following terms:

(i) PERT

(ii) CPM

(iii) Precedence diagram

(iv) Slack

(v) Critical path

(vi) Events

(vii) Activities

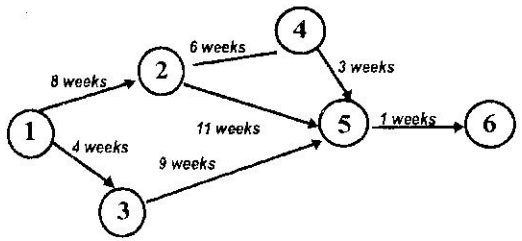
b. Given the following diagram, determine:

(i) The length of each path

(ii) The critical path

(iii) The expected length of the project

(iv) Amount of slack time for each path



(c) Using the network diagram:

(i) Compute the Earliest starting time(ES) and the Earliest finishing time(EF).

(ii) Compute the latest starting time(LS) and the Latest finishing time (LF).

(Provide solution)

Question 107:

The following table shows the consumer price indexes and whole sales price indexes for some commodities given for the years 1990 - 2000

Year	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000
Consumer price index	101.4	100.8	112.1	113.2	114.7	115.3	118.4	120.4	119.6	123.3	122.5
Whole sale price	100.3	102.1	115,7	112.4	100.3	112.4	112.4	116.6	118.2	119.7	118.8

(i) Construct various inderes (a) Various parts you know (Provide solution)

Question 108:

a. Distinguish between qualitative techniques and quantitative techniques of fore casting, and for each classification mention and briefly describe any three methods known to you

b. The ABC stores Ltd has kept records of its sales (in square yards) each year, along with the number of permits for new houses in its area.

YEAR	1999	2000	2001	2002	2003	2004	2005	2006	2006
PERMITS	18	15	12	10	20	28	35	30	20
SALES	13,000	12,000	11,000	10,000	14,000	16,000	19,000	17,000	13000

'i Use the least squares method to fore cast sales for 2008, assuming that 25 permits for new houses are to be issued next year.

ii What is the type and degree of association between permits for new houses and the sales of ABC STORES Ltd? (Provide solution)

Question 109:

a. Describe any five objectives of management in designing and installing a quality control programme.

b. Mention and briefly describe any four consequences of poor quality.

c. A quality inspector took twelve samples, each with five observations, of the length of time (in minutes) to process a loan application at a credit union, and computed the respective means of the samples as follows:

SAMPLE E	1	2	3	4	5	6	7	8	9	10	11	12
MEAN	10.28	10.17	10.07	10.09	10.31	10.16	10.12	10.27	10.34	10.30	10.05	10.

(i). Use the above information to set up a 3- sigma control chart to control the means of future loan processing times. It is known from previous experience that the standard deviation of the process is 0.02 minutes.(Note: Do all your calculations to 2 decimal places as necessary).

(ii). Mention your condition that indicates that a process is out of control when using control chart. (Provide solution)

Question 110:

a. What is production management?

b. Differentiate between production management and project management.

a. What is production management?

b. Differentiate between production management and project management.

c. Fit a trend line to the following data by the semi-average method and hence predict the likely sales of the company by 2005.

YEARS	1980	1981	1982	1983	1984	1985	1986
SALES	102	105	114	110	108	116	112

(Provide solution)

Question 111:

ABC of planning a new operation which will involve the following activities and time duration.

ACTIVITY	DESCRIPTION	DURATION a m B	PREDENT ACTIVITY
I	Design Logistic Solution	4 2 6	
II	Refine logistic Solution	3 4 5	I

III	Prefare planning Application	5 3 7	I	
IV	System Requirement	3 2 1	III	
v	Suppliers Briefing/Negotiation	2 2 6	I	
VI	Obtain planning Content	7 5 9	III	
VII	Mechanical/Handling Equipment	2 3 4	III.II	
IX	Building contrinction	1 4 1	V	
IX	Tractor procurement Curement	20 12 40	VI, VII	
x	Trailar procurement	2 4 6	VIII, IX	

Required
Draw the network for these tasks. (Provide solution)

Question 112:

a. What do you understand by forecasting? Are there any differences in mea between the concept forecasting and predicting? State the relevance of forecasti production

b. Fit a straight line trend of the Y=a+bx by method of least squares t following time series data. Compute the trend value and hence predict the valu production by 1976, 2000.

YEAR	1960	1961	1962	1963	1964	1965	1966	1967	196
PRO(000)	11	13	15	14	15	16	16	17	18

(Provide solution)

Question 113:

A Nigerian operational research consultant has formulated the marketing decision situation shown in this matrix format below for a shoe making firm in Lagos,

	STABLE(*SI*) ₦	TURBULENT(S2) ₦	UNSTABLES(S3) ₦
Maintain old shoe design	4500	1,*500*	1,750
Diversity into *new* product line	250	3500	-7,100
Acquire more Property	3,500	4500	650
Probability	0.3	0.5	0.2

(a) Analyze the situation completely with all the available criteria assuming a criterion of realism of 60%.

(b) A company maintains a chequering account at bank with a special provision for automatic borrowing. If the account balance falls below N2, 000 the bank automatically transfers N5000 into the account as a loan. The probability distribution of the company's net daily deposits and withdrawals are as follows.

Daily Deposits	Probability distribution	Daily withdrawals	Probability distribution
1000	0.21	2000	0.04
1500	0.37	2500	0.09
2000	0.25	3000	0.18
2500	0.17	3500	0.26
		4000	0.28
		4500	0.15

Required to simulate'10'days to determine:

(i) The average bank balance

(ii) The average time between '5000 loan transfer into the account, assuming that the initial balance is N2000 and Random

Deposit numbers: 83, 70, 06, 12, 59, 46, 54, 04, 51, 99 Random.

Withdrawal numbers:46,64,09,48,97,22,29,01,40,75

(Provide solution)

Question 14:

(a) Describe one technique used in determining an initial feasible allocation in a transportation problem.

(b)The ABC Company must by the end of the year open another plant to support its present three plants Ijebu-ode, Enugu, Ibadan

	BENIN	PORT-HARCOURT	ILORIN	SUPPLY
Ijebuode	3	6	12	300
Enugu	5	7	6	400
Ibadan	8	2	5	590
Demand	400	480	410	1290

(Provide solution)

Recommend an appropriate site using ; an initial feasible allocation technique.

Question 115:
(a) List; the assumptions of Markov Analysis

(b) The Undergraduates of Marketing Department in LASU conducted a market survey of three (3) competing products. Bourn vita, Milo and Nescao Sand discovered that they are faced with the following probability situations (100 customers were surveyed)

Of 100 customers currently using brand A, 80 will stay with A, 13 will switch to brand B, and 7 will switch to brand C.
- Of 100 customers currently using brand B, 67 will stay with B, 15 will switch'to brand A, and 18 will switch to brand C Of 100 customers currently using brand C, 90 will stay with C, 4 will switch j Over to brand A, and 6 will switch to brand B.

The current market share is (0.40, 031, 029)

Required:
(i) Develop the survey matrix for the three brands ;
(ii) Calculate the period market share for, Bournvita, Milo and Nescao ,
(Iii) Calculate the stable market share (Provide solution)

Question 116:
(a) What is a decision tree? State three advantages of decision trees and three states of decision environments.

(b) Given the following situation about Henry Investment PLC:

DECISION ALTERNATIVE	STATE OF NATURE		
	N1	N2	N3
Al	20000	15500	30000
A2	15000	25000	35000
A3	25000	18000	22000

Establish the kind of decision that will be taken if we employ:
(i) Hurwitz's criterion (let 85 **be** 75%)
(ii) Laplace criterion
(iii) Minimax criterion (Provide solution)

Question 117:
What is Project Management? State any two (2) techniques used in Project Management analysis and different between them.
(b)The table below shows the most optimistic, realistic and pessimistic time.

Activity Sequence	Time duration in days A	M	B
A	3	4	5
B	4	6	14
C	4	11	12
D	6	8	10
E	8	10	24
F	5	7	9
G	4	5	6
H	3	4	5

Estimate
(i) The expected time for each activity
(ii) Standard deviation for each activity
(Iii) . Standard deviation of the path of the activities. (Provide solution)

Question 118:
Suppose the Lagos State Bulk Purchasing Company is faced with some irresistible offers by one of its suppliers in order to encourage the company buy larger quantities of Product X. For any quantity purchased up to 400 units, the price per unit is N100.00 any quantity up to 800 units would attract a discount of 15%. For

any quantity up 1200 unit or more, the supplier is willing to give a discount of 20%. The Annual Demand of the product is put at 90,000 unit. Ordering costs are estimated at N45.00 per order while carrying costs include: Insurance N35.00 Storage costs N15.00 and Taxes & Rates N15.00.
Determine if it is profitable for the company to take advantage of the discount offers. If so, which of the price ranges would you recommend? Show your working clearly.

(Provide Solution)

Question 119:
The Lagos State University Council has recently approved the extension of the faculty of business administration building to include extra lecture theaters and offices. Relevant information relating to the tasks are provided below:

Table 1

Tasks	A	B	C	D	E	F	G	H	I	J
Proceeding Tasks	-	-	-	B	A	C	D,F	E	H,G	I
Time Duration In weeks	2	3	1	5	3	4	2	3	5	6

Required
(i) Draw the Network Diagram for the project
(ii) Determine the critical path and the project completion time
(iii) What are the unique features of the critical path method
(iv) In what ways is the Critical Path Method (CPM) different from the project
Evaluation review Techniques(PERT)

B. List and explain the criteria for assessing the effectiveness of the production system. (Provide solution)

Question 120:
Suppose the sales manager of Nigerian Breweries Ltd has given you the following sales information for the second and third quarter of year 2010.
Note: 1 month = 1 Period
Table 2

	2 Quarter 2010			3 Quarter 2010			
Month	April	May	June	July	Aug	Sept	October
Sales	480	500	490	600	525	620	?
Weights	0.10	0.14	0.15	0.18	0.20	0.23	

Required
(i) Use a three period moving average to predict the likely sales for August and September 2010.
(ii) Predict the sales for September 2010 using a five-period moving average.
(iii) Use the weighted moving average forecasting method to predict the likely sales figure for the month of October.
(iv) What advantage, if any, does the weighted moving average method have over the simple average method of forecasting? (Provide solution)

Question 121:
An MTN customer service manager intends to use the exponential smoothing method to forecast the number of calls he is likely to receive in the coming peak period of December 2010. The forecast for November was 5500 calls. The calls actually received in November was 5000. At a smoothing constant of x = 0.45, assist the manager in making his forecast for December 2010.
(Provide Solution)

Question 122:
The Managing Director XYZ Company, Miss Onyinyechi Nwachukwu is faced with the problem of how to optimally assign her four field sales supervisors to four geographical sales territories in an attempt to strengthen the company's dominance in these territories. An assessment of the strength of each supervisor in units of sales achievable per day has been carried out and results are as shown below:

	Estimated Sales in each Territory			
Sales Supervisor	A	B	C	D
1	82	39	60	50
2	72	84	80	92
3	59	44	30	76
4	60	80	90	42

Required:
(a) Assign the Sales Supervisors to the sales territories in the most optimally manner using the Branch and Bound Method. What is the value of your allocation?
(b) List the steps involved in solving an assignment problem when the Branch and Bound method is adopted.

(Provide Solution)

The Production Manager of Company XYZ, Miss Biodun Idowu has six jobs to process through the job shop on the available three machines (A, B & C). Each job has a specific routing and timing pattern. Suppose you have been consulted to Assist

in recommending an optimal sequencing pattern for the jobs through the machines, the following information may be required by you in respect of the job.
Tables 3

JOBS	SEQUENCES & TIME	TOTAL TIME	DUE DATE
1	C(2), A(3), B(2)	7	10
2	A(3), C(2), B(3)	8	9
3	B(2), A(1)	3	12
4	B(3), C(4), A(3)	10	13
5	C(3), A(3)	6	8
6	A(3), B(3), C(3)	9	10

Required:
(a) Optimally sequence the jobs through the job shop using the First Come, First Served (FCFS) RULE.
(b) Determine the following from the table 3 above
(i) The Number of late jobs, if any
(ii) The Average flow time
(iii) The Minimum lateness
Resolve any tie using First in System, First Served (FISFS) rule.
(c) Explain the concept of Job Sequencing and serie s. (Provide Solution)

Question 123:
a. The Performance of the Nigerian Manufacturing sector has not been too impressive in recent years. Identify and briefly discuss the problems and challenges facing the sector. What recommendations would you proffer as solution to these problems?
b. Using concrete examples, differentiate between a Continuous Production System and an Intermittent Production System.
c. List and explain at least five (5) function of the production manager.

(Provide Solution)

Question 124:
Consider the following linear programming problem:

Maximize: $z = -2x - x_2 + 3x_3 - 5x_4 - 8x_5$

Subject to:
$3x_1 - 5x_2 - 9x_4 + x_5 - 2x_5 \leq 6$
$2x_1 + 3x_2 + x_3 + 5x_4 + x_5 \leq 9$
$x_1 + x_2 - 5x_3 - 7x_4 + 11x_5 \leq 10$
$x_i \geq 0$ (i = 1,...,5).

(i) Write out the dual of this problem.
(ii) Solve the primal problem for optimal solution.
(iii) Determine the optimal solution to the dual.

(Provide Solution)

Question 125:
(a) Define the following precisely:
(i) A linear program
(ii) Proportionality test and additivity test.
(iii) Slack variables and artificial variables.

(Provide Solution)

(b) Examine the following problem for degenerary:

Max $z = x_1 + 5x_2$

Subject to
$5x_1 + 6x_2 \leq 30$
$3x_1 + 2x_2 \leq 12$
$-x_1 + 2x_2 \leq 10$
$x_i \geq 0$ (i = 1,2)

(c) Show that the dual of the dual problem is the primal problem.

(Provide Solution)

Question 126:
A metal products company produces four products as outputs, all made of metals: serving trays, ash trays, bookshelves and lunch boxes. It uses two commodities as inputs, sheet metal and labour. Input-Output relationships, resource available and sales prices of outputs are shown below:

Outputs / Inputs	Serving tray	Ash tray	Book Shelves	Lunch box	Resource (input) availabilities
Sheet metal	5	1	15	5	100
Labour	3	2	7	5	125
Sales price Per unit (N)	2	1	10	4	

Formulate and solve the problem as a linear programming problem.

(Provide Solution)

Question 127:
Consider a small factor which produces picture frames, cabinets and figurines. To produce those items only wood and labour are assumed to be required, but there are restrictions on the amounts of woods and labour available. The requirements, restrictions, and values of outputs are summarized below:

Units of	Units of wood	Unit value labour	of output
Per picture frame	3	1	8
Per cabinet	4	3	19

Per figurine	1	3	7
Available material	25	50	

Formulating as a linear program we have

Max $8x_1 + 19x_2 + 7x_3$

Subject to $3x_1 + 4x_2 + x_3 \leq 25$

$x_1 + 3x_2 + 3x_3 \leq 50$

$x1 \geq 0$ (i = 1,2,3).

Where x_1, x_2, x_3 respectively are the numbers of picture frames, cabinets and figurines produced in a given time period. Adding is lack variables we get the following tableaus:

	x1	x2	x3	x4	x5	
0	-8	--19	-7	0	0	
25	3	4	1	1	0	= M1
50	1	3	3	0	1	
475/4	25/4	0	9/4	19/4	0	
25/4	¾	1	⅛	¼	0	– M2
125/4	-5/4	0	9/4	-3/4	1	
150	5	0	0	4	1	
25/9	8/9	1	0	1/3	-1/9	=M3
125/9	-5/9	0	1	-1/3	4/9	

X1	X2	X3	X4	X5	
3	4	1	1	0	25
1	3	3	0	1	50
= M_1					
-8	-19	-7	0	0	0
¾	1	¼	¼	0	25/4
-5/4	0	9/4	-3/4	1	125/4
M_2					
25.4	0	-9/4	19/4	0	475/4
8/9	1	0	1/3	-1/9	25/9
-5/9	0	1	-1/3	4/9	125/9
= M_3					
5	0	0	4	1	150

i) Determine pivot matrices F_1, F_2 such that $M_2 = F_1M_1$, $m_3 = F_2M_2$ and hence, determine Q such that $M_3 = Qm_1$.

ii) What is the dual of the initial problem? Determine an optimal solution to the dual from the information given above.

iii) Interpret the dual problem. What are the dual activities and dual variables and dual variables?

Provide Solution

uestion 128:

(a) Given the following sequence of tableaus and information that some tries in M* are incorrect, indicate which columns of M* contain error.

14	-1	2	-3	1	0	0	0	0
2	-3	2	1	5	1	0	0	0
4	2	0	2	1	0	1	0	0

4	4	1	2	-4	0	0	1	0
6	-6	-3	3	1	0	0	0	1

M* =

20	3	5/2	0	0	0	1	½	0
0	-11/5	23/10	0	0	1	-7/5	9/10	0
0	3/5	-1/5	0	1	0	1/5	-1/5	0
2	6/5	1/10	1	0	0	2/5	1/10	0
0	-46/5	-21/10	0	0	0	-7/5	-1/10	1

Formulate, but do not solve. An assembled item consists of two metal parts. .e milling work can be done on different machines, milling machines, turret lathes, or an automatic turret lathes. The maximum output of each type of machine, if devoted exclusively to making one of the parts for one hour, is shown in the following table:

	Number of Machines	Maximum output per machine per hour	
		first part	second part
Milling machines	3	10	20
Turret Lathes	3	20	30
Aut. Trurret Lathes	1	30	80

Divide the work time of each machine to obtain the maximum number of completed ms per hour. (Hint: Apply the 5-step procedure in order to maximize the total mber of first and second parts. Note that the total number of first parts produced uals the number of second parts.)

Provide Solution