ILLUSTRATED MATHEMATICS

FOR

MANAGEMENT SCIENCES

VOL. 1

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PREFACE

This book is written for students, of management sciences. The most important meaning, and consequences of the basic statement are stated, restated, and discussed to make sure that the students understand them.

The material was developed over several years of classroom experience.

As is visual in a mathematics text, the exercises at the end of each section are vital parts of the material.

There are also problems that require the student to use the concept in constructing mathematical models for described situation. This is the process that transforms mathematics from our abstract intellectual exercise into a useful tool for solving everyday problem The computational sections in each of the chapters of the book have been very richly illustrated with worked examples. The book will serve as a companion to all students of Management Sciences.

Bello & Olateju

DEDICATION

TO

LATE
PA SUNMONU BELLO
AND EUNICE OLATEJU

TABLE OF CONTENT

.0 INDICES AND LOGARITHMS:

ILLUSTRATIONS/HINTS

n can be any positive integer while x and y can be any integer

1 **INDICES:** Facts, Laws and Formulae to make the topic easy, are stated below:

$b^x \times b^y = b^{x+y}$

$b^x \div b^y = b^{x-y}$

$(b^x)^y = b^{xy}$

$b^{x/y} = \sqrt[y]{b^x}$ OR $(\sqrt[y]{b})^x$

$b^{1/2} = \sqrt{b}$

$b^{1/n} = \sqrt[n]{b}$

$b^{-y} = \frac{1}{b^y}$

$b^0 = 1$

$$b^{-x/y} = \frac{1}{\sqrt[y]{b^x}}$$

NOTE: b^x → Index (pl = Indices), power etc

→ Base [They has to be the same for the law to be useful]

1.1 Worked examples on indices, with illustration on how the formulae stated above are use.

Example 1

$2^7 \times 2^4 = 2^{7+4}$

$= 2^{11}$

$b^x \times b^y = b^{x+y}$

Here: $b = 2$

$x = 7$

$y = 4$

Example 2

$3^8 \div 3^5 = 3^{8-5}$

$= 3^3 = 3 \times 3 \times 3$

$= 27$

$b^x \div b^y = b^{x-y}$

$b = 3$

$x = 8, y = 5$

Example 3

(a) $(5^4)^6 = 5^{4 \times 6}$

$= 5^{24}$

$(b^x)^y = b^{xy}$

$b = 5$

$x = 4, y = 6$

(b) $(256^{1/4})^2 = 256^{1/4 \times 2} = 256^{1/2}$

$= \sqrt{256} = \underline{16}$

$b = 256, x = 1/4, y = 2$

$b^{x/y} = (\sqrt[y]{b})^x$

$= \sqrt[y]{b^x}$

$\sqrt{64}$

$\sqrt{4 \times 4 \times 4}$

4

(b) $27^{\frac{1}{3}} = (\sqrt[3]{27})^1$

$(\sqrt[3]{3 \times 3 \times 3})^1$

$(3)^1$

$= 3^1 = \underline{81}$

Example 5

$64^{1/2} = \sqrt{64}$

$= \sqrt{8 \times 8}$

$= 8$

Example 6

(a) $81^{1/4} = \sqrt[4]{81}$

$= \sqrt[4]{3 \times 3 \times 3 \times 3}$

$= \underline{3}$

(b) $64^{1/6} = \sqrt[6]{64}$

$= \sqrt{2 \times 2 \times 2 \times 2 \times 2 \times 2}$

$= \underline{2}$

Example 7

(a) $2^{-4} = \frac{1}{2^4}$

$\frac{1}{2 \times 2 \times 2 \times 2}$

$= \frac{1}{16}$

(b) $3^{-3} = \frac{1}{3^3}$

$= \frac{1}{3 \times 3 \times 3}$

$= \frac{1}{27}$

$x = 2,$

$y = 3$

$b = 27,$

$x = 4,$

$y = 3$

$b^{1/2} = \sqrt{b}$

$b = 64$

$b^{1/2} = \sqrt{64}$

$b^{1/n}$ $b = 81,$ $n = 4$

Note: Look for the number that will multiply itself 'n' times to give the base number.

$b = 64$

$n = 6$

$b^{-y} = \frac{1}{b^y}$

$b = 2$

$y = -4,$

Note: If the index of a number is negative, then it implies the inverse of the number.

$10000^1 = 1$

$2^0 = 1$

mple 9

$= \frac{1}{\sqrt[3]{64^2}}$

$= \frac{1}{(\sqrt[3]{4 \times 4 \times 4})}$

$= \frac{1}{4^2}$

$= \frac{1}{16}$

mple 10

$(1/3)^y = 81$

$3^{-y} = 81$

$3^{-y} = 3 \times 3 \times 3 \times 3$

$3^{-y} = 3^4$

$-y = 4$

$y = -4$

$8^{x-1} = 16$

$(2^3)^x \quad 2^4$

$2^{3x-3} = 2^4$

$3x - 3 = 4$

$3x = 4 + 3$

$\therefore x = \frac{7}{3}$

$= 2\frac{1}{3}$

mple 11

$4^{x-1} \times 8^{x+1}$

$\times (2^2)^{x-1} \times (2^3)^{x+1}$

$\times 2^{2x-2} \times 2^{3x+3}$

$b^0 = 1$

$\therefore$ Anything raise to the power of zero is 1

$b^{-xy} = \frac{1}{\sqrt[y]{b^x}}$

$b = 64$

$x = 2, y = 3$

Either x or y could take the negative sign

Write the left hand side in index form

$b^{-y} = \frac{1}{b^y}$

convert 81 to index form with base 3.

Since the bases are equal the indices should also be equal.

Make sure they have the same base,

$8 = 2^3 = 2 \times 2 \times 2$

$16 = 2 \times 2 \times 2 \times 2$

Make sure the bases are similar first

$4 = 2^2$

$8 = 2^3$

$= 2^{x+5-2+5x+3}$

$= 2^{6x+1}$

Example 12

(a) $2^{2y} - 6(2^y) + 8 = 0$

$\therefore (b)^2 - 6(b) + 8 = 0$

$\therefore b^2 - 6b + 8 = 0$

$\therefore b^2 - 4b - 2b + 8 = 0$

$\therefore (b^2 - 4b) - (2b - 8) = 0$

$\therefore b(b - 4) - 2(b - 4) = 0$

$\therefore (b - 4)(b - 2) = 0$

$\therefore b - 4 = 0$ OR $b - 2 = 0$

$\therefore b = 4$ or $b = 2$

$\therefore b = 4$ or 2

BUT

$b = 2^y$

When $b = 4 = 2^2$

$\therefore 2^2 = 2^y$

$\therefore 2 = y$

$\therefore y = 2.$

When $b = 2 = 2^1$

$\therefore 2^1 = 2^y$

$\therefore 1 = y$

$\therefore y = \underline{1}$

(b) $5^{2y} + 2^2 \times 5^{y+1} - 125 = 0$

$\therefore (5^y)^2 + 4(5^{y+1}) - 125 = 0$

$\therefore (5^y)^2 + 4(5^y \times 5) - 125 = 0$

$(5^y)^2 + 4 \times 5(5^y) - 125 = 0$

$b^2 + 20b - 125 = 0$

From quadratic formula

$x = \dfrac{b \pm \sqrt{b^2 - 4ac}}{2a}$

$b^x \times b^y = b^{x+y}$

$(b^x)^y = b^{xy}$

$(2^y)^2 = 2^{2y}$

Let $b = 2^y$

$(2^y)^2 = 2^{2y}$

Let $b = 2y$

Solve the resulting equation by factorization or the use of quadratic formulae i.e.

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

We are using factorisation in solving this.

NOTE: - x - = +

+ x + = +

- x + = -

+ x - = -

From the above we take b to be equal to 2^y

From the above $b^x \times b^y = b^{x+y}$

$\therefore 5^{y+1} = 5^y \times 5$

Let $5^y = b$ as in the above example (12a).

where,

b = x (the quantity we're looking for)

$\therefore b = \dfrac{-(20) \pm \sqrt{(20)^2 - 4(1)(-125)}}{2(1)}$

$= \dfrac{-20 \pm \sqrt{400 + 500}}{2}$

$= \dfrac{-20 \pm 30}{2}$

$\therefore b = \dfrac{-20 + 30}{2}$ OR $\dfrac{-20 - 30}{2}$

$= \dfrac{10}{2}$ OR $\dfrac{-50}{2}$

$= 5$ or -25

But

$b = (5)^y$

When $b = 5$

$\therefore b = 5 = 5^y$

$\therefore 5^1 = 5^y$

$\therefore y = \underline{1}$

12 PRACTICE EXERCISE

QUESTIONS ON INDICES

Find the values of

(1) $(-32)^{-4/5}$ (2) $(1000)^{-4/3}$

(3) $(625)^{-3/4}$ (4) $(27x3)^{-2/3}$

(5) $(16)^{-3/4}$ (6) $(1/3)^0$

(7) 17^1 (8) $(16/25)^{-1/2}$

(9) $16^{3/2}$

Solve the equations

10. $5^x = 625$
11. $25^{(x+1)} = 625$
12. $4/2^x = 4 \cdot 2^x$
13. $(2)^{3x} = 0.25$
14. $2^{2x} + 4(2^x) - 32 = 0$
15. $3^{2x} - 4(3^{x+1}) + 27 = 0$
16. $2^{2x} - 4 = 0$

$\therefore$ the coefficient of b^2 in the above equation = a (in the quadratic formula) = 1

$\therefore$ the coefficient of b in the equation = b (in the formula = 20

$\therefore$ The coefficient of constant c in the quadratic formula = -125 (in the equation)

$\therefore x = b, a = 1, b = 20, c = -125$

Substitute for the above in the quadratic equation.

N.B: Details of the derivation and uses of the quadratic formula will be seen under equation.

NOTE: - 25 has no value in indices

1.2.1 **Relationship Between Indices and Logarithms.**

The logarithm to base b of a number **N** is the index **x** to which b must be raised to give **N**. Hence, if $N = b^x$, therefore x is the logarithm to the base b of N

So, indices and logarithms are related in the sense that $\log_b N = x$ is equivalent to $N = b^x$ exhibit properties which are closely related to those of the law of indices.

1.2.2 Facts, laws and principles of logarithms to make the topic easy are stated below;

Let $N = b^x$, $M = b^y$

$\therefore NM = b^x \times b^y = b^{x+y}$

$\therefore N/M = b^x / b^y = b^{x-y}$

$\therefore N^y = b^{xy}$

$\therefore \text{Log}_b N^y = xy$

$\therefore \text{Log}_b N^y = y \log_b N$

INDEX FORM	LOGARITHMIC FORM
$b^x \times b^y = b^{x+y}$	$\text{Log}_b N + \text{Log}_b M = \text{Log}_b NM$
$b^x \div b^y = b^{x-y}$	$\text{Log}b^x \quad \text{Log}b^M \quad \text{Log}b^{(x/M)}$
$(b^x)^y = b^{xy}$	$\text{Log}_b N^y \quad y\log_b N$
$b^1 = b$	$\text{Log}_b b = 1$
$N = b^x$	$\text{Log}_b N = x$
$b^0 = 1$	$\text{Log}_b 1 = 0$

1.2.3 Change of Base.

From the above, $N = b^x \therefore \log b^N = x$

Because: $N = b^x$

$\therefore \log_c N = \log_c b^x$

$\therefore \log_c N = x \log_c b$

$\therefore x \log_c b = \log_c N$

$\therefore x = \dfrac{\log_c N}{\log_c b}$

substitute for $\log_b N = x$ in the above.

$$x = \frac{\log_c N}{\log_c b}$$

$$\therefore \log_b N = \frac{\log_c N}{\log_c b}$$ is the rule for changing from one base to the other.

1.2.4 Worked examples on logarithms, with illustration on how the formulae and laws are applied in solving problems.

Example 13

Change the following from their index form to their logarithmic forms:

(a) $32 = 2^5$

$\therefore \log_2 32 = 5$ in the logarithmic form

(b) $27 = 3^3$

$\therefore \log_3 27 = 3$ is the logarithmic form

(c) $16 = 256^{1/2}$

$\therefore \log_{256} 16 = 1/2$

(d) $1 = 149^0$

$\therefore \log_{149} 1 = 0.$

(e) $7 = 7^1$

$\therefore \log_7 7 = 1$

$N = b^x$

$\therefore \log b^N = X$, $N = 32$, $b = 2$ (base), $x = 5$ (index)

$N = 27$, $b = 3$, $x = 3$

$N = 16$, $b = 265$, $x = 1/2$

$N = 1$, $b = 149$, $x = 0$

Note anything raise to power zero is one.

$N = 7$, $b = 7$, $x = 1$

Recall $\log b^b = 1$,

$\therefore \log_7 7 = 1.$

Example 14

Change the following from their logarithmic form to their index form.

(a) $\log_3 81 = 4$

$\therefore 81 = 3^4$ (index form)

b $\log_{10} (0.00001) = -5$

$\therefore 0.00001 = 10^{-5}$

(index form)

c $\log_x 64 = 3$

$\therefore 64 = 4^3$ (index form)

d $\log_{1/2} (0.125) = 3$

$\therefore 0.125 = (1/2)^3$

(index form)

Example 15

Solve the equations

(a) $\text{Log}_x 64 = 6$

$\therefore 64 = x^6$

$\therefore 2^6 = x^6$

Note $2^6 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$

64

2^6 x^6

2 x

x = 2

b $\frac{1}{2} \text{Log}_x 81 = 2$

$\text{Log}_x 81 = 2$

$\text{Log}_x 9 = 2$

$\text{Log}_3 9 = 2$

9 x^2

$\therefore 3^2 = x^2$

$\therefore 3 = x$

$\therefore x = 3$

$\log b^x = x$

$\therefore N = b^x$

N = 81, b = 3, x = 4

N = 0.00001

b = 10, x = -5

N = 64

b = 4, x = 3

N = 0.125,

$b = 1/2$, x = 3

Recall:

$\text{Log}_b N = x$

$\therefore N = b^x$

Note: If the indices are equal the base must also be equal and vice versa

$\therefore 64 = 2^6$

$\therefore 2^6 = x^6$ x = 2

$y\log_b N = x$

$\therefore \text{Log}_b N^y = x$

Note: $81^{1/2} = 9$ (From indices).

Recall: If the indices are equal the bases must be equal and vice versa.

Express b in terms of x if $\text{Log}_3 (b-1) = x$.

Solution

$\text{Log}_3 (b-1) = x$

$\therefore (b-1) = 3^x$

$\therefore b - 1 = 3^x$

$\therefore b = 3^x + 1$

$= \mathbf{3^x + 3^0}$

Example 16

Show that $\text{Log}_a b \times \log_b a = 1$

Solution

$\text{Log}_a b \times \text{Log}_b a = 1$

$\therefore \text{Log}_{ab}^{(b \times a)} = 1$

$\therefore \text{Log}_{ab} (ab) = 1$

$\therefore \text{Log}_{ab} ab = 1$

$\therefore 1 = 1$

Example 17

$\log_3 27 + 2\log_3 9 - \log_3 54$

$\therefore \log_3 27 + 2\log_3 9 - \log_3 54$

$= \log_3 27 + \log_3 9^2 - \log_3 54$

$= \log_3 \left[\frac{27 \times 9^2}{54}\right]$

$= \log_3 \left[\frac{27 \times 9 \times 9}{54}\right]$

$= \log_3 (81/2) = \log_3 81 - \log_3 2$

$= \log_3 3^4 - \log_3 2$

$= 4 \log_3 3 - \log_3 2$

$= 4 \times 1 - \log_3 2$

$= \mathbf{4 - \log_3 2}$

Recall:

$\text{Log}_b N = x$,

$\therefore N = b^x$

$N = (b-1)$

$b = 3$

$1 = 3^0$

Note: $\text{Log}_b b = 1$

$\therefore \text{Log}_{ab} ab = 1$

To prove any equation the LHS = RHS

$\therefore 1 = 1$

LHS = Left hand side

RHS = Right hand side

$\text{Log}_b N + \text{Log}_b M$

$= \text{Log}_b NM$

$\text{Log}_b N - \text{Log}_b M$

$= \text{Log}_b (N/M)$

Note: $\log_b b = 1$

$\therefore \log_3 3 = 1$

Example 18

$\log_b 12.5 + \log_b 2$

$\log_b (12.5 \times 2)$

$\log_b 25$

$\log_b 5^2 = 2\log_b 5$

$= 2 \times 1 = \underline{2}$

$\log_b N + \log_b M = \log_b NM$

$\log_b b = 1$

$\therefore \log_b 5 = 1$

Example 19

$\log_b b^{10} \quad 10\log_b b$

10×1

10

$\log_b b = 1$

Example 20

Given that $\log_{10} 2 = 0.3010$, $\log_{10} 3 = 0.4771$ and $\log_{10} 5 = 0.6990$

a). $\log_{10} 37.5$

$\therefore \log_{10} 37.5 = \log_{10}(75/2)$

$= \log_{10} 75 - \log_{10} 2$

$= \log_{10}(25 \times 3) - \log_{10} 2$

$= \log_{10} 25 + \log_{10} 3 - \log_{10} 2$

$= 2\log_{10} 5 + \log_{10} 3 - \log_{10} 2$

$= 2 \times 0.6990 + 0.4771 - 0.3010$

$= 1.3980 + 0.4771 - 0.3010$

$= 1.8751 - 0.3010$

$= \underline{1.5741}$

$\log_{10} 37.5$ is the same as $\log_{10}(75/2)$

because $75/2 = 37.5$

$\log_b(N/M) = \log_b N - \log_b M$

$\log_b NM = \log_b N + \log_b M$

Note: $75 = 25 \times 3$

$= 5^2 \times 3$

Substitute

$\log_{10} 5 = 0.6990$

$\log_{10} 3 = 0.4771$

$\log_{10} 2 = 0.3010$

b. $\log_{10} 35 - \log_{10} 17.5$

$= \log_{10}(35/17.5)$

$= \log_{10} 2$

$= \underline{0.301}$

$\log_b N - \log_b M = \log_b(N/M)$

c. $\log_{10} 60 + \log_{10} 25$

$= \log_{10}(12 \times 5) + \log_{10}(5 \times 5)$

$= \log_{10}(4 \times 3 \times 5) + \log_{10}(5^2)$

$= \log_{10} 4 + \log_{10} 3 + \log_{10} 5 + 2\log_{10} 5$

$= 2\log_{10} 2 + \log_{10} 3 + \log_{10} 5 + 2\log_{10} 5$

$60 = 12 \times 5 = 4 \times 3 \times 5$

$25 = 5 \times 5$

$\text{Log}_b NM = \log_b N + \log_b$

$= 0.6020 + 0.4771 + 2.0970$

$= \underline{3.1761}$

d $\log_{10}\sqrt{32} = \log_{10} 32^{1/2}$

$= 1/2 \log_{10} 32$

$= 1/2 \log_{10} 2^5$

$= 1/2 \times 5\log_{10} 2$

2.5×0.3010

$= \underline{0.7525}$

Example 21

a) $\log_{10}(x^2 - 4x + 7) = 2$

Change from log form to index form

$\therefore (x^2 - 4x + 7) = 10^2$

$\therefore x^2 - 4x + 7 = 100$

$\therefore x^2 - 4x + 7 - 100 = 0$

$\therefore x^2 - 4x - 93 = 0$

From quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\therefore x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-93)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{16 + 372}}{2}$$

$$= \frac{4 \pm \sqrt{388}}{2}$$

$$\therefore x = \frac{4 + 19.70}{2} \quad \text{OR} \quad \frac{4 - 19.70}{2}$$

= 11.85 OR - 7.85

$+ \log_b P + \log_b Q + \ldots$

$\log_{10} 4 = \log_{10} 2^2 = 2\log_{10} 2$

$\log_{10} 5 + 2\log_{10} 5 = 3\log_{10} 5$

$= 3 \times 0.6990$

Recall

$\sqrt{b} = b^{1/2}$

$32 = 2 \times 2 \times 2 \times 2 \times 2$

$= 2^5$

Note: $\text{Log}_b N = x$

$\therefore N = b^x$

$N = x^2 - 4x + 7$

$x = 2$

$b = 10$

Check example

12(a and b) for detail on the working.

a = coefficient of $x^2 = 1$

b=-4

c=-93

Substitute in the formula.

$\sqrt{388}$ can be solve using common log thus

No	Log
$\sqrt{388}$	$\frac{2.5888}{2}$
1.2944	19.70

Antilog

$x^2 - 5x - 6 = 2^3$

$x^2 - 5x - 6 = 8$

$x^2 - 5x - 6 - 8 = 0$

$x^2 - 5x - 14 = 0$

$(x^2 + 2x) - (7x + 14) = 0$

$x(x + 2) - 7(x + 2) = 0$

$(x + 2)(x - 7) = 0$

$x + 2 = 0$ or $x - 7 = 0$

$x = \underline{\textbf{-2 or 7}}$

c. $\text{Log}_{10}(x - 8) = 1 - \log_{10}(x + 1)$

$\therefore \log_{10}(x - 8) = \log_{10}10 - \log_{10}(x - 1)$

$\therefore \log_{10}(x - 8) = \log_{10}({}^{10}/_{x+1})$

$\therefore x - 8 = {}^{10}/_{x+1}$

$\therefore 10 = (x + 1)(x - 8)$

$= x^2 - 7x - 8$

$0 = x^2 - 7x - 8 - 10$

$\therefore x^2 - 7x - 18 = 0$

$\therefore x^2 + 2x - 9x - 18 = 0$

$\therefore (x^2 + 2x) - (9x + 18) = 0$

$\therefore x(x + 2) - 9(x + 2) = 0$

$\therefore (x + 2)(x - 9) = 0$

$\therefore x + 2 = 0$ OR $x - 9 = 0$

$\therefore$ **x = -2 OR 9**

$x^2 - 5x - 6 = 0$

$b = 2$

$x = 3$

The remaining part is solve by factorization

$+2x - 7x = 5x$ the middle term

$(+2)(-7) = -14$ the constant

Recall

$\log_b b = 1$

Note $\log_b(N/M)$

$= \log_b N - \log_b M$

Now, $N = 10$

$M = x + 1$

$\log_{10}$ cancel out

Factorize $(+2)(-9) = -18$

$+2x - 9x = -7x$

1 Solve the following equations

(a) $\log_{10}(10 + 9x) - \log_{10}(11 - x) = 2$

(b) $\log_2(x - 3) + \log_2 x = 2$

2 Given that log2 = 0.3010

log3 = 0.4771 and log 7 = 0.8451

evaluate the following

(a) log 6 (b) log 21 (c) log 42

3 Express the following in index form

(a) log a = b (b) log a + 3 logb = 3

(c) log a + 1 = 0

4 Solve the equation: $(2x)^2 - 3 \times 2^x + 2 = 0$

5 Evaluate the following

(a) Log_9 99 (b) Log_7 707 (c) Log_3 8

2.0 SURDS

2.1 Surds are irrational numbers such as $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$ etc. They can be expressed in many different forms. Below are facts, rules and means of making surds easy to manipulate.

i. $\sqrt{a} \times \sqrt{a} = \sqrt{a^2} = \sqrt{a \times a} = a$

ii. $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$

iii. $\sqrt{a} \times \sqrt{a} \times \sqrt{b} = \sqrt{a \times a \times b} = a\sqrt{b}$

iv. $\sqrt{a} + 2\sqrt{a} + \sqrt{a} = 4\sqrt{a}$

v. $\frac{a}{\sqrt{b}} = \frac{a}{\sqrt{b}} \times \frac{\sqrt{b}}{\sqrt{b}} = \frac{a\sqrt{b}}{b}$ *(Rationalization)*

vi. $\frac{a}{b\sqrt{c}} = \frac{a}{b\sqrt{c}} \times \frac{\sqrt{c}}{\sqrt{c}} = \frac{a\sqrt{c}}{bc}$ *(Rationalization)*

vii. $\frac{a}{\sqrt{a} + \sqrt{b}} = \frac{a}{\sqrt{a} + \sqrt{b}} \times \frac{\sqrt{a} - \sqrt{b}}{\sqrt{a} - \sqrt{b}} = \frac{a[\sqrt{a} - \sqrt{b}]}{a - b}$
(Rationalization)

viii. Conjugate pairs of surds are
*1 $(a + \sqrt{b})$ and $(a - \sqrt{b})$
*2 $[a\sqrt{b} - \sqrt{c}]$ and $[a\sqrt{b} + \sqrt{c}]$
*3 $[a\sqrt{b} - c\sqrt{d}]$ and $[a\sqrt{b} + c\sqrt{d}]$

*1-3 The multiplication of any pair of the above will lead to a rational number, from the knowledge of difference of two squares.

Note:
$\frac{\sqrt{a}}{\sqrt{a}} = \frac{\sqrt{b}}{\sqrt{b}} = \frac{\sqrt{c}}{\sqrt{c}} = 1$

The products of conjugate pairs of surds give rational number, from the knowledge of difference of two squares.

2.2 Basic Forms

Basic forms of surds occur when the number under the square root sign is a rational number, which is a multiple of other numbers, is converted into its simpler form.

2.2.1 Examples

i $\sqrt{18} = \sqrt{2 \times 9} = \underline{3\sqrt{2}}$

ii $\sqrt{288} = \sqrt{144 \times 2} = 12\sqrt{2}$

iii $\sqrt{500} = \sqrt{100 \times 5}$
$= \sqrt{10 \times 10 \times 5} = 10\sqrt{5}$

iv $\sqrt{54} = \sqrt{3 \times 3 \times 3 \times 2}$
$= \sqrt{3 \times 3} \times \sqrt{3 \times 2} = 3\sqrt{6}$

v $\sqrt{200} = \sqrt{2^2 \times 5^2 \times 2}$
$2 \times 5 \times \sqrt{2} = 10\sqrt{2}$

2.2.2

Conversion from basic forms to single surd, is the opposite of the above operations. These are illustrated by the following examples.

i $3\sqrt{2} = \sqrt{3 \times 3 \times 2} = \sqrt{18}$

ii $7\sqrt{2} = \sqrt{7 \times 7 \times 2} = \sqrt{49 \times 2} = \sqrt{98}$

iii $15\sqrt{2} = \sqrt{15 \times 15 \times 2}$
$\sqrt{15 \times 30}$
$= \sqrt{450}$

Note: $\sqrt{a} \times \sqrt{a} = a$
$\sqrt{18} = \sqrt{3 \times 3 \times 2}$
$= \sqrt{3} \times \sqrt{3} \times \sqrt{2} = 3\sqrt{2}$
This is the basic form.

$\sqrt{a} \times \sqrt{a} = \sqrt{a} \times \sqrt{a} = a$
$288 = 12 \times 12 \times 2$
$= 12 \times 12 \times 2 = 12\sqrt{2}$
*As above or you break it down into its factors:

2	500
2	250
5	125
5	25
5	5
	1

$\sqrt{500} = \sqrt{2^2 \times 5^3}$
$= \sqrt{2^2 \times 5^2 \times 5}$
$= 2 \times 5 \times \sqrt{5} = 10\sqrt{5}$
This is the reverse of the fact that $\sqrt{a} \times \sqrt{a} = a$
So, $a = \sqrt{a \times a}$
from above $\sqrt{18} = \sqrt{9 \times 2}$
$= \sqrt{18} = 3\sqrt{2}$
Now, work from the back to the front.

iv $2\sqrt{2} = \sqrt{2 \times 2 \times 2} = \sqrt{4 \times 2}$
$= \sqrt{8}$

v $11\sqrt{3} = \sqrt{11 \times 11 \times 3} = \sqrt{121 \times 3}$
$= \sqrt{363}$

2.3.0 Similar Surds.

These are surds which are multiples of the same surd or those with the same digit under the square root sign. They could be added or subtracted from each other

2.3.1 *Examples of similar surds are listed below:*

i $2\sqrt{2} = \sqrt{8}$
ii $3\sqrt{2} = \sqrt{18}$
iii $4\sqrt{2} = \sqrt{32}$
$5\sqrt{2} = \sqrt{50}$ e.t.c

others are

$2\sqrt{3} = \sqrt{12}$
$3\sqrt{3} = \sqrt{27}$
$4\sqrt{3} = \sqrt{48}$
$5\sqrt{3} = \sqrt{72}$ e.t.c

2.3.2 Addition of Similar Surds

i $\sqrt{2} + 2\sqrt{2} + \sqrt{2}$
$= \mathbf{4\sqrt{2}}$

ii $3\sqrt{2} + 4\sqrt{2} + 5\sqrt{2}$
$= \mathbf{12\sqrt{2}}$

iii $3\sqrt{3} + \sqrt{3} + 6\sqrt{3}$
$= \mathbf{10\sqrt{3}}$

iv $3\sqrt{3} + 10\sqrt{3} + 6\sqrt{3}$
$= \mathbf{19\sqrt{3}}$

v $3\sqrt{3} + \sqrt{3} + 2\sqrt{2} + \sqrt{2}$
$= \mathbf{4\sqrt{3} + 3\sqrt{2}}$

In handling This aspect one need to be versed in multiplication.

FROM THE ABOVE

$\sqrt{a} + 2\sqrt{a} + \sqrt{a}$
$= 4\sqrt{a}$

* The alphabelt under the square root sign is a, and it is the same all through that is why we can add them together this same rule is applicable in all other addition processes provided the digit under the square root signs are the same.

2.3.3 Subtraction of similar surds.

This follow the same procedure as those of addition which are explained above. below are the examples.

i $4\sqrt{2} - \sqrt{2}$
$= \mathbf{3\sqrt{2}}$

ii $10\sqrt{2} - 6\sqrt{2}$
$= \mathbf{4\sqrt{2}}$

iii $12\sqrt{3} - 7\sqrt{3}$
$= \mathbf{5\sqrt{3}}$

iv $4\sqrt{7} - 2\sqrt{7}$
$= \mathbf{2\sqrt{7}}$

v $5\sqrt{7} - \sqrt{7} - 3\sqrt{7}$
$= 5\sqrt{7} - 4\sqrt{7}$
$= \sqrt{7}$

This is the same as $4 - 1 = 3$, then add the root 2 to it, to become $3\sqrt{2}$

2.3.4 Combination Process

This implied process involving addition and subtraction in the same operation.

e.g

i $7\sqrt{2} - \sqrt{2} + 5\sqrt{2} - 6\sqrt{2}$
$= 7\sqrt{2} + 5\sqrt{2} - \sqrt{2} - 6\sqrt{2}$
$= 12\sqrt{2} - 7\sqrt{2}$
$= \mathbf{5\sqrt{2}}$

This is the same as $7 - 1 + 5 - 6$
$= 7 + 5 - 1 - 6$
$= 12 - 7 - 1 - 6$
$= 12 - 7 = 5$
Then add the root 2, for it to become $5\sqrt{2}$. Follow the same procedure

ii $10\sqrt{2} - 3\sqrt{2} + 11\sqrt{2} - 6\sqrt{2}$
$= 10\sqrt{2} + 11\sqrt{2} - 3\sqrt{2} - 6\sqrt{2}$
$= 21\sqrt{2} - 9\sqrt{2}$
$= \mathbf{12\sqrt{2}}$

iii $10\sqrt{5} - 4\sqrt{5} + 2\sqrt{5} - 6\sqrt{5}$
$= 10\sqrt{5} + 2\sqrt{5} - 4\sqrt{5} - 6\sqrt{5}$
$= 12\sqrt{5} - 10\sqrt{5}$
$= \mathbf{2\sqrt{5}}$

2.4.0 Conjugate Surds

These are two surds whose products give a rational number. conjugate pair of surds are base on the difference of two squares.

2.4.1 *Examples of conjugate pairs of surds are:*

1a $(a+\sqrt{b})$ and $(a-\sqrt{b})$

Note: Product

$$\begin{array}{r} a+\sqrt{b} \\ \times\ a-\sqrt{b} \\ \hline -a\sqrt{b}-b \\ a^2+a\sqrt{d} \\ \hline a^2 \qquad -b \end{array}$$

b $[3+\sqrt{2}]$ and $[3-\sqrt{2}]$

PRODUCT:

$(3+\sqrt{2})(3-\sqrt{2})$

$= 3[3-\sqrt{2}]+\sqrt{2}[3-\sqrt{2}]$

$= 9-3\sqrt{2}+3\sqrt{2}-2$

$= 9-2$

$= \underline{7}$

2 a $[a\sqrt{b}-\sqrt{c}]$ and $[a\sqrt{b}+\sqrt{c}]$

$= a\sqrt{b}\,[a\sqrt{b}+c]-\sqrt{c}\,(a\sqrt{b}+\sqrt{c})$

$= a^2b + a\sqrt{bc} - a\sqrt{bc} - c$

$= a^2b - c$

b $5\sqrt{2}-3\sqrt{3}$ and $5\sqrt{2}+3\sqrt{3}$

Product:

$$\begin{array}{r} 5\sqrt{2}-3\sqrt{3} \\ \times\ 5\sqrt{2}+3\sqrt{3} \\ +\ 15\sqrt{6}-27 \\ \hline 50-15\sqrt{6} \\ \hline 50-27 \end{array}$$

$\therefore \quad 50 - 27 = \underline{23}$

Conjugate Surds are the same but the middle signs are reversed i.e if one is positive the other will be negative and vice versa.

Note that the product of the two pairs give a rational number which is 7.

The other examples will also give a rational number.

$\therefore \quad 3\sqrt{3} \times 3\sqrt{3}$

$= 27$

$\therefore \quad 5\sqrt{2} \times 3\sqrt{3}$

$= 15 \times \sqrt{2} \times 3$

$= 15\sqrt{6}$

c $(a\sqrt{b}-c\sqrt{d})$ and $(a\sqrt{b}+c\sqrt{d})$

Product:

$(a\sqrt{b}-c\sqrt{d})(a\sqrt{b}+c\sqrt{d})$

$$\begin{array}{r} = a\sqrt{b}-c\sqrt{d} \\ \times\ a\sqrt{b}+c\sqrt{d} \\ \hline ac\sqrt{bd}-c^2d \\ a^2b-ac\sqrt{bd} \\ \hline a^2b \qquad -c^2d \end{array}$$

$(a\sqrt{b}-c\sqrt{d})(a\sqrt{b}-c\sqrt{d}) = a^2b - c^2d$

2.5.0 *Rationalization the denominator of Surds*

This could be said to be the removal of surds from the denominator of fractions in which they occur.

2.5.1 Rationalization of simple surds.

Below are examples of ralionalization of simple surds.

1. Rationalize the following

(a) $\frac{3}{\sqrt{3}}$ (b) $\frac{17}{\sqrt{7}}$ (c) $\frac{8}{\sqrt{5}}$

Solution

a. $\frac{3}{\sqrt{3}} = \frac{3}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$= \frac{3\sqrt{3}}{3}$

$= \underline{\sqrt{3}}$

b $\frac{17}{\sqrt{7}} = \frac{17}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}}$

$= \frac{17\sqrt{7}}{7}$

Note:

$-15\sqrt{6} + 15\sqrt{6}$

$= 0$

$c \times \sqrt{d} \times c \times \sqrt{d}$

$= c^2d$

$a\sqrt{b} \times c\sqrt{d}$

$= ac\sqrt{bd}$

$a\sqrt{b} \times a\sqrt{b}$

$= a^2b$

$ac\sqrt{bd} - ac\sqrt{bd} = 0$

Note:

$\frac{a}{\sqrt{b}} = \frac{a}{\sqrt{b}} \times \frac{\sqrt{b}}{\sqrt{b}}$

$= \frac{a\sqrt{b}}{b}$

$\therefore \frac{a}{a} = 1 = \frac{\sqrt{3}}{\sqrt{3}}$

Recall

$\sqrt{a} \times \sqrt{a} = a$

c. $\frac{8}{\sqrt{5}} = \frac{8}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}$

$= \frac{8\sqrt{5}}{5}$

2.5.2 *Rationalizing the denominator with its conjugate pair.*

Below are examples of this type.

Rationalize the following:-

(a). $\frac{3}{3+\sqrt{2}}$ (b) $\frac{3}{\sqrt{5}-1}$ (c) $\frac{2}{\sqrt{2}+1}$

Solution

a. $\frac{3}{3+\sqrt{2}} = \frac{3}{3+\sqrt{2}} \times \frac{3-\sqrt{2}}{3-\sqrt{2}}$

$= \frac{3[3-\sqrt{2}]}{7}$

b. $\frac{3}{\sqrt{5}-1} = \frac{3}{\sqrt{5}-1} \times \frac{\sqrt{5}+1}{\sqrt{5}+1}$

$= \frac{3[\sqrt{5}+1]}{5-1}$

$= \frac{3[\sqrt{5}+1]}{4}$

$= 0.75[\sqrt{5}+1]$

c. $\frac{2}{\sqrt{2}+1} = \frac{2}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1}$

$= \frac{2(\sqrt{2}-1)}{2-1}$

$= 2(\sqrt{2}-1)$

Note

In the examples the surds h
been removed from th
denomination.

The conjugate pair of $3+$
is
$3-\sqrt{2}$.

Now, remember that
$\frac{3-\sqrt{2}}{3-\sqrt{2}} = 1$ and 1 time an
number is that same numbe

$3+\sqrt{2}$
$\times 3-\sqrt{2}$
$-3\sqrt{2}-2$
$9+3\sqrt{2}$
$9-2 = 7$

Recall:
$-3\sqrt{2}+3\sqrt{2}$
$= 0$

The conjugate pair of $\sqrt{5}+1$ is
$\sqrt{5}-1$. Now follow the same
Procedure.

The conjugate pair of $\sqrt{2}+1$ is
$\sqrt{2}-1$

Difficult and special forms of rationalization.

The examples are given below:

a $2\sqrt{3} - \frac{6}{\sqrt{3}} + \frac{3}{\sqrt{27}}$

$2\sqrt{3} - \left[\frac{6}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}\right] + \left[\frac{3}{\sqrt{27}} \times \frac{\sqrt{27}}{\sqrt{27}}\right]$

$2\sqrt{3} - \left[\frac{6\sqrt{3}}{3}\right] + \left[\frac{3\sqrt{27}}{27}\right]$

$2\sqrt{3} - 2\sqrt{3} + \frac{3 \times 3 \times \sqrt{3}}{27}$

$2\sqrt{3} - 2\sqrt{3} + \frac{\sqrt{3}}{3}$

$\frac{\sqrt{3}}{3}$

$\sqrt{27} = \sqrt{3 \times 3 \times 3}$
$= 3\sqrt{3}$
$2\sqrt{3} - 2\sqrt{3} = 0$

b $\left[\sqrt{2} + \frac{1}{\sqrt{2}}\right]^2$

$= \left[\sqrt{2} + \frac{1}{\sqrt{2}}\right]\left[\sqrt{2} + \frac{1}{\sqrt{2}}\right]$

$\sqrt{2}\left[\sqrt{2} + \frac{1}{\sqrt{2}}\right] + \frac{1}{\sqrt{2}}\left[\sqrt{2} + \frac{1}{\sqrt{2}}\right]$

$= 2 + 1 + 1 + \frac{1}{2}$

$= 4\frac{1}{2}$

Note

$\sqrt{2} \times \sqrt{2} = 2$

$\sqrt{2} \times \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{\sqrt{2}} = 1$

$\frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = \frac{1}{2}$

c Given that $\sqrt{3} = 1.7321$ and $\sqrt{5} = 2.2361$ evaluate

i. $\frac{1}{\sqrt{5}-\sqrt{3}}$

ii. $\frac{\sqrt{3}+1}{\sqrt{3}-1}$

Solution

i. $\frac{1}{\sqrt{5}-\sqrt{3}} = \frac{1}{2.2361 - 1.7321}$

$= \frac{1}{0.504}$

$= 1.9841$

$\simeq 1.98$ (2p.d)

ii. $\frac{\sqrt{3}+1}{\sqrt{3}-1} = \frac{1.7321 + 1}{1.7321 - 1}$

$= \frac{2.7321}{0.7321}$

$= 3.7319$

$\simeq$ **3.73** (2p.d)*

This is just a case of substitution for the values of the given roots.

2.2361
- 1.7321
5040

$\simeq$ Approximation

* 2p.d = 2 places of decimal.

2.6. PRACTICE EXERCISE

1. Convert the following to their basic forms
   a. $\sqrt{300}$ b. $\sqrt{75}$ c. $\sqrt{125}$
2. Convert the basic forms of surds to their single forms
   a. $9\sqrt{2}$ b. $11\sqrt{5}$ c. $19\sqrt{2}$
3. Simplify the following as far as possible
   a. $3\sqrt{2} + 10\sqrt{2} - \sqrt{10}$
   b. $\sqrt{20} + \sqrt{5}$ c. $\sqrt{5} + \sqrt{20} - \sqrt{80}$
4. Rationalize the denominaltors of the following
   a. $\frac{4}{\sqrt{11}}$ b. $\frac{4+\sqrt{2}}{4-\sqrt{2}}$ c. $\frac{3+\sqrt{3}}{3-\sqrt{3}}$
5. Whrite down the conjugate pairs of the following surds and find their products.
   a. $11 + \sqrt{5}$ b. $20\sqrt{2} - 3\sqrt{3}$ c. $7 + \sqrt{3}$
6. Given that $\sqrt{2} = 1.414$; $\sqrt{3} = 1.732$ and $\sqrt{5} = 2.236$, evaluate
   a. $\left[\frac{\sqrt{3}+1}{\sqrt{5}}\right]^2$ b $\left[\frac{1+1}{\sqrt{5}}\right]^3$ c. $\frac{\sqrt{2}+1}{2+\sqrt{2}}$

3.0 CHANGE OF SUBJECT FORMULAE

3.1 Several mathematical formulae usually involve more than one variable or quantity with one of these variables or quantities serving as the subject of the formula or relationship.

In handling these types of mathematical manipulations there are no any known procedure or lay-down principle but, the unwritten rule is WHATEVER YOU DO TO THE LEFT HAND SIDE [L.H.S], YOU DO THE SAME TO THE RIGHT HAND SIDE [R.H.S].

This literally implies that, changing of subject formulae should be tackle as "a pair of twins", so whetever you give to the first, the same should be given to the second, to avoid crises. Details are illustrated in the examples below.

3.2 Simple forms of change of subject formulae.

3.2.1 Examples of the simple forms. Make the letter written in italic the subject in each of the following cases:

a). $v = u + at$ t

b). $a = dr + br$ r

c). $s = {}^{a}/_{1-r}$ r

e). $T = 2\pi \sqrt{L/g}$ g

Solution

a. $v = u + at$ t

$v - u = u + at$ u

$v - u = at$

$\therefore at = v - u$

$\frac{at}{a} = \frac{v - u}{a}$

$\therefore t = \frac{v - u}{a}$

b. $a = dr + br$ r

$a = r(d + b)$

$\therefore \frac{a}{(d + b)} = \frac{r(d + b)}{(d + b)}$

$\therefore \frac{a}{(d + b)} = r$

$\therefore r = \frac{a}{d + b}$

c. $s = \frac{a}{1 - r}$ r

$\therefore \frac{s}{1} = \frac{a}{1 - r}$

$s[1 - r] = a$

$s - sr = a$

$\therefore s = a + sr$

$\therefore s - a = sr$

$\therefore {}^{s-a}/_{s} = r$

$\therefore r = {}^{s-a}/_{s}$

$= {}^{s}/_{s} - {}^{a}/_{s}$

$= \frac{1 - {}^{a}/_{s}}{}$

Subtract 'u' form both sides then, turn the equation as: at = v - u

Now, note that at = v - u so, 'a' is disturbing us from obtaining 't'. To remove the "a" we divide both side by a because the opposite of divison is multiplication and the opposite of addition is subtraction.

dr + br = r (d + b)

Because r is common to them

After that divide both sides by (d + b) which is disturbing us' form getting r. And, obtain the answer.

Write as

$s \times \frac{a}{1 - r}$

s(1 - r) = 1 x a = a. for easy manipulation.

Then, open the brackets and follow the procedure above.

$I \times 100 = {}^{PRT}/_{100} \times 100$

$I \times 100 = PRT$

$\frac{PRT}{PR} = \frac{I \times 100}{PR}$

$T = \frac{100I}{PR}$

c. $T = 2\pi\sqrt{L/g}$ g

Divide both side by 2π

$\therefore \frac{T}{2\pi} = \frac{2\pi}{2\pi}\sqrt{L/g}$

take the square of both sides

$\therefore [{}^{T}/_{2\pi}]^2 = [\sqrt{L/g}]^2$

$\frac{T^2}{4\pi^2} = \frac{L}{g}$

Cross multiply

$gT^2 = 4\pi^2 L$

Divide both sides by T^2

$\frac{gT^2}{T^2} = \frac{4\pi^2 L}{T^2}$

$\therefore g = \frac{4\pi^2 L}{T^2}$

Harder forms of change of subject formulae examples.

If $ST = \sqrt{\frac{(a + T)T}{a}}$ make T the subject

$ST = \sqrt{\frac{(a + T)T}{a}}$

Multiply both sides by a

$aST = a \times \sqrt{\frac{(a + T)T}{a}}$

Repeat the same procedure and bear in mind the un-written rule.

Divide both sides by 2π then, take the square of both sides.

And follow the previous steps.

The denominator on the right hand side is a.

Remove the square root sign by squaring both sides.

Open the bracket and continue the operations as usual.

Square both sides.

$[aST]^2 = [\sqrt{(a + T)T}]^2$

$a^2S^2T^2 = [a + T]T$

$\therefore \quad a^2S^2T^2 = Ta + T^2$

Divide both side by T^2

$$\frac{a^2S^2T^2}{T^2} = \frac{Ta}{T^2} + \frac{T^2}{T^2}$$

$a^2S^2 = {}^a/_T + 1$

Subtract 1 form both sides

$a^2S^2 - 1 = {}^1/_T + 1 - 1$

$\therefore \quad a^2S^2 - 1 = {}^a/_T$

Multiply both sides by T

$\therefore \quad T[a^2S^2 - 1] = {}^a/_T \times T$

$T[a^2S^2 - 1] = a$

Divide both side by $(a^2S^2 - 1]$

$$\frac{T[a^2S^2 - 1]}{[a^2S^2 - 1]} = \frac{a}{[a^2S^2 - 1]}$$

$$\therefore \quad T = \frac{a}{[a^2S^2 - 1]}$$

b. Make g the subject of the relation:

$$\frac{t}{L} = \sqrt{\frac{4\pi^2}{g}}$$

take the square the both sides

$$\therefore \left(\frac{t}{L}\right)^2 = \left(\sqrt{\frac{4\pi^2}{g}}\right)^2$$

$$\therefore \quad \frac{t^2}{L^2} = \frac{4\pi^2}{g}$$

Multiply both sides by g.

$$\frac{g \times t^2}{L^2} = \frac{4\pi^2 \times g}{g}$$

Again multiply through by L^2

$$\frac{gt^2 \times L^2}{L^2} = 4\pi^2 L^2$$

First step to liberate g, is to square both sides of the equation because g is under square root sign from surds

$\sqrt{a} \times \sqrt{a} = a$

$$\therefore \left(\sqrt{\frac{4\pi^2}{g}}\right)^2 = \frac{4\pi^2}{g}$$

Note: This question looks like example 'e' above check

$$\therefore \quad g = \frac{4\pi^2 L^2}{t^2}$$

c. Make R the subject of the formula:

$v = \pi L [R^2 - (R - t)^2]$

$v = \pi L [R^2 - (R - t)^2]$

$= \pi L [R^2 - (R^2 - 2Rt + t^2]$

$= \pi L [R^2 - R^2 + 2Rt - t^2]$

$= \pi L [2Rt - t^2]$

$= 2\pi RLt - \pi Lt^2$

Add πLt^2 to both sides.

$v + \pi Lt^2 = 2\pi RLt - \pi Lt^2 + \pi Lt^2$

$v + \pi Lt^2 = 2\pi RLt$

$v + \pi Lt^2 = 2\pi RLt$

$2\pi RLt = v + \pi Lt^2$

Divide through by $2\pi Lt$

$$\frac{2\pi RLt}{2\pi Lt} = \frac{v + \pi Lt^2}{2\pi Lt}$$

$$R = \frac{v + \pi Lt^2}{2\pi Lt}$$

$$= \frac{v}{2\pi Lt} + \frac{t}{2}$$

First expand the brackets on the R.H.S:

$[R - t]^2 = R^2 - 2Rt + t^2$

Then, multipy through by the negative sign. $+ R^2 - R^2 = 0$

Multiply through again by πL

Dividing through by $2\pi Lt$ leaves R on the R.H.S,

Substitution into changed formulae

This is easily done, by first changing the subject of the relation to the required one, before substituting the given values

a). Make g the subject of the formula

$$d = \frac{gv^2t^2}{v^2 + gt}$$

Hence, calculate the value of g when

$d = 7$, $v = 5$ and $t = 3$

Solution

$$d = \frac{gv^2t^2}{v^2 + gt}$$

Cross multiply

$\therefore gv^2t^2 = d[v^2 + gt]$

$gv^2t^2 = dv^2 + dgt$

Subtract dgt from both sides

$\therefore gv^2t^2 - dgt = dv^2 + dgt - dgt$

$\therefore gv^2t^2 - dgt = dv^2$

Factorise L.H.S.

$\therefore gt[v^2t - d] = dv^2$

Divide both sides by $t[v^2t - d]$

$$\therefore \frac{gt[v^2t - d]}{t[v^2t - d]} = \frac{dv^2}{t[v^2t - d]}$$

$$\therefore g = \frac{dv^2}{t[v^2t - d]}$$

$$= \frac{dv^2}{v^2t^2 - dt}$$

$d = 7$, $v = 5$ and $t = 3$.

$$g = \frac{dv^2}{v^2t^2 - dt}$$

$$= \frac{7 \times 5^2}{5^2 \times 3^2 - 7 \times 3}$$

$$= \frac{175}{225 - 21} = \frac{175}{204}$$

$\simeq$ **0.86** (2 places of decimal)

b. The formula $F = {}^{9c}/_{5} + 32$ Shows the relationship between the temperature in degree centigrade (C).

i. Make C the subject of the relation.

ii. a. Find C when F = 77

b. Find F when C = 30

$$\frac{d}{1} \times \frac{gv^2t^2}{v^2 + gt}$$

$dgt - dgt = 0$

Factorise L.H.S

$gt[v^2t - d] = gv^2t^2 - dgt$

Substitue for the values i.e replace the values in the formula for g, to find the value of g.

i. $F = {}^{9c}/_{5} + 32$

Multiply through 5

$5 \times F = 5 \times {}^{9c}/_{5} + 5 \times 32$

$\therefore 5F = 9c + 160$

subtract 160 from both sides

$5F - 160 = 9c + 160 - 160$

$\therefore 9c = 5F - 60$

$$c = \frac{5f - 160}{9}$$

ii. a). From above

$$C = \frac{5f - 160}{9}$$

So, when F = 77

$$C = \frac{5 \times 77 - 160}{9}$$

$$= \frac{385 - 160}{9}$$

$= \mathbf{25^0c}$

b). F when c = 30

From above:

$F = {}^{9c}/_{5} + 32$

$= 9 \times {}^{30}/_{5} + 32$

$= 54 + 32$

$= \underline{\mathbf{86F}}$

Convert the equation into lateral equation by multiplying by 5.

Note, the answer can as well be written as $c = {}^{5}/_{9}F - {}^{160}/_{9}$

From the formula for c replace F with 77 to find c.

Recall the given formula, then replace c with 30

3.3 PRACTICE EXERCISE

1. Make the letters in italic subject of the relations:
   a. $v = u + at$ a
   b. $T = 2\pi\sqrt{L/g}$ L
   c. $a = dr + br$ b

2. Make k the subject of the formula
   $$a = \frac{k\sqrt{t}}{b} + s$$

3. Make r the subject of the relation:
   $$s = \frac{3r}{2a - r}$$
   Hence, find the value of r when s = 5 and a = 3.

4. Make A the subject of the formula
   $d = 3 - {}^{4}/_{5}\sqrt{A}$
   Hence, calculate the value of A when d = 4

5. If $s = (5u + {}^{1}/_{2} F)(t - 6)$
   What is the value of s when t = 4, u = - 3 and f = - 4

   b. Make a the subject of the formula
   If $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

4.0 INTEREST & DEPRECIATION

4.1 INTEREST

Interest is the money charged for borrowing money from a lender or paid to somebody who invests in a business concern, such as the bank e.t.c. It is always expressed as a percentage of the actual money saved or loaned, otherwise refer to as PRINCIPAL over a period of time at an agreed or specified rate. It could be simple or compounded interest depending on the terms of agreement.

4.1.1 Simple interest

This types of interest is calculated on the principal and added at the end of an agreed period of time .The basic formula use in calculating simple interest: $I = \frac{P \times R \times T}{100}$

I = Interest => simple interest, is the amount accrue.

P = Principal => actual money borrowed or lend usually in $, ₦, C e.t.c.

R = Rate The percentage charged

T = Time => the agreed period usually stated in years to easy calculation.

4.1.1.1 Calculating Simple Interest

a. Find the simple interest on N6000 for 4 years at 9% per annum

Solution: I = Interest = x
P = Principal = N6000
R = Rate = 9%
T = Time = 4 years.
$I = {}^{PRT}/_{100}$
$x = \frac{N6000 \times 9 \times 4}{100}$
= **N 2160.00**

List the given parameters. Then use formula to determine the unknown.
x = Interest
Recall the formula $I = {}^{PRT}/_{100}$ and substitute the values into the formula to get I.

b. Fine the simple interest on $ 1500 at 12% per annum for 3 years.

Solution
I = Interest = x
P = Principal = $ 1,500
R = Rate = 12%
T = Time = 3 years.

Recall:
$I = \frac{PRT}{100}$
$x = \frac{\$1500 \times 12 \times 3}{100}$
= **$ 540.**

As in "a" above.

c. Mr Audu saved N18,000.00 with a cooperative society. If simple interest is paid yearly at $4^1/_2$ % per annum find the amount which he has in the society at the end of 6 years.

Solution
P = Principal = N18,000
R = Rate = $4^1/_2$%
T = Time = 6 years
$I = \frac{P \times R \times T}{100}$

Note: $4^1/_2\% = {}^9/_2\%$
For this question you calculate the interest and add it to the principal. Add it to the principal.
Amount =P+ I.
See compound interest.

$I = \frac{N18000 \times 9 \times 6}{100 \times 2}$
= **N4,860.00**

The amount Mr. Adudu has with the cooperative at the end of 6 years = Principal + Interest = N18,000 + N4860 = N22860.00

d. Find the amount, if simple interest is paid yearly at 12% for 3years 4 months on a principal of N1,500.00

Solution: I = Interest
P = Principal = N1,500.00
R = Rate = 12
T = 3years 4 months
= $3^1/_3$yrs. = ${}^{10}/_3$yrs
$\therefore I = \frac{PRT}{100}$
$\therefore I = \frac{N1500 \times 12 \times 10}{100 \times 3}$
= N600
Amount = P + I
= N1500 + N600
= **N2100.00**

Note
The time is always express in years so, convert 3 years 4 months to $3^4/_{12} = 3^1/_3$ because12 mouths make one year.

4.1.2 Calculating the Principal.

a) Find the principal which earn an interest of N3000 at 15% for 12 years.

Solution: I = N3000.00
R = 15%
T = 12 year
P = ?
$P = \frac{I \times 100}{RT}$

From:
$I = {}^{PRT}/_{100}$ make P the subject of the relation.
$I \times 100 = PRT$
$\therefore P = \frac{I \times 100}{RT}$
Then, substitute for the v

$= \frac{N3000.00 \times 100}{15 \times 12}$

$\therefore P = N1,666.667$

$\cong$ **N1,666.67**

b. A trader saved a certain amount of money for a period of $7\frac{1}{2}$ years at the rate of 10%. if the said amount yield an interest of N15,000.00,.Find the actual amount saved.

Solution:

$T = 7\frac{1}{2}$ years $= \frac{15}{2}$ years

$R = 10\%$

$I = N15,000$

$\therefore P = \frac{I \times 100}{RT}$

$= \frac{N15,000.00 \times 100 \times 2}{15 \times 10}$

$=$ **N20,000.00**

c. Mr. Fadahunsi invested a certain amount of money in a business concern, at the rate of $12\frac{1}{2}\%$ for 5 years 6 months. If the investment fetch him N60,000.00, Find the initial amount invested?

Solution: $I = N60,000.00$

$R = 12\frac{1}{2}\%$

$= \frac{25}{2}\%$

$T = 5$ years 6 months

$= 5\frac{1}{2}\%$

$P = \frac{I \times 100}{RT}$

$= \frac{N60,000 \times 100}{\frac{25}{2} \times \frac{11}{2}}$

Note:

The actual amount save is the principal.

From above: $P = \frac{I \times 100}{RT}$

Then, substitute as above for the given values

Remember that the time has to be in years and not in months so, 5 years 6 months is equivalent to $5\frac{1}{2}$ years i.e

$5 + \frac{6}{12}$

$5 + \frac{1}{2} = 5\frac{1}{2}$

$= 11/2$

$= \frac{N60,000 \times 100 \times 2 \times 2}{25 \times 11}$

N87,272.73

1.1.1 Calculating rates and time.

a) Calculate the rate at which N3000 would be invested, for it to earn an interest of N600 in 5 years?

Solution: Rate = ?

Principal = N3,000.00

Interest = N600.00

Time = 5 years

$R = \frac{I \times 100}{PT}$

$= \frac{N600 \times 100}{N3000 \times 5}$

$=$ **4%**

b) Find the rate at which N30,000 would be invested for it to earn an interest of N25,000 in 4 years.

Principal = N30,000.00

Interest = N25,000.00

Time = 4 years

Rate $= \frac{I \times 100}{PT}$

$= \frac{N25,000 \times 100}{N30,000 \times 4}$

$=$ **20.83%**

Recall: $I = \frac{PRT}{100}$

make R, the subject of the relation:

$I \times 100 = PRT$,

$\therefore R = \frac{I \times 100}{PT}$

Now, substitute the given values into the formula to obtain the rate.

Note, rate is always express in percentage (%).

From the above

$R = \frac{I \times 100}{P \times T}$

Substitute the given values in the equation.

Recall:

$I = \frac{PRT}{100}$

Now,its the turn of T to be the subject of the formula

C. How long will it take for the sum of N50,000 invested at the rate of 25% to earn N12,500 as interest.

Solution

Principal = N50,000.00
Interest = N12,500.00
Rate = 25%
Time = $\frac{I \times 100}{PR}$

$\therefore T = \frac{N12500 \times 100}{N50,000 \times 25}$

= **1Year**

4.1.2 COMPOUND INTEREST

Most savings or investments give compound interest,not simple interest because the interest is added to the principal at regular intervals,such that the principal increases as the interests is added to it.

So,if a certain sum of money P is invested for n number of years at the rate of r % per annum compound interest,the amount A,at the end of n years can be obtain from the formula

$A = P[1+{}^{r}/_{100}]^{n}$

4.1.2.1 Calculating Amount (without making use of the formula)

(a) The Model College Teachers cooperative society pays interest at 15% per annum compounded yearly.What will be the amount in Mr Oloyede's account at end of 3 years if he saved N12,000 with the society.

$I \times 100 = PRT$

$\therefore T = \frac{I \times 100}{PR}$

Flip back to chapter three
details on change of subj
formulae if you are still
doubt.

Note: that how long,find t
period etc has to do with tim

Solution

For the first year:

Principal = N12,000
Rate = 15%
Interest = $\frac{N12,000 \times 15}{100}$
= N1800

At the end of the first year A = N12,000+N1800
= N13,800.00

Second year:

Principal = N13,800.00
Rate = 15%
Interest = $\frac{N13,800 \times 15}{100}$
= N2070.00

Amount = N13,800+N2070
= N15870.00

Third year

Principal = N15870.00
Rate = 15%
Interest = $\frac{N15870 \times 15}{100}$
= N2380.50

Amount = N15870 + N2380.50
= N18250.50

The amount in the account at the end of the third year.

A = P + I
i.e Amount = Principal + interest.
So the amount at the end of the first year, is the interest for the first year which will be 15% of N12,000 plus the principal.

Note, principal for the second year is the amount at the end of the first year. And the amount at the end of the second year will serve as principal for the third year,the amount at the end of the third year is the required amount.

b The sum of N100,000.00 is invested by the trustees of N.U.T cooperative society at 10% per annum compound interest,the interest being added half-yearly. Find the amount after 2years.

Solution

For First Half of the Year:

Principal = N100,000
Rate = 10%
Interest = $\frac{P \times R}{100}$
= $\frac{100,000 \times 10}{100}$
= N10,000.00

Amount = N100,000 + N10,000
= 110,000.00

For the second half of the year:

Principal = N110,000.00
Rate = N 10%
Interest = $\frac{N110000 \times 10}{100}$
= 11,000
Amount = 110,000 + N11,000
= N121,000.00

Third Half of The Year:

Principal = N121,000.00
Rate = 10%
Interest = $\frac{N121,000 \times 10}{100}$
= N12,100.00

Since the compound intere
is added half-yearly,it mean
it has to be added twice in
year and four times in tw
year.

Note: That the rate is alway
constant for the preceedin
year or half yearly as th
case may be Amount is th
amount after 2 years since
will serve as the principal if th
money is still in the
investment.

Fourth & Final Half, which makes it two

Principal = Amount
= N121,000 + N12,100
= N133,100.00
Rate = 10%
Interest = $\frac{N133,1000 \times 10}{100}$
= N13,310.00

Amount = 133,100 + N13,310
= N146,410.00

A man saves N50,000 for his child at the rate of 5% compound interest per annum. Find the amount the child will be having at the end of the fifth year.

Solution

First year

Principal, = N50,000.00
Rate = 5%
Interest, = $\frac{N50,000 \times 5}{100}$
= N2,500.00

Amount, = N50,000 + N2,500
= N52,500.00

Second year

Principal = N52,500.00
Rate = 5%
Interest = $\frac{N52500.00 \times 5}{100}$
N2,625.00

We are going to follow the same steps we used in solving the questions in the above examples a and b.

Interest$_1$ = Interest for the first year

Principal$_1$ = First principal

Amount$_1$ = Amount at the end of the first year etc.

Though, the amount can be found, with out using the formula but it will be monotonous as in example 'c'

Amount = N52500 + N2625
= N55,125.00

Third year

Principal = N55,125.00
Rate = 5%
Interest = $\frac{\text{N55125} \times 5}{100}$
= N2,756.25

Amount = N5756.25

Fouth year

Principal = N57881.25
Rate = 5%
Interest = $\frac{\text{N57881.25} \times 5}{100}$
= N2.894.06

Amount = N57881.25 + N2894.06
= N60,775.31

Fifth year

Principal = N60,775.31
Rate = 5%
Interest = $\frac{\text{N60,775.31} \times 5}{100}$
= N3038.77

Amount = N60,775.31+ N3038.77
= N63,814.08 => Amount at the end of the fifth

However with numbers ab
5 they are better solved u
formula unless it is stipul
that any other method sh
be use.

Calculating amount with formula

a) The sum of N120,000 was saved as a gift for a child during his twelveth year birthday by the father at the rate 7½%. Calculate the amount in the boy's account after 10 years.

Solution

P = Principal = N120,000.00
r = Rate = 7½%
n = Number of years = 10
A = Amount = Principal + Interest
$P[1 + r/100]^n$
A = $N120,000 [1 + 7.5/100]^{10}$
= $N120,000 [1 + 0.075]^{10}$
= N247,323.79 is the amount in the boy's account after 10 years.

Recall:
$A = P[1 + r/100]^n$ The use of the formula also saves time and space.

b Calculate the interest paid on a fixed deposit of N35,000, which is invested for a period of 6 years at a yearly compound interest rate of 12% per annum.

Solution

A = Amount = $P[1 + r/100]^n$
P = Principal = N35,000.00
r = Rate = 12%
n = number of years = 6
A = $N35000 [1 + 12/100]^6$
= N69083.79
I = Interest
I = A - P
= N69083.79 - N35.000.00
= **N34083.79**

With the formula just substitude and get your answers.
A = P + I
I = A - P
From simple change of formula.

4.1.2.3 DISCOUNTING

It was observed that in real life interests are compounded more frequently than annually as shown in the previous examples, if its paid k times in a particular year, then each year will be $^{1}/_{k}$ and for n year we would have nk period times, implying that

$A_c = P[1+ {}^{r}/_{k}]^{nk}$

P = Present value of the investment worth (refer to as principal in the above examples).

r = Rate in percentage

n = Number of years

k = Number of times the interest is paid per year eg. semi-annually quarterly. etc.

A_c = Accumulated value of P. The compound interest is given by Ac-P (i.e Accumulated value on P less the present value of P.)

4.1.2.3.1 CALCULATION INVOLVING DISCOUNTING (COMPOUNDED INTEREST)

Find the compound interest on N1,000.00 compounded quartely at 6% for 2 years

Semi-annually mean twice in a year,
Quarterly, mean the interest is compounded four times in a year.
Monthly stands for once in a month and twelve times in a year etc.

Ac = Accumulated value on P

P = Present worth of the investment N1,000.00

R = Rate = N6% =0.06

K = Number of times the interest is paid = 4

N = Number of years = 2yrs

$Ac = N1,000.00 [1 + {}^{0.06}/4]^{8}$

$N1,000.00 [1+0.015]^{8}$

N1,126.49

Compound interest

Ac -P

N1,126.49-N1,000.00

N 126.49

4.1.2.3.2 CALCULATION INVOLVING PRESENT VALUE OF AN INVESTMENT

Find the present value of N10,000.00 due in 15 yrs compounded monthly at 12%

Solution

A_c = Accmulated value of

P = Present value of investment = ?

R = Rate = 12% =0.12

K = Number of times the interest is paid = 12

n = Number of years = 15yrs

$Ac = P[1+ {}^{r}/_{k}]^{nk}$

$N10,000.00 = P [1+ {}^{0.12}/12]^{15x12}$

$P [1+ 0.01]^{180}$

$P[1.01]^{180}$

P [5.9958]

$P = {}^{N10,000.00}/5.9958$

N1667.83

compound interest =Ac-P

k = 4[because the interest is paid quarterly i.e four times in years]

∴ nk = 2 x 4 = 8

$Ac = P[1+ {}^{r}/_{k}]^{nk}$

$r = {}^{6}/100 = 0.06$

Substitute into the formula and get your answer as shown .

Note - If the interest is compounded quarterly for 3yrs; k = 4 n =3

∴ nk = 3x 4 =12. etc.

$A_c = P[1+ {}^{r}/_{k}]^{nk}$

Since the interest is paid monthly, it means the interest will be paid 12 times in one yrs . k=12 from change of subject. $P = {}^{Ac}/[1+ {}^{r}/_{k}]^{nk}$

Substitute for

Ac, r,k and n to obtain the value of P or shown

Note: N10,000.00 = P[5.9958]

$P = {}^{N10,000.00}/5.9958$

DISCOUNTING (RATE)

What rate of compound interest will be required semi-annually for 4 yrs with an investment of N4,000.00 which will yield N6,000.00

Solution:

$Ac = P(1+I)^{nk}$

Method 1

Ac= Accumulated value of

P = N6,000

P = Present value of investment

= 4,000.00

n = Number of times the interest is paid per yrs = 2

k = number of times the intrest is paid per year = 2

$\therefore N6,000 = N4,000.00\ [1+r/2]^{2\times4}$

$= N4,000.00\ [1+r/2]^{8}$

$= N4,000.00\ [2+r]^{8}$

$\therefore N6,000.00 = N\ 4,000\ [^{2+r}/_{2}]^{8}$

$1.5 = [^{2+r}/_{2}]^{8}$

$1.5 = [^{2+r}/_{2^8}]$

$1.5 \times 2^8 = 2 + r^8$

$384 = (2 + r)^8$

$\log 384 = \log (2 + r)^8$

$\therefore 2.5843 = 8 \log (2 + r)$

$\therefore \frac{2.5843}{8} = \log(2 + r)$

$\therefore 0.3230 = \log (2 + r)$

Take second function of log

$2.104 = 2+r$

$2.104 - 2 = r$

$\therefore 0.104 = r$

by 5.9958]

Note: A = P+I

$= P+^{PRT}/100$

$= P(1+^{RT}/100)$

I= Interest

$I = {}^{r}/_{k}$

n = T = Time is years

$\therefore Ac = P(1+I)nk$

k=2 (semi-annually) n=4

Ac = N6,000.00

$N6,000.00 = N4,000\ [^{2+r}/2]^{8}$

Divide both side by N4,000

$[^{2+r}/2]^{8}$

$1.5 = [^{2+r}/2]^{8}$

Remember that $[^{2+r}/2]^{8} = [^{2+r}/2^{8}]^{8}$

$15x2^8\ [2+r]^8$

$384 = (2+r)^8$

to resolve the express take the log of both sides.

Note that the log of 384 = 2.5843 but log $(2+r)^8$ can be express as 8 log (2+r) Having obtain 0.3230 = log (2+r) take second function of

$\therefore$ The rate = 10.4%

Method 2

$A_c = P\ [1 + I]^{nk}$

$N6000 = N4,000\ (1 + I)^{2\times4}$

$\frac{N6000}{N4000} = [1 + I]^{8}$

$\therefore 1.5 = [1 + I]^8$

Take log of bothsides

$\log 1.5 = \log [1+I]^8$

$0.1761 = 8 \log (1+I)$

$\frac{0.1761}{8} = \log (1+I)$

$0.0220 = \log (1+I)$

Take the second function of log

$1.0520 = 1+I$

$1.0520 - 1 = I$

$0.0520 = I$

$\therefore I = \frac{r}{k} = 0.0520$

$\therefore r = 2 \times 0.0520$

$= 0.104$

$\therefore$ Rate = 10.4%

4.2 DEPRECIATION

Depreciation is the reduction or loss in the value of assets. It is usually expressed in percentage right from the time of purchase of the item or at the beginning of the year.

Apart from depreciation, we will also talked about other methods of investing money such as shares etc.

logarithm from the right hand side of the equation as shown.

Note:- The value of r obtain is o.104 you multiply it by 100 to get the actual rate in percantage same applies is the second method to.

Recall, $I = {}^{r}/_{k}$

k = 2

$I = {}^{r}/_{2}$

I x 2 = r

for the two methods you have to

i. take the logar thms of both sides

ii. take second function of log

depreciation.

(a) Uche's TV set costing N17,000 depreciates by 10% in the first year and by 7.5% in the second year. Find it value after 2 years.

Solution:

First year:

Cost Price = N17,000.00

Depreciation rate = 10%

Depreciation value = $\frac{N17000 \times 10}{100}$

= N1,700.00

∴ Actual value at the end of the first year

= N17,000.00 - N1,700.00

= N15,300.00

Second year begins

Cost Price = N15,300.00

Depreciation rate = 7.5%

Depreciation value = $\frac{N15300 \times 7.5}{100}$

= N1,147.5

∴ Actual value after 2 years

= N15,300.00 - N1,147.50

= N114,152.50

(b) Mr. Owolabi bought a second hand car at an unspecified amount. If the car was bought by the initial owner for N220,000.00 and the car depreciates at a constant rate of 10% per year. Find the amount if the car was resold to him after 3 years.

In terms of depreciation, it is important to note that the rate may not be constant as we have in the previous section under simple and compound interest.

So, the specified rate of depreciation has to be deducted at the end of the year to have the actual value for the preceeding year.

Therefore, the actual cost at the end of a particular year equals cost price at the beginning of the year less depression value.

Solution:

First Year

Cost Price = N220,000.00

Rate of Depreciation = 10%

Value of Depreciation = $\frac{N220000 \times 10}{100}$

= N22,000.00

Actual value at the end of the first year = N220,000 - N22,000

= N198,000.00

Second Year

Cost Price = N198,000

Depreciation rate = 10%

Depreciation value = $\frac{N198,000 \times 10}{100}$

= N19,800.00

Actual value at the end of second year = N198,000 - N19,800

= N178,200.00

Third Year:

Cost Price = N178,200.00

Depreciation rate = 10%

Depreciation value = $\frac{N178,200 \times 10}{100}$

= N17,820.00

The amount the car was sold after 3 years = N178,200 - N17,820

= **N160,380.00**

This is a hypothetical case of the rate being constant, and it should be treated as such.

C. Mr. Akapo's sport car costs N500,000.00. Its value depreciates by 25% in the first year, 20% in the second year and 15% in each of the following years. Find its value after 4 years

Solution:

Calculate the actual value of depreciation based on the rate of depreciation for the year.

First Year

Cost Price = N500,000.00

Rate of Depreciation = 25%

Value of Depreciation = $\frac{N500000 \times 25}{100}$

= N125,000.00

∴ Actual value at the end of the first year = N500,000 - N125,000

= N375,000.00

Second Year

Actual Value = N375,000

Rate of Depreciation = 20%

Value of Depreciation = $\frac{N375,000 \times 20}{100}$

= N75,000.00

∴ Actual value at the end of second year = N375,000 - N75,000

= N300,000.00

Third Year:

Actual Value = N300,000

Rate of Depreciation = 15%

Value of Depreciation = $\frac{N300,000 \times 15}{100}$

= N45,000.00

Actual value at the end of the third year = N300,000 - N45,000

= N255,000.00

Fourth Year:

Actual Value = N255,000

Rate of Depreciation = 15%

Value of Depreciation = $\frac{N255,000 \times 15}{100}$

= N38,250.00

Actual value at the end of the fourth year = N255,000 - N38,250

= **N216,750.00**

Shares and calculations involving shares:

Shares are sold to members of the public by private or public owned companies, in order to create room for expansion of the companies capital base or technological know - how.

Share are bought or sold by sharesholders at the stock exchange. Shareholders collect dividend, based on the number of the shares they have in the company. The company decides the amount it would pay out of its profit to the sharesholders as dividend at the end of the year.

It is also important to note that the difference between the buying price and the selling price of a share is refer to as the spread. However, by selling a share,

a shareholder will make profit and the brokers too will make profit.

(a) Mr. Oluwole invested N20,000.00 in N5.00 ordinary shares of a company. He bought the shares at N10.00 each.
   i) How many shares did he bought?
   ii) If a dividend of N2.50 per share is declared, how much dividend does he receive.

Solution:

i) Amount invested = N20,000.00
   Selling price = N5.00
   Buying price = N10.00

∴ Number of share brought
$$= \frac{N20,000.00}{N10.00}$$
$$= \underline{\mathbf{2,000}}$$

ii) Dividend on the shares bought by Mr. Oluwole
= 2000 x N2.50
= **N5,000.00**

B. Mrs. Ike bought 2000 shares of N5.00 each from the co-operative society of her school at N7.00 per share. Find:
i) The amount invested on the purchase of the shares
ii) The total amount of spread of the shares she bought.

In determing the number o
shares bought you divide th
total amount invested by th
actual money use in buying th
share:
i.e Total amount Investment
The rate at which the shar
was sold

The company will pay dividen
base on the number of shar
bought = number of share
dividend per share

iii) If at the end of the year, a profit of 25% per share was shared, what was her total dividend.

Solution:

Number of shares bought = 2000
Selling price = N5.00
Buying price = N7.00

i) ∴ Amount invested = 2000 x N7.00
= **N14,000.00**

ii) Total amount of spread
= (N7 - N5) X 2000
= N2 X 2000
= **N4,000.00**

iii) Since the company sells the share at N5.00, so a profit of 25%
$$= \frac{25}{100} \times N5$$
= N5/4
= **N1.25k**
∴ Profit = N1.25 per share.
∴ Since she bought 2000 shares, her dividend = 2000 x N1.25
= **N2,500.00**

(Which of the following investments produce the better advantage interms of percentage yield on investment?
i) N10 shares at N15, which gives a dividend of 20% at the end of the year
ii) N50 shares at N70, which gives a dividend of 14% at the end of the year.
iii) N75 shares at N90.00, which gives a dividend of N25 per share at the end of the year.

The selling price per share is the amount the company that is selling the share brought, it to the stock exchange ie. the price which they will reckon with while, the buying price is the price at which the share holder bought the share.

These two should not be miscontrude.
∴ *Amount invested
= No of shares x Buying Price

* Spread
= Buying price - Selling price

∴ Total spread
= [Buying price - Selling price] x No. of share purchased.

The spread goes to the brokers.

Solution:

i) Percentage dividend = 20%
Selling price = N10.00
Buying price = N15.00
∴ The dividend is 20% of N10.00

$= \frac{20}{100} \times N10$

$= N2.00$

∴ The percentage yield

$= \frac{N2}{N15} \times 100\%$

$= \mathbf{13.33\%}$

ii) Selling price = N50.00
Buying price = N70.00
Percentage dividend = 14%
∴ The dividend $= \frac{14}{100} \times N50$

$= N7.00$

∴ The percentage yield $= \frac{N7}{N70} \times 100\%$

$= \mathbf{10\%}$

iii) Buying price = N90.00
Selling price = N75.00
Dividend = 25%
∴ Percentage yield $= \frac{25}{90} \times 100\%$

$= 27.78\%$

Therefore, (iii) produces a better yield than (i) or (ii).

Yield is the percentage retu
on unit investment.

$= \frac{\text{Dividend per share}}{\text{Buying price per share}} \times 100$

ANNUITIES

An annuity is a sequence of fixed equal payment or the sequential addition of a specified sum of money paid at regular interval to form a lump sum. Examples of annuity include payment of rent; insurance premiums etc. The accunmulated value A_c of an ordinary simple annuity is given by

$$A_c = \frac{R(1+i)^n - 1}{i}$$

The present value of P is given by P

$$R\,\frac{1-(1+i)^{n}}{i}$$

Where

R = Periodic Payment of the annuity

A_c = Accumulated value on the amount of an annuity.

P = Present value or discounted value.

3.1 CALCULATION INVOLOING ANNUITIES (ACCUMULATED VALUE)

Find the accumulated value of an ordinary simple annity of N6,000 per year for 5 years compounded monthly at 7%.

Solution

A = Accumulated value = ?
R = Periodic payment of the annuity = N6,000

A_c represent the accumulated value on the amount of an annuity to distinguish it from A which has been use for amount in the previous sections.

$$A_c = \frac{R[(1+i)^n - 1]}{i}$$

$$i = \frac{r}{k} = \frac{\text{rate}}{\text{No. of times interest is paid per year}}$$

i = Interest = $^{7}/_{1}$ = $^{0.07}/_{12}$

= 0.0058

Number of years = 5

$$n = \frac{N6000\,[(1+0.00058)^{60}-1]}{0.0058}$$

$$= \frac{N6000\,[(1.00583)^{60}-1]}{0.0058}$$

$$= \frac{N6.000\,[0.4176]}{0.0058}$$

= **N432,000**

4.2.3.2 CALCULATING THE DISCOUNTED VALUE OF AN ANNUITY.

Find the discounted value of an annuity of N600 a month for 2 years at 10% rate of interest.

Solution

P = Discounted Value/ Present Value

R = Perodic Payment of the annunity = N600

i = interest = 0.00833

$$P = \frac{N600\,(1-(1+^{0.1}/_{12})^{-24}}{^{0.1}/_{12}}$$

$$= \frac{N600\,(1-(1+0.00833)^{-24}}{0.00833}$$

$$= \frac{N600\,(1-(1.00833)^{-24})}{0.00833}$$

$$= \frac{N600\,(1-0.8195)}{0.00833}$$

$$= \frac{N600 \times 0.1805}{0.00833}$$

= **N13,001.20**

= $\frac{0.07}{12}$

= 0.0058

n = number of years substitute into the equation above

= 5 x 12 = 60

because its on a monthly basis.

Recall:

$$P = \frac{R[1-(1+i)^{-n}]}{i}$$

$i = {}^{7}/_{k} = 0.00833$

n = 2 x 12 = 24

Substitute into the formula for P above as shown.

AMORTIZATION

An interest bearing debt is assumed to be amortized when the principal and interest are discharged through an equal payment made at equal intervals of time.

The periodic equal payment R is given by

$$R = A_c \frac{i}{1-(1+i)^{-nk}}$$

For example, a loan of N10,000.00 is to be amortized with equal quarterly payment over 2 years if interest is at 12% find the quarterly payment and construct an amortization schedule illustrating the payments made.

Solution

R = Periodic equal payment

A_c = Accumulated value of loan = N10,000

i = Interest = $^{7}/_{k}$ = $^{0.12}/_{4}$ = 0.03

$$\therefore R = N10,000 \times \frac{0.03}{1-[1+0.03]^{-8}}$$

N1,424.50

Recall that

$$R = A_c \frac{i}{1-(1+i)^{-nk}}$$

= i = 0.3, nk = 2 x 4 = 8

A = N10,000.00

Substilute into the formula as shown.

5. The table showing the amortization

Payment Number	Periodic Payment (₦)	Interest at 0.03 (₦)	Principal Repaid (₦)	Outstanding Prinicpal (₦)
				10,000.00
1	1424.50	300.00	1124.50	8875.50
2.	1424.50	266.27	1158.23	7717.27
3.	1424.50	231.52	1192.98	6524.29
4.	1424.50	195.73	1228.77	5291.47
5.	1424.50	158.86	1265.64	4029.83
6.	1424.50	120.89	1303.61	2726.22
7.	1424.50	81.79	1342.71	1383.51
8.	1424.50	41.51	1382.99	0

4.3 PRACTICE EXERCISE:

1. Find the simple interest on N30,000 for 5 years at 12% per annum.

2. Find the principal which earn an interest of N5,000 at 10% for 6 years.

3. Calculate the rate at which N50,000 would be invested to earn an interest of N12,500 in 2 years.

4. Owotutu cooperative society pays compound interest at 10% per annum yearly. What will be the amount in your account at the end of five years, if you save N6,000.00 with the society.

actual value after 4 years, if its depreciates at constant rate of 10% per year.

6. An investors bought 1000, N10 shares at N15. He receives a dividends of N1.20k per share at the end of the year. If he later sold the shares at N15.50. Calculate his total gain on the transaction.

7. Find the compound interest earned on N500.00 for 2years 2 months at 11.3% compounded monthly.

8. A N4000 loan is to be amortized with equal quarterly payment over 2years. If interest is at 10%, find the quarterly payment and construct an amortization schedule.

5.0 SERIES

5.1 Arithmetic Progression (AP)

Facts, and formulae to make the topic easy are stated below:

First term = a
Common difference = d
Last term = L
Number of terms = n
The linear sequence = $T_n = L$
Sum of the sequence = S_n

$\therefore T_n = a + (n-1)d$

$\therefore S_n = \frac{1}{2}n[2a + (n-1)d]$
$= n/2 [a + L]$
$= n/2 [a + T_n]$

Note: If three items x, y and z are in arithmetic progression, it implied that their common difference "d" can be represented as:

$d = y - x$
or $d = z - y$
$\therefore d = y - x = z - y$
$\therefore y - x = z - y$
$y + y = z + x$
$\therefore 2y = z + x$
$\therefore y = \frac{z + x}{2}$

$\therefore$ Arithmetic mean $= \frac{1st + last\ term}{2}$

$= \frac{a + L}{2}$

The difference betwee
successive numbers in th
series = the comm
difference.

Calculating the nth term (or last term) of a sequence.

(a) Find the T_8, T_{11} and T_{19} of each of the following series:

i) 4, 8, 12, 16 _ _ _

ii) 2, 5, 8, 11 _ _ _

iii) -3, 1, 5, 9 _ _ _

Solution:

i) Sequence: 4, 8, 12, 16 _ _ _
$a = 4$
$d = 4$
$T_n = a + (n-1)d$

When n = 8
$T_8 = 4 + [8-1]\ 4$
$= 4 + (7)4$
$= 4 + 28$
$= \underline{32}$

When n = 11
$T_{11} = 4 + (11-1)\ 4$
$= 4 + (10)4$
$= 4 + 40$
$= \underline{44}$

When n = 19
$T_{19} = 4 + (19-1)\ 4$
$= 4 + (18)4$
$= 4 + 72$
$= \underline{76}$

Recall: Changes of subject of formulae.

So, what you are going to be doing here basically is substitution into the formulae

Note: The first digit or number is always referred to as a.
Therefore a = 4
d = common difference
= 8 - 4 = 12 - 8
= 16 - 12 = 4
$T_n = a + (n-1)\ d$
Now, substitute for the various values.

ii) Sequence! 2, 5, 8, 11 _ _ _

$a = 2$

$d = 3$

$T_n = a + (n-1)d$

When n = 8

$\therefore T_8 = 2 + [8-1)\ 3$

$= 2 + (7)3$

$= 2 + 21$

$= \underline{23}$

When n =11

$\therefore T_{11} = 2 + (11-1)\ 3$

$= 2 + (10)3$

$= 2 + 30$

$= \underline{32}$

When n =19

$\therefore T_{19} = 2 + (19-1)\ 3$

$= 2 + (18)3$

$= 2 + 54$

$= \underline{56}$

ii) Sequence! -3, 1, 5, 9, _ _ _

$a = -3$

$d = 1 - (-3) = 1 + 3 = 4$

$Tn = a + (n-1)d$

When n = 8

$\therefore T_8 = -3 + [8-1)\ 4$

$= -3 + (7)4$

$= -3 + 28$

$= \underline{25}$

Use the same proce
applied above.

For this series of numb
2
d = 5 - 2
= 8 - 5
= 11 - 8
= 3

And with the given n sub
for the various data.

For this series of numbe
a = -3
d = 1 - (-3)
= 5 - 1
= 9 - 5
= 4

When n =11

$\therefore T_{11} = -3 + (11-1)\ 4$

$= -3 + (10)4$

$= -3 + 40$

$= \underline{37}$

When n =19

$\therefore T_{19} = -3 + (19-1)\ 4$

$= -3 + (18)4$

$= -3 + 72$

$= \underline{69}$

B. The first term of a linear sequence is 70 and the common difference is -5, find the 15th term of the sequence.

Solution:

Given: a = 70 - first term
d = -5 - common diff.
n= 15 - last term

$T_n = a + (n-1)\ d$

$\therefore T_n = 70 + (15-1)\ (-5)$

$= 70 + (14)(-5)$

$= 70 - 70$

$= \underline{0.}$

C. Find the last term of a series comprising of nineteen term, starting with 7 which differs from each other by 5.

With the given values of n
substitute into $T_n = a + (n-1)\ d$

First list out the given
parameters then substitute
into:
$T_n = a + (n-1)d$
to look for the last term or T_n

a = First term - = 7
d = Common difference = 5
n= Last term = 19
$T_n = a + (n-1)\,d$
$\therefore T_n = 7 + (19-1)\,5$
$= 7 + (18)5$
$= 7 + 90$
$= \underline{97}$

5.1.2. **Calculating the common difference of an AP.**

(a) The first term of a linear sequence is 3 and the 21st term is 83. Find the common difference.

Solution:

a = First term = 3
d = Common Difference = ?
n = number of term = 21
$T_n = 83$
$\therefore T_n = a + (n-1)d$
$\therefore d = \frac{T_n - a}{n-1}$
$\therefore d = \frac{83 - 3}{21 - 1}$
$= 80/20$
$= \underline{4}$

(b) The first term of an AP is -1 and the fifth term is 11. Find the common difference.

Solution:

a = First term = -1
d = Common difference = ?
n = 5

Note:

"Starting with" in this questi
means first term while "diffe
means common difference

Simply make d the subject
the relation.
$T_n = a + (n-1)d$
$\therefore T_n - a = (n-1)d$
$\therefore \frac{Tn - a}{(n-1)} = d$

Hence!

$$d = \frac{T_n - a}{n-1}$$

Now, substitute and get you
answer.

T 11
d $\frac{Tn - a}{n-1}$
$\frac{11 - (-1)}{5 - 1}$
$\frac{11 + 1}{4}$
3

(c) The first term of a series is $3\frac{1}{2}$ and the 10th term is 17. Find the common difference.

Solution

Hint a = first term = $3\frac{1}{2}$
d = common difference = ?
n = number of terms = 10
Tn = 17
d = $\frac{Tn - a}{n-1}$
$= \frac{17 - 3\frac{1}{2}}{10 - 1} = \frac{17 - 3.5}{9}$
$= 13.5/9$
$= 1.5$
$= 1\frac{1}{2}$

Same as illustrated is the above example

Calculating the first term of an AP

(a) The 8th term of an AP is 54 and the common difference is 7. Find the first term of the series.

Solution:

a = first term = ?
d = common difference = 7
n = number of terms = 8
l = 8th term = 54
l = a + (n-1)d

Make "a" the subject of the relations.
$T_n = a + (n-1)d$
from change of subject formulae
$a = T_n - (n-1)d$

$\therefore$ a = Tn - (n - 1)d
= 54 - (8-1)7
= 54 -(7)7
= 54 - 49
= **5**

(b) Given that the 11th term of an AP is - 46 and the common difference is - 5. Find the first term.

Solution:

a = first term = ?
d = common difference = -5
n = number of terms = 11
T_{11} = 11th term = -46
$\therefore$ Tn = a + (n-1)d
$\therefore$ a = T_n - (n - 1)d
= -46 - (11-1)-5
= -46 -(-50)
= -46 + 50
= **4**

(c) The 100th term of an AP is 300 and the common difference is 3. Calculate its first term.

Solution:

a = first term = ?
d = common difference = 3
n = number of terms = 100
T_{100} = 100th term = 300
$\therefore$ T_n = a + (n-1)d
$\therefore$ a = T_n - (n - 1)d
= 300 - (100-1)3
= 300 -(99) 3
= 300 - 297
= **3**

In mathematics, avoid jumping into conclusion. You can only do that after following the right procedures and using the right formula.

Note: a = T_n -(n-1)d from above.

As above a = T_n - (n-1)d

1.4 Calculating the sum of an A.P.

(a) Find the sum of the A.Ps in section 5.1.1a, up to the 19th terms.

Solution:

Hints:

i) The A.P is 4, 8, 12, 16 _ _ _
a = first term = 4
d = common difference = 4
n = number of terms = 19
S_n = n/2 [2a + (n-1) d]
S_{19} = 19/2 [2x4 + (19-1) 4]
= 19/2 [8 + 72]
= $\frac{19}{2}$ x 80
= **760**

ii. The AP is 2, 5, 8, 11 _ _ _
a = first term = 2
d = common difference = 3
n = number of terms = 19
Sn = Sum of the AP = ?
= n/2 [2a + (n-1)3]
= 19/2 [2x2 + (19-1) 3]
= 19/2 [4 + 54]
= $\frac{19}{2}$ x 58
= 19 x 29
= **551**

iii. The AP is -3, 1, 5, 9 _ _ _
a = first term = -3
d = common difference = 4
n = number of terms = 19
S_n = Sum of AP = ?
= n/2 [2a + (n-1)d]
= 19/2 [2x-3 + (19-1) 4]

From the APs in the said section 5.1.1.a, you can easily determine the first term, the common difference and the number of terms has been given in the question.

Now, substitute in the formula
S_n = n/2 [2a + (n-1) d]

For each of the questions i - iii

= 19/2 [-6 + 72]
= 19 x 66
2
= **627**

(b) Find the sum of first 13th terms of a linear sequence where the first term is 5 and last term 89.

Solution: Hints

a = first term = 5
L = last term = 89
n = number of terms = 13
S_{19} = n/2 [a + L]
= 13/2 [5 + 89]
= 13 x 94
2
= 13 x 47
= **611**

(c) The first term of an A.P is one-third of the fifth term. Find the sum of the first twelve terms of the series given that one sixth of the fifth term is 6.

Solution:

Hints:

a = first term = 1/3 T_5
T_5 = fifth term
= 6 x 6 = 36
∴ T_5 = 36
∴ a = 36/3
= **12**

Here you make use of the relation.
Sn = n/2 [a + L]
and substitute for the values.

Here you need to be very careful and reason along the line.
$T_6 \div 3 = \frac{T_6}{3} = a$
But 1/6 T5 = 6
∴ T_6 = 6 x 6
∴ $a = \frac{T_6}{3}$
= 36/3
= 12

= a + 4d
36 = 12 + 4d
36 12 = 4d
d = 24/4
= 6
= 12 d = 6 n = 12
12/2 [2 x 12 + (12-1)6)
6 [24 + 66)
6 x 90
540

CALCULATIING INCREMENT AND THE TOTAL PACKAGE OF A WORKER.

The first step on salary grade leve 4 is N10,000.00 and the last step is N13,000.00 if there are six steps with equal annual increment on this level. determine the annual increment and the total salary earned by a man who spend six years on level 4

Solution
First term = a = N10,000.00
Last term = in = N13,000.00
Common difference = d =?
Number of term = n = 6
N13,000 = N10,000 + (6-1)d
=N10,00 + 5d
N13,000 - N10,000 = 5d

d = common difference
T_5 = a + (5-1) d
36 = 12 + 4d
∴ 24 = 4d
d = 6

Now substitute in
S_n = n/2 (2a + (n-1)d)

In = a + (n -1)d
a = N10,000
n = 6 d = x
Tn = N13,000
Substitute into the formulate obtain the value of d, which is N600.00
Recall:
Sn = 1/2 (2a + (n-1)d)
or

$\therefore 5d = N3000$

$\therefore d = \frac{N3000}{5}$

$= \underline{\mathbf{N600.00}}$

Total salary = Sn

Common difference = N600

$\therefore Sn = {}^{6}/_{2}\,[2 \times 10000 + (6-1)N600]$

$= 3[N20,000 + 5 \times N600]$

$3[N20,000 + N3,000]$

$= N69,000.00$

5.2 GEOMETRIC PROGRESSION (G.P):

Facts and formulae to make the topic easy are stated below:

First term = a

Common ratio = r

Number of terms in the G.P = n

The nth term of the G.P or last term

$= Tn = ar^{n-1}$

The sum of n-terms of a G.P, when the common ratio is greater than 1.

$$S_n = \frac{a[r^n - 1]}{r - 1} \quad r > 1$$

The sum of n - terms of a GP when the common ratio is less than 1.

$$S_n = \frac{a[1 - r^n]}{1 - r} \quad r < 1$$

If the sum of the n-terms of a G.P increases towards infinity, The sum of such a G.P is

$$S\alpha = \frac{a}{1 - r} \quad r \longrightarrow \alpha$$

$Sn = {}^{n}/_{2}\,(a + T_n)$

using:

$Sn = {}^{n}/_{2}\,(9 + T_n)$

$= {}^{6}/_{2}\,(10,000 + 13,000)$

$= 3 \times N23,000.00$

$= N69,000.00$

is the total package on grade level 04.

b/a = c/b where a, b, and c are sequence of numbers

$n - 1\sqrt{\frac{T_n}{a}}$

Calculating the nth term or the last term of a G.P.

(a) The first term of a G.P is 8 and its common ratio being 3. Find the 9th term of the G.P

Solution:

a first term = 8

r common ratio = 3

n number of terms = 9

T_n nth term

ar^{n-1}

T_9 $8 \times 3^{9-1}$

8×3^8

52488

(b) Calculate the 11th term of a G.P whose common ratio is -5 and the first term 15.

Solution:

a first term = 15

r common ratio = -5

n number of terms required = 11

T_n ar^{n-1}

T_{11} $15 \times (-5)^{11-1}$

$15 \times (-5)^{10} = 15 \times 5^{10}$

146484375

1.46×10^8

From the above, the T_n of a GP = ar^{n-1}

substitute for the values:

a = 8 r = 3 n = 9

Recall

$T_n = ar^{n-1}$

The values are:

a = 15, r = -5, n = 11

You can leave the answer in index form bearing in mind that since the index is even, the answer will be positive but if the index is odd and carries a negative sign as the one in question, the answer will be negative.

(c) The first term of a GP is 4, while the common ratio is $3^1/_2$. Find the 8th term of the series.

Solution:

Hint:

a = first term = 4

n = number of term = 8

r = common ratio = $3^1/_2$

$T_n = ar^{n-1}$

$\therefore T_8 = 4 \times (3.5)^{8-1}$

$= 4 \times 3.5^7$

$= \underline{25735.72}$

5.2.2 Calculating the common ratio of a G.P.

(a) The fifth term of a GP is 7×10^4 and the first term is 7. Find the common ratio.

Solution:

a = First term = 7

n = the required term = 5

r = common ratio = ?

T_5 = The fifth term = 7×10^4

$$\therefore r = \sqrt[n-1]{\frac{Tn}{a}}$$

$$= \sqrt[5-1]{\frac{7 \times 10^4}{7}}$$

$$= \sqrt[4]{10^4}$$

$= 10$

$Tn = ar^{n-1}$

The values are:

$a = 4,\ n = 8,\ r = 3^1/_2$

From change of subject formulae.

$Tn = ar^{n-1}$

$\therefore ar^{n-1} = Tn$

$\therefore r^{n-1} = \frac{T_n}{a}$

$$\therefore r = \sqrt[n-1]{\frac{T_n}{a}}$$

Now, substitute for the values and obtain the answer.

It could be positive or negative.

(b) The common ratio of a GP is negative. If the first term is 15 and the 11th term is 3×5^{11}. Find the common ratio.

Solution:

a = First term = 15

r = common ratio = ?

$T_{11} = 3 \times 5^{11}$

n = 11

$$r = \sqrt[n-1]{\frac{T_n}{a}}$$

$$= \sqrt[11-1]{\frac{3 \times 5^{11}}{15}}$$

$$= \sqrt[10]{5^{10}}$$

$= \pm 5$

$= -5$ (Because the question say it is negative).

(c) What is the value of the common ratio of a G.P whose first term is 6 and the 7th term 2×3^7.

Solution

Hint

a = first term = 6

T_7 = seventh term = 2×3^7

$$r = \sqrt[n-1]{\frac{T_n}{a}}$$

Note:

$$\therefore r = \sqrt[n-1]{\frac{T_n}{a}}$$

Can be use to obtain both positive and negative ratios. So, its the question that will state, if the value is positive or negative.

Now, substitute for the given values to obtain the answer.

The question did not specify if the ratio is positive or negative, so we assume its positive

$$r = \sqrt[n-1]{\frac{T_n}{a}}$$

$= 7 - 1 \sqrt{\frac{2 \times 3^7}{6}}$

$= 6\sqrt{3^6}$

$= \underline{3}$

5.2.3 Calculating the first term of a G.P.

(a) Given that the fifth term of a G.P is 1250 and its common ratio is 5. Calculate the first term of the series.

Solution

Hints

a = first term = ?

n = nth term = 5

r = common ratio = 5

T_5 = fifth term = 1250

$a = \frac{T_n}{r^{n-1}}$

$= \frac{1250}{5^{5-1}}$

$= \frac{1250}{625}$

$= \underline{2}$

b. The last term of a twelve membered G.P is 3×5^{12} and the common ratio is 5. Find the first term of the series

Solution

Hints:

a = first term =?

T_{12} = twelfth term = 3×5^{12}

r = common ratio = 5

$a = \frac{T_n}{r^{n-1}}$

Substitute as usual and the answer.

From:

$T_n = ar^{n-1}$

$\therefore a = \frac{T_n}{r^{n-1}}$

Substitute and obtai answer.

Recall

$a = \frac{T_n}{r^{n-1}}$

Subtitute and get your ans

a =? n = 12 r = 5 T_{12} = 3 x

$= \frac{T_{12}}{r^{12-1}}$

$= \frac{3 \times 5^{12}}{5}$

$= \underline{15}$

c. Find the first term of a GP whose S_5 = 1562 and r = 5

Solution

Hints

r = common ratio = 5

n = number of term = 5

S_5 = sum of five terms = 1562

$\therefore a = \frac{S_n(r-1)}{r^{n-1}}$

$= \frac{1562\,(5-1)}{5^5 - 1}$

$= \frac{6248}{3124}$

$= \underline{2}$

5.2.4 Calculating the sum of G.Ps

(a) Calculate the sum of the GP

$6 + 60 + 600 \ldots\ldots\ldots 6 \times 10^6$

Solution

Hints:

a = first term = 6

r = common ratio = 10

l = last term = 6×10^6

$= ar^{n-1}$

n = number of term in the series = 7

S_7 = Sum of the GP

$= \frac{a(r^n - 1)}{r - 1}$ $r > 1$

Here

$r > 1$

$\therefore Sn = \frac{a[r^n - 1]}{r - 1}$

From change of subject formula

$\therefore S_n = \frac{a[r^n - 1]}{r^n - 1}$

$\therefore a[r^n - 1] = S_n[r - 1]$

$\therefore a = \frac{S_n[r - 1]}{r^n - 1}$

Substitute and get the value of the first term.

Here the r and n were not given directly, but you need to find them from the given data so, r can be found from

$\frac{b}{a} = \frac{c}{b} \ldots\ldots\ldots$

$\therefore r = \frac{60}{6} = \frac{600}{60}$

$= 10 = 10$

n can be found from:

a = 6 r = 10

$S_7 = \frac{6(10^7 - 1)}{10 - 1}$

$= \frac{6(10^7 - 1)}{9}$

$= \frac{2(10^7 - 1)}{3}$

$= \underline{\mathbf{6,666,666}}$

b. Calculate the sum of the tenth terms of the geometrical progression: 5, $-2\frac{1}{2}$, $1\frac{1}{4}$ - - -

Solution

Hint

a = first term = 5

n = number of terms

r = common ratio = $\frac{b}{a} = \frac{c}{b}$

$= \frac{-2.5}{5} = \frac{1.25}{-2.5}$

$= \mathbf{-0.5}$

S_n = sum of terms $= \frac{a(1 - r^n)}{1 - r}$ $r < 1$

$S_{10} = \frac{5(1-(-0.5)^{10})}{1-(-0.5)}$ $r < 1$

$= 5\left(\frac{1 - 9.765625 \times 10^{-4}}{1.5}\right)$

$= \frac{4.995}{1.5}$

$= \underline{3.33}$

$6 \times 10^6 = ar^{n-1}$

$= 6 \times 10^{n-1}$

$\therefore 6 \times 10^{n-1} = 6 \times 1$

divide both side by

$\frac{6 \times 10^{n-1}}{6} = \frac{6 \times 10}{6}$

$10^{n-1} = 10^6$

take log 10 both si

$\therefore n-1 = 6$

$\therefore n = 6 + 1$

$= 7$

Subtitute the value now a
obtain the answer.

Here you are require to fi
the r first and substitute
above.

Find the sum to infinity of GP whose first term is 24.75 and common ratio 0.75

Solution:

a = first term = 24.75

r = common ratio = 0.75

S_∞ = sum to infinity of GP

$\frac{a}{1 - r}$

$\frac{24.75}{1 - 0.75}$

$\frac{24.75}{0.25}$

99

USE OF G.P IN INVESTMENT

The following sum of money was received from an investment at the end of each year by a man: N12,800.00, N6,400 N3200, N1,600

a. What is the mans income form the investment at the end of the 10th year
b. What is the total amount of money realisable form this investment at the end of the 10th year
c. Determine the sum realizable throughtment the life of the investment.

Here, recall the formula for finding the sum to infinity. Substitute and obtain the answer.

i.e $S_\infty = \frac{a}{1 - r}$

First term = a = N12,800
Common ratio = r = 0.5
Number of years = n = 10
$\therefore T_{10} = N12,800\ (0.5)^{10-1}$
$= N12,800\ (0.5)^{9}$
= N12,800 x 0.001953125
= **N25.00**

b. $S_{10} = N12,800\ \frac{(1-0.5^{10})}{1-0.5}$
$= \frac{N12,800\ (0.999023437)}{0.5}$
= N25574.99
= **N25575.00**

e. $S\infty = {}^{a}/_{1-r}$
$= \frac{N12,800}{1-0.5}$
$= \frac{N12,800}{0.5}$
= N25,600.00

5.3 Calculations involving annuity.

(a) Find the amount an annuity of N100 payable yearly for 11 years at 10% interest per annum.

Solution:

Annuity ===> N100
Rate of Increase = 1.1
Number of years = 11
Amount of an annuity
= sum of GP = $a\ [r^{n} - 1]$
$r - 1$
$= \frac{N100\ (1.1^{11} - 1)}{1.1 - 1}$

n = 10
r = common rate
$= \frac{6400}{12.800}$
$= {}^{1}/_{2} = 0.5$
$T_{r} = ar^{n-1}$

$S_{n} = \frac{a(1 - r^{n})}{1-r}$
where r<1
substitute as usual.

Sum to infinite = sum realisable throughout the life of the investment = $\frac{a}{1-r}$

Note

The last annuity (i.e the eleven amount) dropped will not yield interest so, it will just be N100, while
$10^{th} = N100\ (1.1)$
$9^{th} = N100(1.1)^{2}$ etc
$1^{st} = N100(1.1)^{10}$
The rate of increase will serve as the common ratio, while the number of years will be n. Then we use the formula for finding the sum of a GP when r >1

0.1
N1853.00 (to the nearest naira)

Abdulwasiu purchase a car worth N192000. He pays N96,000 and agreed to pay the remaining in six instalments. If compound interest is paid at 4% per annual, what is his annual instalment to the nearest naira.

Solution

Worth of item = N192,000
Amount paid = N96,000
Balance = $N96000 \times (1.04)^{6}$
$N96000 \times (1.04)^{6} = x\ \frac{(1.04^{6} - 1)}{1.04 - 1}$
$= 6.63x$
$\frac{N96,000(1.04)^{6}}{6.63} = x$
$\equiv N18,321.36 = x$
N18,320 = x (Amount paid to the nearest naira).

Find the amount arising from annuity of N500 per annum payable yearly for 12 years, if the compound interest is paid at 5% per annum.

Solution

Annuity per annum = N500
Period of payment = 12years
Rate 5% (i.e 1.05)
Amount = $\frac{N500(1.05^{12} - 1)}{1.05 - 1}$
$= \frac{N500(1.80 - 1)}{0.05}$
= **N8000**

$r - 1$
From: rate 10%: $1 + \frac{10}{100}$
$= \frac{100 + 10}{100}$ $= \frac{110}{100}$
$= 1.1$
In this case rate will always be greater than 1 except in some special cases.

The balance of payment
N
= 192,000
= -96,000
= 96,000
Therefore, N96,000 plus compound Interest of 4%per annum will now be spread into six instalments. We now represent the amount paid per instalment as Nx.

Subtitute for the parameters in the equation $S_{n} = \frac{a(r^{n} - 1)}{r - 1}$

1. The first term of an AP is 5 and the common difference is 3. find the 11^{th} term of the series.

2. The last term of an AP is 54, the first term is 5 and 7 being the common difference. How many term are in the series.

3. The first term of a GP is 9 and the common ratio is 9. Find the 7^{th} term of the sequence

4. Calculatethe the sum of an AP comprising of thirteen terms with the first being -5 and the common difference is 5.

5. Given the GP -5, 2.5, -1.25..... calculate:

   (a) The 8^{th} term of the sequence.

   (b) The sum of the first five terms.

   (c) The sum to the inifinity of the GP.

6. The n^{th} term of a series is $\log_2(n+3)$. Find the difference between the 13^{th} term and the first term. (Hint = Substitute for n)

7. Calculate the sum of the GPs
   (i) 3; 30; 300..... 3×10^5
   (ii) 5, 50, 500..... 5×10^5
   (iii) 7, 49, 343..... 7^{11}

8. The n^{th} term of a sequence is given by, $3 \times 2^{n-2}$, what are the first five terms of the sequence.

9. The first step on salary grade 12 is N26,000.00 and the last step is N32,000.00. If there are 5 steps with equal annual increment on this level. Determine the annual increment and the total salary earned by a woman who spent 5 years on this level.

10. The following sum of money was received from an investment by a man at the end of each year: N5,000.00, N10,000.00, N20,000.00, N40,000.00 .....
    (a) Whats the man's income from the increment at the end of the 12th year
    (b) What is the total amount of money realisable from the investment.
    (c) Determine the sum realisable throughout the man's life time.

6.0 VARIATION

6.1 Variation could be said to be an act of varying the values of items in specified manner. Mathematically there are four types of variation namely direct, inverse, joint and partial variation.

Example of these forms of variation are found in our day to day activities .

6.2 **Direct Variation**

This type of variation occurs when the items or parameters involve increases with each other.i.e the increase of A leads to the increase in the value of B. So, if A varies directly as B,it would be written as: $A \alpha B$.

α = proportionality sign.

The removal of the proportionality sign between A and B will lead to the introduction of a constant k as below:

$A \alpha B$

$A = kB$ k = Constant

The next step is to look for the actual value of k or the constant, which is the determining factor in the relationship.

∴ A= KB

∴ A/B= KB/B

∴ A/B= K=Constant

Now, its important to state that after obtaining the value of the constant (k)

k= Constant or you can use any other alphabet for the constant but not A or B

* see change of subject.

or as the case may be you then substitute for it in the equation A = kB,to get the relationship or the connection between A and B.

Simple examples of Direct Variation.

If $a \alpha b$, and a = 3 when b = 36. find b when a = 6.

Solution

$a \alpha b$

$a = kb$ k = Constant

$a/b = k$

$a = 3$ when b = 36

$a = \frac{3}{}$

$b = 36 = k$

$k = 1/12$

$a = \frac{1}{12} \times b$

$= b/12$

when a = 6

$a = 1/12 \times b$

$6 = \frac{b}{12}$

$6 \times 12 = b$

$72 = b$

$b = \underline{72}$

This is a simple case of direct variation so write out the expression introducing the constant and look for its value substitute the value obtain for k in a = kb to obtain the relationship between a and b.

b The length of a rectangle varies directly as its area. When the length is 9cm,the area is 36 cm^2 Find the length when the area is 400cm^2

Solution

L = length

A = Area

$\therefore L \alpha A$

$\therefore L = KA$

$\therefore k = L/A$ =Constant

L = 9cm A = 36 cm^2

$\therefore k = 9cm/36cm^2$

= 1/4cm

when A = 400cm^2 k = 1/4 cm

L = KA

= 1/4cm x 400 cm^2

= **100cm**

3 If the table below, illustrates the relationship $x \alpha y$.Find a and b

x	1	2	3	a	11
y	21	42	63	168	b

Solution

$x \alpha y$

$\therefore x = ky$

$\therefore x/y = k$ =Constant

when x =2, y = 42

$k = x/y = \frac{2}{42}$

$k = \frac{1}{21}$

$\therefore x = ky$

$= \frac{1y}{21}$ OR $\frac{y}{21}$

Use letter to represent lengt
and area respectively a
shown in the working An
solve as above

you can pick any of the know
values to obtain your k fro
which you can proceed
obtain the data or a straight li
graph of the relationship can b
plotted and the values of a an
b obtain from the graph.

(a) when y = 168

x = 1/21 x 168

= 8

(b) when x = 11

x = 1/21y

11 = y/21

y =11 x 21

= 231

MORE EXAMPLES ON DIRECT VARIATION.

... If a α b and a = 15 when b = 1/3 find the formula connecting a and b find b when a = 40

Solution

a α b

$\therefore a = kb$

a/b = k Constant

a = 15, b = 1/3

$k = \frac{15}{1/3}$

15 x 3 = 45

The law or formula connecting a and b

a = kb

= 45b

$\therefore a = 45b$

b when a = 40

from: a = 45b

40 = 45b

$\therefore b = \frac{40}{45}$

$= 2\frac{2}{3}$

As already explained,the formula connecting a and b can be obtain by substituting the value of a in the relation a = kb. Then substitute

(b) If a varies directly as the square of b and a = 4 and b = 4 find a when b = 8

Solution

$a \propto b^2$

$\therefore a = cb^2$ [where c = constant] a = 4 , b = 4

$c = \frac{a}{b^2} = \frac{4}{4^2} = \frac{4}{4 \times 4} = \frac{1}{4}$

when b = 8

$a = \frac{1}{4} \times b^2$

$= \frac{1}{4} \times 8^2$

$= \frac{1}{4} \times 8 \times 8$

$= \underline{16}$

c. If y varies as the cube of x; y = 3 when x = 3 Find y, when x = 9

Solution

$y \propto x^3$

$\therefore y = c x^3$

$y/x^3 = c$ = constant

y= 3 when x = 3

$\therefore c = 3/3^3 = 3/ 3 X 3X 3$

$= 1/ 3X3$

$= \underline{1/9}$

$\therefore y \propto x^3$

$\therefore y = 1/9x^3$ when x = 9

$y = \frac{1}{9} \times 9x 9X9$

$= \underline{81}$

This example illustrates th
fact that you can use an
alphabet to represent you
constant

c = constant, follow th
instructions

INVERSE VARIATION

In this type of variation, as one of the parameters increases the other decreases, i.e as the value of A increases that of B decreases. So, as A varies inversely as B this would be written as

$A \propto \frac{1}{B}$

$A = k \times 1/_B$

$A = K/B$ [where k is Constant]

Therefore, the constant K for inverse variation is K = AB as against that of the direct in which K = A/B

CALCULATIONS INVOLVING INVERSE VARIATION

(a) $a \propto 1/b$ a =15 when b = 8 Find the law; a when b = 12 b when a = 20

Solution

i $a \propto 1/b$

$\therefore a = k/b$

$\therefore ab = k$ = Constant

a = 15 b = 8

$\therefore k = ab$

$= 15 X 8$

$= 120$

The law is a = **120/b**

ii when b = 12

$\therefore a = 120/b$

$= 120/12$

$= \underline{10}$

Remember the most important thing about any types of variation is the constant, So look for it first or look for k first.

Substitute for b from the law Note as b increases to 12[from10] a decrease to 10[from15] etc.

iii b when a = 20

$a = 120/b$

$\therefore 20 = 120/b$

$\therefore 20b = 120$

$b = 120/20$

$= \underline{6}$

b A varies inversely as b^2 if a = 1/6 when b = 3 find the relationship between a and b. Hence, Find b when a = 8 and a when b = 6

Solution

$a \propto 1/b^2$

$a = c/b^2$ c = Constant

$\therefore c = ab^2$

$a = 1/6$ $b = 3$

$\therefore c = ab^2$

$= \frac{1}{6} \times 3x3$

$= 1.5$

i The relationship between a and b^2 is $a = 1.5/b^2$

ii b when a= 8

$\therefore a = 1.5/b^2$

$8 = 1.5/b^2$

$8b^2 = 1.5$

$\therefore b^2 = 1.5/8$

$b = \sqrt{1.5/8}$

$= \underline{0.433}$

iii a when b = 6

$a = 1.5/b^2$

$= 1.5/6^2$

$= 1.5/36$

$= \underline{0.042}$

$ab^2 = 1.5$

$b = \sqrt{1.5/a}$

b when b is 3. Find a when b = 1.5 and b when a = 0.6

Solution

$a \propto 1/b^2$ d = Constant

$a = d/b^2$

$b = 3$ $b = 3$

$d = ab^2$ $9 \times 3^2 = 243$

i a when b = 1.5

d constant = 243

$a = d/b^2$

$243/1.5^2$

$\underline{72}$

b when a = 0.6

$a = 243/b^2$

$ab^2 = 243$

$b^2 = 243/0.6$

$b = \sqrt{405}$

$= 20.40$

JOINT VARIATION

This type of variation a quantity often depend on two or more other quantities we have the following type of combination.

Combination of two (or more) direct quantities .

$A \propto BC$ K = Constant

$A = KBC$ (where k is constant)

$A = k$

BC

Combination of one (or more)direct and one inverse quantities

Use any letter of your choice as the constant.

Make b the subject of the relation.

$b = \sqrt[3]{\frac{243}{a}}$

$A \alpha B/C$ K = Constant

$\therefore A = KB/C$

$\therefore AC/B = K = \text{Constant}$

iii Combination of two (or more) inverse quantities.

$A \alpha 1/BC$ K = Constant

$\therefore A = K/BC$

$\therefore ABC = K = \text{Constant}.$

The above are the various forms of joint variation and how the different constant are obtained.

6.4.1 Calculations Involving Joint Variation (direct quantities).

(a) $A \alpha BC$. when A = 15, B = 12 and C = 6. Find A when B = 4 C = 4.5 also B if A = 50 and C = 16.

Solution

$A \alpha BC$

$A = KBC$

K = Constant

$\therefore A/BC = K = \text{Constant}$

A = 15, B = 12 C =6

$\frac{15}{12 \times 6} = \frac{5}{24} = K$

i A when B = 4 C = 4.5

$A = KBC$

$= \frac{5}{24} \times 4 \times 4.5 = \underline{3.75}$

ii B when A = 50 C =16

$\therefore A = 5/24 \times B \times C$

$50 = 5/24 \times B \times 16$

$\therefore 50 \times 24 = 5 \times B \times 16$

This type of joint variation h
semblance with dire
variation, the basic differen
being that there are t
variables, on the left ha
side.First look for the value
K from the first set of valu
given.

5×16

$B = \frac{60}{4}$

$= \underline{15}$

$M \alpha E^2T$. when E = 3, M = 90 and T = 5. Find M when E = 7, T = 10. And also T when E = 11 and M = 3630

Solution

$M \alpha E^2T$

$M = KE^2T$ [where k is constant]

$\frac{M}{E^2T} = k$

Constant, E = 3, M = 90, T = 5

$K = \frac{M}{E^2T} = \frac{90}{3^2 \times 5}$

$= {}^{90}/_{45}$

$= \underline{2}$

$M = KE^2T = 2E^2T$

i M when E = 7 T = 10

$\therefore M = 2E^2T$

$= 2 \times 7^2 \times 10$

$= \underline{980}$

ii T when E = 11 and M =3630

$M = 2E^2T$

$M = 2E^2T$

$\therefore 3630 = 2 \times 11^2 \times T$

$\therefore T = \frac{3630}{2 \times 11^2}$

$= \underline{15}$

The more the values on the right handside should not bother you for it makes the working easy. Just follow the earlier instructions.

(One or more quantities direct and one or more quantities inverse).

a. $A \alpha {}^{B}/_{C}$ when A = 30, B = 5 and C = 6. Find A when B = 9 and c = 8.

Solution

$A \alpha {}^{B}/_{C}$

$\therefore A = k {}^{B}/_{C}$ k = Constant

$\therefore {}^{AC}/_{B} = k$ = Constant

A = 30 B = 5 C = 6

$\therefore k = {}^{AC}/_{B} = {}^{30 \times 6}/_{5}$

$= 36$

$\therefore A = {}^{36B}/_{C}$

B = 9 C = 8

$A = \frac{36 \times 9}{8}$

$= \underline{\mathbf{40.5}}$

b. $A \alpha B/C^2$ when B = 5 C = 3 and A = 20. Find the formula connecting A, B and C Hence, Find A when B = 6 and C = 4. Also C when A = 21.6 and B = 15.

Solution

$A \alpha \frac{B}{C^2}$

$\therefore A = {}^{KB}/_{C^2}$ K = Constant

$\therefore {}^{AC^2}/_{B} = K$ = Constant

A = 20 B = 5 C = 3

$\therefore K = \frac{AC^2}{B} = \frac{20 \times (3)^2}{5}$

$= 4 \times 9$

$= \underline{\mathbf{36}}$

Introduce K (i.e. the constant), find its value and substitute back into the equation.

A $36B/C^2$

i A when B = 6 and C = 4

$A = \frac{36 \times 6}{4 \times 4}$

$= \frac{9 \times 3}{2}$

$= 27/2$

$= \underline{\mathbf{13.5}}$

ii C when A = 21.6 and B = 15

$A = \frac{36B}{C^2}$

$AC^2 = 36B$

$\therefore C^2 = \frac{36B}{A}$

$C = \sqrt{\frac{36B}{A}}$

$= \sqrt{\frac{36 \times 15}{21.6}}$

$= \underline{\mathbf{5}}$

Calculation involving joint variation (all inverse)

If A varies inversely as p and r. A = 10 when p = 2 and r = 3. Find the relationship between A, p and r. Hence, find the value of A when p = 12 and r = 4. Also p when A = 80 and r = 15.

Solution

$A \alpha {}^{1}/_{pr}$

$A = {}^{K}/_{pr}$ K = Constant

A = 10 P = 2 r = 3

Make c the subject of the formula

$\therefore C = \sqrt{\frac{36B}{A}}$

see change of subject formula for detail

Remember: The first set of values given are to enable you determine the value of the constant.

$\therefore K = Apr$

$= 10 \times 2 \times 3$

$= \underline{\mathbf{60}}$

The formula connecting A, p and r is $A = {}^{60}/_{pr}$

i. A when p = 12 and r = 4

$\therefore A = {}^{60}/_{pr}$

$= \frac{60}{12 \times 4}$

$= 1{}^{1}/_{4}$

ii. p when A = 80 and r = 15

$A = {}^{60}/_{pr}$

$Apr = 60$

$p = 60/Ar$

$p = \frac{60}{80 \times 15}$

$= \mathbf{1/20}$

b $A \alpha {}^{1}/_{BCD}$. when A = 0.5, B = 10 C = 8 and D = 6. find

i A when B = 2, C = 3 and D = 4.

ii B when A = 10, C = 6 and D = 12.

iii C when B = 20, D = 1 and A = 30.

Solution

$A \times 1/BCD$

$A = K/BCD$ k = contant

$\therefore K = ABCD$

when A = 0.5, B = 10 C = 8 and D = 6

$\therefore K = ABCD$

$= 0.5 \times 10 \times 8 \times 6$

$= \underline{\mathbf{240}}$

constant and move on to so
the problem (from change
subject. formula.)

Make p the subject first
substitute to obtain the ans

Don't forget to check for
values of the constant first
foremost even, if you are
ask to write the form
connecting them.

$\therefore 240 = ABCD$

$240 = A(2)(3)(4)$

$240/2x3x4 = A$

$\therefore A = 10$

(ii) B when A = 10, C = 6 and D = 12

$240 = ABCD$

$= 10 \times B \times 6 \times 12$

$\therefore B = 240/12 \times 10 \times 6$

$= 1/3$

(iii) C when B = 20 D = 1 and A = 30

$240 = ABCD$

$= 30 \times 20 \times C \times 1$

$C = 240/600$

$= \underline{\mathbf{0.4}}$

PARTIAL VARIATION

This type of variation consists of two or more parts which are added together to form a simple equation. The name partial variation was derived from ' partial or partially varies as' which is synonymous with this type of equation.

An important feature which is commonly associated with this type of variation is that it constants of more than one constants, which could be found simultaneously or otherwise.

A typical example of this variation is A = C + kB where C and K are constants.

8.5.1 **CALCULATIONS INVOLVING PARTIAL VARIATION.**

(a) A is partly constant and partialy varies as B. When

A= 56, B =12 and when A = 120 B =20. Find

(i) The relationship between A and B

(ii) A when B = 16

(iii) B when A = 240

Solution

A = C + BK

First: A = 56 B = 12

$\therefore$ 56 = C + 12K

Second: A = 120 B = 20

120 = C + 20K

Combine equation 1 & 2

56 = C + 12K -------- (1)

120 = C + 20K --------- (2)

Subtract 1 from 2

120 = C + 20K ------ (2)

(-) 56 = C + 12K ------ (1)

64 = 8K

64/8 = K

8 = K

$\therefore$ K = **8**

Substitute 8 for K in 1.

56 = C + 12K

56 = C + 12 x 8

56 = C + 96

56- 96 = C

$\therefore$ C = -40

Note

2 and 1, can be taken to be 2+1= 3 Therefore A is partly constant 'and' partly varies as B can taken to be A = C + BK where C and K are constants respectively.

However the first two sets of numbers are to help determine the values of the constants by subtitution.

Now:

C = -40 K = 8

i The relationship between A and B is

A = 8B - 40

ii A when B = 16

A = 8B - 40

= 8 x 16 - 40

= **88**

iii B when A = 240

A = 8B - 40

240 = 8B - 40

$\therefore$ 240 + 40 = 8B

280 = 8B

$\therefore$ 8B = 280/8

= **35**

Put in the values you obtain for C and K back into the equation A = C + BK to get the formula connecting A and B or the relationship between them. A = -40 + 8B

= 8B - 40

b Union meetings are so organise that the time taken for a meeting is partly constant and partly varies as the square of the number present. If there are 20 members the meeting lasts only 1 hour 30 min but with 25 members it takes exactly 3 hours. How long will it last if there are 40 members

Solution

$t = C + Km^2$

t = time

m = members

c and k = contants.

First: m = 20

t = 1 hr 30min = **90 minutes**

$90 = c + k(20)^2$

90 = c + 400k

Reduce the values in hours to minute for easy manipulation, as follows. 1 hour= 60 min.

Second: m = 25 t = 3hrs = 3 x 60 = 180min

$180 = c + k(25)^2$

$= c + 625k$

Subtract (1) from 2

180 = c + 625k ii

(-) 90 = c + 400k i

90 = 225k

$\therefore k = 90/225$

$= \underline{0.4}$

Substituting 0.4 for k in (1)

90 = c + 400k

= c + 400(0.4)

= c + 160

90 - 160 = c

$\therefore c = -70$

c = -70 k = 0.4

$t = 0.4m^2 - 70$

with 40 members.

$t = 0.4(40)^2 - 70$

= 0.4 x 1600 - 70

= 640 - 70

= 570min

= **9 hours 30mins.**

c. If A is partly constant and partly varies as P. When A = 15, P = 21 and when A =21; P = 24, find

(i)The law of variation between A and P.

(ii) A when P = 33

(iii) P when A = 63

*Note this 9 hours 30 minutes may not be spend at a stretch, this includes the time spend on consultation before, during and after the meeting in question.

Solution

A = C + KP

When A = 15 P = 21

15 = C + 21K 1

When A = 21 P = 24

21 = C + 24K 2

(-) 15 = C + 21K

6 = 3K

$\therefore K = 6/3$

$\therefore k = \underline{2}$

Substituting 2 for k in equation 2

21 = c + 24k

21 = c + 24 (2)

= c + 48

21 - 48 = c

c = -27

k = **2**

i The law of the variation between A and P is A = 2P - 27.

ii A when P = 33

A = 2P - 27

= 2(33) - 27

= 66 - 27

= **39**

iii P when A = 63

A = 2P - 27

63 = 2P - 27

63 + 27 = 2P

90 = 2P

2P = 90

P = 90/2

= **45**

Repeat the same procedures above

6.4.2 Special Cases of Variation

Some examples will be solve below:

(a) $a \alpha b$ and $b \alpha c^2$. What is the relationship between a and c

Solution

$a \alpha b$ 1

$b \alpha c^2$ ..... 2, But b = b (the same) $\therefore$

$a \alpha c^2$

(b) $X \alpha y^2$ and $y \alpha 1/ab$; How does x vary with ab

Solution

$X \alpha y^2$

$y \alpha 1/ab$; $y = y$

$\therefore X \alpha (1/ab)^2$

$X \alpha \frac{1}{(ab)^2}$

(c) $a \alpha mc$; $m \alpha 1/E$. $c \alpha E/T$ and $\theta \alpha T$ How does a varies with E:

Solution

$a \alpha mc\theta$

$m \alpha 1/E$. $c \alpha E/T$, $\theta \alpha T$

$\therefore a \alpha [1/E^2] [E/T] [T]$

$a \alpha [1/E^2 \times E/T \times T]$

$a \alpha 1/E$

Just have it in mind that b
but not $b \neq B$

$a \alpha c^2$ because $a \alpha b$ a
αc^2 from simple logic

Simple substitution fo
various parameters with

6 PRACTICE EXERCISE

1 The three variables a,b and c are related by Joint Variation when a = 3 and b = 4, c=36 find c when a = 3 and b = 6 if (i) $c \alpha ab$ (ii) $c \alpha ab^2$ (iii) $c \alpha b/a^2$ (iv) $c \alpha b/a$ (v) $c \alpha b^2/a$

2 E varies partly as d and partly as d^2, when d = 8
E = 105.6 and when d = 10 E= 162. Find
(i) The relationship between d and E
(ii) E when d = 7 (iii) d when E = 264.

3 $a \alpha b$ and $b \alpha c^3$ find the connection between a and c. When a = 50 c = 3. What will be the value of a when c = 7.

4 (a) $c \alpha d^2$ and c = 27 when d = 6 state the law of the variation and find c when d = 10 and d when c = 243
(b) $a \alpha 1/b$ when a = 15, b = 4. Find the law connecting a and b, and a when b = 4.

5. The volume of a gas [Vcm^3] varies directlyas its absolute temperature [TK] and inversely as its pressure [p KN/ m^2]. A Volume of 195cm^3 of gas was collected at a pressure of 100 KN/m^2 and temperature of 303k. Find its Volume at s.t.p (standard temperature and pressure 273k and 101.3 KN/m^2)

7.0 ALGEBRAIC EXPRESSION AND EQUATION

7.1 Algebra is a branch of mathematics which deals with the use of letters, symbols, e.t.c. to represent quantities.This branch of mathematics will not be tedious, if one bears in mind that the letter so picked represents a particular expression.

However, to avoid distortions, some letters of the English alphabets are not frequently used e.g 0 and 1

It should also be noted that mathematical expressions are words or phrases which expresses important clue(s) about a given problem e.g.

5 years ago a man was 25 years old. How old is he now? While, an equation couldbe said to be a mathematical statement confirming that two (or more) expressions are equal eg 3x + 1 = 10 etc.

7.2 USE OF LETTERS

As explained above, the letters use in algebra may be the first letter of the quantity to be represented eg. H - Hydrogen, M for Man; C for Cats etc.

i Simple examples on use of letters

(a) Express t - Naira and Z kobo in kobo.

Solution

N1 => 100 kobo

Nt = t x 100 kobo

t Naira + z kobo

= 100t kobo + Z kobo

= **(100t + z) kobo**

Since
N1 => 100k
N2 => 2 x 100k
N3 => 3 x 100k etc.
Nt = t x 100k etc

(b) Which number is 19 times as big as 20 (ii) 19 times as big as B.

Solution

i n = the number that is 19 times as big as 20.

= 19 x 20

= **380**

Let'n' represent the unknown number we are looking for. For a number to be 1 times as big as 20, the number will be 1x20 = 20. 2 times as big as 20 = 2x20 = 40 etc.

ii for a number to be 19 times as big as B,

it must be = 19 x B

=**19B**

(c) How many kobo is in (a) ₦8 (ii) ₦P

Solution

i ₦1 = 100k

∴ ₦8 = 100k/₦1 x ₦8

= **800k**

Since
100k = N1
200k = N2
3 x 100k = N3 etc.

If more less divides, If less more divides.

ii ₦1 = 100k

∴ ₦P = 100k/₦1 x NP

= 100k x P

= **100Pk**

7.2.2 Harder examples on use of letters.

(a) A car travels at a speed of Skm/h for h-hours. How far does it go?

Solution

The speed = $Skmh^{-1}$

Total time taken = h - hours

∴ Distance = how far does it goes

= Skm/hour x h - hour

= Sh km

b. A rectangle has one of its sides as (4b + 8)cm and a perimeter of (12b + 8). Find (i) the other side (ii) Area of the rectangle.

Solution

Given side = (4b + 8)cm

Perimeter = (12b + 8)cm

∴ (12b + 8) = 2(L + B)

= 2[(4b + 8) + B]

= 2(4b + 8 + B)

= 8b + 16 + 2B

∴ 12b + 8 - 8b - 16 = 2B

4b - 8 = 2B

∴ 2B = 4b - 8

$$\therefore B = \frac{4b - 8}{2} = \frac{2[2b - 4]}{2}$$

= **(2b - 4)cm**

ii. Area of the rectangle

Area = L x B

= (2b - 4) (4b + 8)

= $8b^2$ - 32

= **8(b - 2)(b+2)cm^2**

Remember

Speed = Distance travelled / Time taken

∴ Distance travelled = Speed x Time taken

Perimeter of a rectangle

∴ (12b + 8) = 2(L + B)

= 2[(4b + 8) + B

L = Length

B = Breadth

Area = L x B

* Could be either length breadth, so use your discret in substituting for any of t sides.

Note that b and B are not t same and cannot be add together or subtracted fr each other.

$$\begin{array}{r} 2b - 4 \\ \times\ 4b + 8 \\ \hline + 16b - 32 \\ 8b^2 - 16b \\ \hline 8b^2 \quad - 32 \end{array}$$

$8[b^2 - 4] = 8 [b + 2][b - 2]$

Three years ago, a father was three times as old as his son, in five years time, the father will be twice as old as his son. What will be the sum of their ages in four years time?

Solution: Present ages

F Father's age

S Son's age

3 years ago

F - 3 = 3(S - 3)

F = 3s - 9 + 3

F = 3s - 6 i

In 5 years time

F + 5 = 2(s + 5)

= 2s + 10 ii

From above, substitute 3s - 6 for F in equation ii F + 5 = 2s +10

(3s - 6) + 5 = 2s + 10

∴ 3s - 6 + 5 = 2s + 10

3s - 1 = 2s + 10

∴ 3s -2s = 10 + 1

S = 11

Substitute 11 for s in equation i

F = 33 - 6

= 3(11) - 6

= 33 - 6

= 27

Son's present age = 11 years, Father present age = 27 years

In four years time the sum of their ages will be [(11 + 4) + (27 + 4)] = 46 years

Use letters to represent the unknown so, let F represent Father, S for Son.

However it should be substracted from the present while years to come should be added to the present ages as shown

Year ago = -

in years time = +

Present = N ; 1

7.3 Groupings with respect to open and removal of brackets.

Expressions containing more than one term can be grouped together with respect to the term. Basically, this is done by collecting like terms together or introducing bracket(s) to encompass those that have common factor(s) eg

$3a + 5b - a + 3b = 3a - a + 5b + 3b$

$= (3a - a) + (5b + 3b)$

$= \underline{2a + 8b}$

After the introduction of the bracket, by grouping like terms together, in removing the brackets terms outside the bracket multiply the ones inside the bracket to clear the brackets but care should be taken in handling the signs.

7.3.1 Calculation involving grouping with respect to opening and removal of brackets.

(a) $4m - 7n - 3m + 9n.$

Solution

$4m - 7n - 3m + 9n$

$= 4m - 3m + 9n - 7n$

$= (4m - 3m) + (9n - 7n)$

$= \underline{\mathbf{m + 2n}}$

(b) $6x - 9x + 5y + 6x + y$

$= 6x - 9x + 6x + 5y + y$

$= (6x - 9x + 6x) + (5y + y)$

$= (12x - 9x) + (6y)$

$= 3x + 6y$

First collect like terms toge
i.e those quantities v
semblance

Then: introduce bracket m
can not simplified any furt

Here after opening
removing the bracket $\underline{3x}$
obtained has a common f
which is 3.

$\frac{3(5x}{3} + \frac{6y)}{3}$

$3\ \underline{(x + 2y)}$

(c) Remove brackets from the following and simplify where possible.

(i) $3(a + b - 3)$

(ii) $-3(4m - n + 3)$

(iii) $6(a^2 - 2a - 3) - 3a(2a - 5)$

(iv) $6x - 2x(3 - (x - 8))$

(v) $2a(3a - 1) + 7(a^2 + 2)$

Solution

i $3(a + b - 3) = \underline{3a + 3b - 9}$

ii $-3(4m - n + 3)$

$= \underline{-12m + 3n - 9}$

iii $6[a^2 - 2a - 3] - 3a(2a - 5)$

$6a^2 - 12a - 18 - 6a^2 + 15a$

$6a^2 - 6a^2 - 12a + 15a - 18$

$3a - 18 = 3\left[\frac{3a}{3} - \frac{18}{3}\right]$

$3[a - 6]$

iv $6x - 2x[3 - (x - 8)]$

$6x - 2x[3 - x + 8]$

$6x - 6x + 2x^2 - 16x$

$(6x - 6x) + (2x^2 - 16x)$

$2x^2 - 16x$

$2x\left[\frac{2x^2}{2x} - \frac{16x}{2x}\right]$

$\underline{2x[x - 8]}$

So that, final answer will have a bracket as show.

*This process is known as factorization.

Recall:

$+ \times + = +$

$- \times + = -$

$+ \times - = -$

$- \times - = +$

Multiply through by the figure outside the bracket to remove the bracket.

in (iii) multiply each bracket by the figure outside it, then collect like terms together, add or subtracted them as the case may be.

In this case you start by opening the innermost bracket first and follow suit in that order.

$6x - 6x = 0$

v $2a[3a-1]+7(a+2)$

$= 6a^2 - 2a + 7a^2 + 14$

$= 6a^2 + 7a^2 - 2a + 14$

$= \mathbf{13a^2 - 2a + 14}$

7.3.2 EXPANSION OF BRACKETS

In simple term, expansion of brackets means multiplying brackets by each other or one another. And this could be achieved in two ways.

1. $(a+b)(c+d) =$

$$\begin{array}{r} a+b \\ \times \quad c+d \\ \hline ad+bd \\ ac+bc \\ \hline ac+ad+bc+bd \end{array}$$

2. $(a+b)(c+d)$

$= a(c+d) + b(c+d)$

$= ac + ad + bc + bd.$

OTHER EXAMPLES

Expand the following:

(i) $(a+b)(a+2b)$

(ii) $(3a+b)(a-2b)$

(iii) $(3x+y)^2$

(iv) $(3x-y)^2$

(v) $(x+2)(x-2)$

You will notice that the two processes led to the same answer. Therefore, we can conviniently state that the expansion of $(a+b)(c+d)$ is $ac + ad + bc + bd$.

Solution

i $(a+b)(a+2b)$

$= a[a+2b]+b[a+2b]$

$= a^2+2ab+ab+2b^2$

$= \mathbf{a^2+3ab+2b^2}$

ii $[3a+b][a-2b]$

$=$

$$\begin{array}{r} 3a+b \\ \times \quad a-2b \\ \hline -6ab-2b^2 \\ 3a^2+ab \\ \hline 3a^2-5ab-2b^2 \end{array}$$

$\therefore [3a+b][a-2b] = \mathbf{3a^2-5ab-2b^2}$

iii. $[3x+y]^2 = [3x+y][3x+y]$

$= 3x[3x+y]+y[3x+y]$

$= 9x^2+3xy+3xy+y^2$

$= 9x^2+6xy+y^2$

iv $[3x-y]^2 = [3x-y][3x-y]$

$$\begin{array}{r} 3x-y \\ \times \quad 3x-y \\ \hline -3xy+y^2 \\ 9x^2-3xy \\ \hline 9x^2-6xy+y^2 \end{array}$$

$\therefore (3x-y)^2 = 9x^2-6xy+y^2$

v $(x+2)(x-2) = x[x-2]+2[x-2]$

$= x^2 - 2x + 2x - 4$

$\therefore (x+2)(x-2) = \mathbf{x^2 - 4}$

7.3 Factorization of Expressions

This is the process of simplifying expressions by looking for their common factor or a dual process of

Use any of the two methods highlighted above.

In the first method it was treated as our normal long division with little modification.

In two one was kept constant while the other was use to multiply out

NOTE:

$[A+B]^2 = A^2+2AB+B^2$

$[A-B]^2 = A^2 - 2AB + 2$

$[A+B][A-B] = A^2 - B^2$

$A^2+B^2 = [A+B]^2 - 2AB$

bringing out the HCF (Highest Common Factor) and dividing through by the same HCF. Below are worked examples:

(a) Factorize the following expression:

i $3a^2 + 6a + 9$

ii $9x + 6x^2$

ii $-8h - 12k$

Solution

i $3a^2 + 6a + 9$

The HCF is 3

$3a^2 + 6a + 9$

$= 3\left[\frac{3a^2}{3} + \frac{6a}{3} + \frac{9}{3}\right]$

$= \underline{\mathbf{3[a^2 + 2a + 3]}}$

Find the HCF first:

$3a^2 = 3 \times a \times a$

$6a = 2 \times 3 \times a$

$9 = 3 \times 3$

HCF = 3 [Because it is t highest factor that is comm to the set of quantities]

Now: Bring it out of a brack and divide through by it.

ii $9x + 6x^2$

HCF = 3x

$9x + 6x^2 = 3x\left[\frac{9x}{3x} + \frac{6x^2}{3x}\right]$

$= \underline{\mathbf{3x[3 + 2x]}}$

$9x = 3 \times 3 \times X$

$6x^2 = 2 \times 3 \times X \times X$

HCF = 3x

iii $-8h - 12k$

The HCF is - 4

$-8h - 12k = -4\left[\frac{-8h}{-4} - \frac{12k}{4}\right]$

$= \underline{\mathbf{-4[2h + 3k]}}$

You can as well use 4 as t HCF as shown below:

$4\left[\frac{8h}{4} - \frac{12k}{4}\right]$

$= \underline{4[-2h - 3k]}$

Note: $= 4[-2h - 3k]$

$= -4[2h + 3k] = -8h - 12k$

i $18a^3b^5c^4 - 6a^3b^6c$

ii $12a^4b^2c^4 + 30a^2c^2x - 18a^2c^2x^2$

ii $16c^3d^6f - 24c^2f + 8cf^2$

Solution

i $18a^3b^5c^4 - 6a^3b^6c$

$= 6a^3b^5c\left[\frac{18a^3b^6c^4}{6a^3b^5c} - \frac{6ab^6c}{6a^3b^5c}\right]$

$= \underline{6a^3b^5c[3c^3 - b]}$

The HCF of $18a^3b^5c^4$ and $6a^3b^6c$ is $6a^3b^5c$.

Next follow the procedures stated above

ii $12a^4b^2c^4 + 30a^2c^2x - 18a^2c^2x^2$

$= \underline{6a^2c^2[2a^2b^2c^2 + 5x - 3x^2]}$

The HCF of $12a^4b^2c^4$ $30a^2c^2x$ and $18a^2c^2x^2$ is $6a^2c^2$

iii $16c^3d^6f - 24c^2f + 8cf^2$

$= \underline{8cf[2c^2d^6 - 3c + f]}$

Divide through by the HCF as in the above example to obtain the answer

c Factorize

i $[3a-b][a+2b]+[3a-b][a+3b]$

ii a^2b^2-16

iii $81a^2 - 25[b+2c]^2$

Solution

i $[3a-b][a+2b] + [3a-b][a+3b]$

$= [3a-b]\left[\frac{(3a-b)(a+2b)}{(3a-b)} + \frac{(3a-b)(a+3b)}{(3a-b)}\right]$

$= [3a-b][a+2b]+[a+3b]$

$= [3a-b][a+2b+a+3b]$

$= \underline{\mathbf{[3a-b][2a+5b]}}$

(3a - b) is the H.C.F. since it can be found on both sides of the positive sign.

ii a^2b^2-16

$= (ab)^2 - (4)^2$

$= \underline{(ab+4)(ab-4)}$

$a^2b^2 = (ab)^2$

$16 = 4^2$

Now recall

$A^2-B^2 = (A+B)(A-B)$

assume the first bracket to be A and the second B.

iii. $81a^2-25(b+2c)^2$
$= (9a)^2-[5[b+2c]]^2$
$= [9a+[5(b+c)][9a-[5[b+c]]$
$= \underline{[9a+5b+5c][9a-5b-5c]}$

This is the final answer because it can not be simplified any longer

d **FACTORIZE:**

i $103 \times 1002+87 \times 1002$
ii $200 \times 97-200 \times 87$
iii $^1/_8 \times 3200 - ^1/_8 \times 800$
iv $0.3 \times 101 + 0.3 \times 99$
v $1a^2 - 11^2$

Solution

i $103 \times 1002+87 \times 1002$
$= 1002[103+87]$
$= 1002[190]$
$= \underline{\mathbf{190380}}$

ii $200 \times 97-200 \times 87$
$= 200 [97-87]$
$= 200[10]$
$= \underline{\mathbf{2000}}$

$A^2 = (ab)^2$
$B^2 = (4)^2$
substitute in $(A+B)(A-B)$
$= (ab+4)(ab-4)$

$A^2-B^2 = (A+B)(A-B)$
Always take the first quantity to be A and the second B
$A^2 = 81a^2 = 9a \times 9a$
$= (9a)^2$
$B^2 = 25[b+2c]^2$
$= 5^2[b+2c]^2$
$= [5[b+2c]]^2$
Substitute as shown. Then open the brackets and add or subtract to obtain the answer

1002 is the HCF

The common factor is 200

iii $^1/_8[3200-^1/_8 \times 800$
$= 1/8[3200-800]$
$= 1/8[2400] = \underline{\mathbf{300}}$

iv $0.3 \times 101 + 0.3 \times 99$
$= 0.3[101+99]$
$= 0.3 \times 200$
$= \underline{\mathbf{60}}$

v 19^2-11^2
$= [19+11][19-11]$
$= [30][8]$
$= \underline{\mathbf{240}}$

FACTORISE

i $27a^3-8b^3$
ii $64d^6 + 27e^3$

Solution

i $27a^3-8b^3$
$= (3a)^3-(2b)^3$
$= [3a-2b][3a]^2+ [3a \times 2b]+[2b]^2$
$= \underline{[3a-2b][9a^2+6ab+4b^2]}$

ii $64d^6+27e^3$
$= (4d^2)^3 + [3e]^3$
$= [4d^2+3e][4d2]^2$
$[4d^2 \times 3e]+[3e]^2$
$= [4d^2+3e]+[3e]^2$
$= \underline{[4d^2+3e][16d^4-12d^2e+9e^2]}$

Common factor is 1/8

$A^3 - B^3 = [A^2 + AB + B^2]$
$A^3+B^3 = [A+B] [A^2-AB+B^2]$
$[A-B]^3 = [A-B][A^2+2AB+B^2]$

$27a^3 = 33a^3=(3a)^3$
$8b^3=2^3b^3=(2b)^3$
Now take $(3a)^3$ to be A and $(2b)^3$ as B

$A=64d^6=4^3d^6 =(4d^2)^3$
$B=27e^3=(3e)^3$

7.4 Equations with Fractions:

Fractions exist in the form a/b where a is refer to as the numerator while b is the denominator. Base on the values of a and b, we could have a proper or improper fraction while mixed fraction occurs as a result of the breaking down of improper fraction into whole number and proper fraction.

Proper fraction occurs when a is less than b eg 1/2, 1/3,2/5 e.t.c while improper fraction results when a is greater than b eg 3/2, 7/3, 5/2 e.t.c. mixed fractions are $1^1/_2$, $2^1/_3$ $2^1/_2$ e.t.c resulting from the convertion of 3/2, 7/2 and 5/2 respectively.

However, equations with fractions occurs when letters (i.e algebraic term) are found in the denominator or numerator of a fraction instead of numbers, series of examples of this will be solve below as an illustration.

7.4.1 Simplify equations with fraction

a Simplify

i 3a/5 + a/5

ii a/2+2a/3

iii b/8+2b/5

Solution

i $\frac{3a}{5} + \frac{a}{5}$

$= \frac{3a+a}{5}$

$= \mathbf{\frac{4a}{5}}$

Fraction = a/b

Proper fraction a < b

Improper fraction a > b

First look for the L.C.M of denominator of the fractio the L.C.M of 5 and 5 because 5 is a factor of 5

ii $\frac{a}{2} + \frac{2a}{3}$

$= \frac{3a + 4a}{6}$

$= \mathbf{\frac{7a}{6}}$

iii $\frac{b}{8} + \frac{2b}{5}$

$= \frac{5b + 16b}{40}$

$= \frac{21b}{40}$

b Simplify

i $\frac{3a}{7} - \frac{a}{7}$

ii $\frac{5b}{8} - \frac{3a}{2}$

iii $\frac{c}{2} - \frac{2c}{7}$

lution

i $\frac{3a}{7} - \frac{a}{7}$

$= \frac{3a - a}{7}$

$= \mathbf{\frac{2a}{7}}$

ii $\frac{5b}{8} - \frac{3a}{2}$

$= \frac{5b - 4(3a)}{8}$

$= \mathbf{\frac{5b - 12a}{8}}$

The L. C. M. of 2 and 3 is 6. Because 2 and 3 are factors of 6. Now, 2 in 6 is 3, 3 x a = 3a. 3 in 6 is 2, 2 x 2a = 4a etc.

The L. C. M. of 5 and 8 is 40.

The L. C. M. is 7.

L. C. M. of 2 and 8 is 8.

This does not simplify further

iii. $\frac{c}{2} - \frac{2c}{7}$

$= \frac{7c - 4c}{14}$

$= \frac{\mathbf{3c}}{\mathbf{14}}$

c. Simplify

i. $\frac{3}{a} + \frac{6}{2a}$

ii. $\frac{7}{4a} - \frac{5}{4b}$

iii. $\frac{4a+2}{3} + \frac{3}{4a}$

Solution

i. $\frac{3}{a} + \frac{6}{2a}$

$= \frac{2(3) + 6}{2a}$

$= \frac{6 + 6}{2a}$

$= \frac{12}{2a}$

$= \frac{\mathbf{6}}{\mathbf{a}}$

ii. $\frac{7}{4a} - \frac{5}{4b}$

$= \frac{\mathbf{7b - 5a}}{\mathbf{4ab}}$

LCM = 14

7c – 4c = 3c.

The L. C. M. of a and 2a

2a.

$\frac{12}{2a} = \frac{12}{2 \times a} = \frac{6}{a}$

The L. C. M. of 4a and 4b

4ab.

$\frac{4a+2}{3} + \frac{3}{4a}$

$= \frac{4a(4a+2) + 3(3)}{12a}$

$= \frac{\mathbf{16a^2 + 8a + 9}}{\mathbf{12a}}$

Harder equations with fraction

i $\frac{a}{a+3} + \frac{a+2}{a+4}$

ii $\frac{a}{a+2} - \frac{a-2}{a-3}$

Solution

$\frac{a}{a+3} + \frac{a+2}{a+4}$

$\frac{a(a+4) + (a+2)(a+3)}{(a+3)(a+4)}$

$\frac{a^2 + 4a + a^2 + 5a + 6}{(a+3)(a+4)}$

$\frac{\mathbf{2a^2 + 9a + 6}}{\mathbf{a^2 + 7a + 12}}$

$\frac{a}{a+2} - \frac{a-2}{a-3}$

$\frac{a(a-3) - (a-2)(a+2)}{(a+2)(a-3)}$

$\frac{a^2 - 3a - (a^2 - 4)}{(a+2)(a-3)}$

$\frac{a^2 - 3a - a^2 + 4}{(a+2)(a-3)}$

$\frac{\mathbf{4 - 3a}}{\mathbf{(a+2)(a-3)}}$

Note

4a + 2 = (4a + 2).

The L. C. M. of 3 and 4a is 12a.

Clear the bracket.

The L.C.M.of (a+3) and (a+4) is (a+3)(a+4).

(a+2)(a+3)

$= a(a+3)+2(a+3)$

$= a^2 + 3a + 2a + 6$

$= a^2 + 5a + 6$

$2a^2 + 9a + 6$

(a + 3) (a + 4) does not simplify any further but can be expanded to give $a^2 + 7a + 12$

i L.C.M = (a+2)(a-3)

ii Open the bracket by expansion.

(a - 2)(a + 2) = $a^2 - 4$

b. Simplify

i. $\frac{3}{a} + \frac{a}{2} + 3$

ii. $7 - \frac{a}{3} + \frac{3}{2a}$

Solution

i. $\frac{3}{a} + \frac{a}{2} + 3$

L.C.M = 2a

$= \frac{6 + a^2 + 6a}{2a}$

$= \mathbf{\frac{a^2 + 6a + 6}{2a}}$

ii. $7 - \frac{a}{3} + \frac{3}{2a}$

L.C.M = 6a

$= \frac{7(6a) - 2a^2 + 9}{6a}$

$= \frac{42a - 2a^2 + 9}{6a}$

$= \frac{-2a^2 + 42a + 9}{6a}$

$= \frac{-1(2a^2 - 42a - 9)}{6a}$

$= \mathbf{-\frac{1}{6a}(2a^2 - 42a - 9)}$

-1 [$2a^2$ - 42a - 9]

= $-2a^2$ + 42a + 9

c. Simplify

i. $\frac{1}{(a-2)} + \frac{2}{(a-3)} + \frac{1}{(a-1)}$

ii. $\frac{1}{(a+1)} - \frac{2}{(a+3)} + \frac{1}{(a+2)}$

Solution

$\frac{1}{(a-2)} + \frac{2}{(a-3)} + \frac{1}{(a-1)}$

i L.C.M

= (a - 1) (a - 2) (a - 3)

$\frac{(a-1)(a-3) + 2(a-1)(a-2) + (a-2)(a-3)}{(a-1)(a-2)(a-3)}$

ii Expand each of the brackets, in the numerator

(a - 1)(a - 3) = a^2 – 4a + 3

2(a - 1)(a - 2) = $2a^2$ – 6a + 4

(a - 2)(a - 3) = a^2 – 5a + 6

iii Collect like terms.

iv. Check if the numerator can factorise if not leave the answer as shown.

ii $\frac{4a + 3 + 2a^2 - 6a + 4 + a^2 - 5a + 6}{(a-1)(a-2)(a-3)}$

$\mathbf{\frac{4a^2 - 15a + 13}{(a-1)(a-2)(a-3)}}$

Simplify

i $\frac{3a}{a-1} - \frac{a}{a+1}$

$\frac{4}{(a-5)} + \frac{(a-5)}{5a}$

Solution

i $\frac{3a}{a-1} - \frac{a}{a+1}$

i L.C.M = (a+1)(a-1)

ii Open bracket and collect like terms

$\frac{3a(a+1) - a(a-1)}{(a+1)(a-1)}$

iii Factorise where possible.

$\frac{3a^2 + 3a - a^2 + a}{(a+1)(a-1)}$

$\frac{2a^2 + 4a}{(a+1)(a-1)}$

$\mathbf{\frac{2a(a+2)}{(a+1)(a-1)}}$

ii. $\frac{4}{(a-5)} + \frac{(a-5)}{5a}$

$= \frac{4(5a) + (a-5)(a-5)}{5a(a-5)}$

$= \frac{20a + (a^2 - 10a + 25)}{5a(a-5)}$

$= \frac{20a + a^2 - 10a + 25}{5a(a-5)}$

$= \frac{a^2 + 10a + 25}{5a(a-5)}$

$= \frac{(a+5)(a+5)}{5a(a-5)}$

$= \frac{(a+5)^2}{5a(a-5)}$

i L.C.M.=5a(a-5)

ii Make a common fracti
with the L.C.M. as t
common denominator.

iii Open brackets a – 5

x a – 5

-5a + 25

$a^2 - 5a$

$a^2 - 10a + 25$

iv Collect like terms

v Factorize where possibl

$(a+5)^2 = (a+5)(a+5)$

$= a^2 + 10a + 25$

7.5 WORD PROBLEM

These are algebraic expressions that arise from sentences.

Below is a list of words and their mathematics connotation.

	Words	Mathematical connotation
1.	Sum of, Addition of, Add – up , Sum, plus, Altogether e.t.c	+
2.	Subtract, Subtraction of, Difference e.t.c	–

Words	Mathematical connotation
Multiple of, Product of, Times Multiplication, etc.	X
Division of, Divide by etc.	÷
Successive Even numbers Differ from each by 2	n + 2 * n ⇒ The first even number eg if n = 2, the next even number will be 2 + 2 = 4, when n = 4, the next even number (4+2) is 6 but n must be an even number.
Successive odd numbers differ from each other by 2	n + 2 gives the next odd number e.g. when n = 7 the next will be (n + 2) = 7 + 2 = 9 etc Here n must be an odd number
Successive numbers differs by 1	n + 1 where n is the first the (n + 1) will be the next etc Here n can be even or odd.
Equal to, same as, gives etc.	=

Note: The unknown can be represent by any alphabet, as explained earlier. Having enumerated the above, we now give examples of how to apply them in solving statements leading to algebraic expressions

7.5.1 Simple word problems

a. If it costs Lekan ₦35 to make a cup of tea, using a sachet of Bournvita and a sachet of Cowbell and Sugar. If the Sugar costs ₦5.00 and the cost of a sachet of Bournvita is twice that of cowbell. Find the cost of the components of the tea.

Solution

Let b represents Bourvita
c represents Cowbell
s represents Sugar
Total cost of tea = N35
∴ b+c+s = N35
b = 2xc = 2c
S = N5
∴ 2c+c+N5 = N35.00
3c = N35-N5
3c = N30
C = $\frac{N30}{3}$
∴ C = N10.00
∴ Cost of Cowbell = N10.00
∴ Cost of Bournvita = 2 x N10
= N20.00
Sugar = N5
Total = **N35.00**

Tea = Sugar + Bournvita
+Cowbell
Sugar =N5
Twice =2x the other
∴ Bournvita = 2x Cowbell
Now, Substitute for b and s wi
2c and N5 respectively

b I am thinking of a number which if 16 is added to it, the result is the same as the square of nine.

Solution

n = Unknown number
$n + 16 = 9^2$
n + 16 = 81
∴ n = 81 – 16
= **65**.

A woman went to market with N1000, she bought ten sachets of indomie, a tin of Milo and exhausted the remaining amount of money on cooking edikaikan and transportation. How much did she spend on cooking and transportation if a sachet of indomie costs N27.00 (wholesale price) and a tin of Milo costs N310.00

Solution

I = Indomie = N27.00 per sachet
M = Milo = N310.00
C = Cooking & transportation
N1000 = 10 x I + M + C
= 10 x ₦30 + ₦310 + C
= ₦300 + ₦310 + C
C = ₦1000 – ₦610
= N390 for cooking and transportation.

Let n represent the unknown

same as above

7.5.2 Harder word problems

a. The perimeter of our classroom is 70m and its area is 300m^2. Find the length and bread of the class.

Solution

Let L stand for Length

B for Breadth

Perimeter = 70m Area = 300m^2

$\therefore 70 = 2(L + B)$

Divide both side by 2

$35 = L + B$ i

$300 = L \times B$

$300 = LB$ ii

From equation i

$L + B = 35$

$\therefore B = 35 - L$

Substitute (35 –L) for B in equation ii

$300 = LB$

$= L(35 - L)$

$= 35L - L^2$

$\therefore L^2 - 35L + 300 = 0$

$\therefore L^2 - 20L - 15L + 300 = 0$

$\Rightarrow (L^2 - 20L) - (15L - 300) = 0$

$\Rightarrow L(L - 20) - 15(L - 20) = 0$

$\Rightarrow (L - 20)(L - 15) = 0$

$\therefore L - 20 = 0$ OR $L - 15 = 0$

$\therefore L = 15$ or 20

When $L = 15$

$300 = LB$

$\therefore B = \frac{300}{15}$

$= 20$

Perimeter = 2(Length + Breadth)

Area = Length x Breadth

$-15L - 20L = -35L$

$-15 \times -20 = +300$

$\therefore L^2 - 15L - 20L + 300 = 0$

$\therefore [L - 15][L - 20] = 0$

See quadratic equation i chapter 8.

Note: Those are the possible values of L and B. But L i always longer than B in a rectangle.

So, L = 20 while B =15

$300 = LB$

$\therefore B = \frac{300}{20}$

$= \underline{15}$.

The Length of the class is 20m while its breadth is 15m

When four times a number is subtracted from the square of the number, the result is 45. What is the number?

Solution

Let n stand for the number

$\therefore n^2 - 4n = 45$

$\Rightarrow n^2 - 4n - 45 = 0$

$\Rightarrow n^2 - 9n + 5n - 45 = 0$

$\Rightarrow (n^2 - 9n) + (5n - 45) = 0$

$\Rightarrow n(n - 9) + 5(n - 9) = 0$

$\Rightarrow (n - 9)(n + 5) = 0$

$\therefore n - 9 = 0$ OR $n + 5 = 0$

$\therefore \mathbf{n = \underline{9}}$

(Because the number has to be a positive integer).

c. A man is 36 years old while his son is 13 years old. How many years ago was the product of their ages 210?

Mark the statement subtracted from the square i.e. $n^2 - 4n$ because it was removed form the square before, obtaining the given result.

We now look for two factors of 45 which when added gives – 4, the numbers are -9 and 5.

Solution

n = some years ago.

Father's age then = 36 – n

Son's age then = 13 – n

Product of their ages then = 210

$= (36 - n)(13 - n)$

$\therefore (36 - n)(13 - n) = 210$

$\therefore$ Open the brackets

$36(13 - n) - n(13 - n) = 210$

$468 - 36n - 13n + n^2 = 210$

$\therefore 468 - 49n + n^2 = 210$

$\therefore n^2 - 49n + 468 - 210 = 0$

$\therefore n^2 - 49n + 258 = 0$

$a = 1 \quad b = -49 \quad c = 258$

now:

substitute for the values in

$$n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$n = \frac{-(-49) \pm \sqrt{(-49)^2 - 4(1)\,258}}{2(1)}$$

$$= \frac{49 \pm \sqrt{2401 - 1032}}{2}$$

$$= \frac{49 \pm 37}{2}$$

$$= \frac{86}{2} \text{ OR } \frac{12}{2}$$

$$= 43 \text{ OR } 6$$

$\therefore$ **n = 6 years ago.**

Since the product of their ages was 210, some years ago.

Let subtract 'n' from their ages.

This type of equation are also refer to as quadratic word problem.

This type of quadratic equation can be solved by using quadratic formula:

$$x = \frac{-b \pm \sqrt{b - 4ac}}{2a}$$

$$x = n$$

a will be the coef. of n^2, b coef of n and e coef. of the constant.

Note:

We cannot pick 43 years because the father hasn't being born then and if we subtract from that of the two of them it will give negative So, six is the ideal answer i.e $30 \times 7 = 210$.

Coefficient = Coef.

1 A rectangle has one side 5cm longer than the other. The area of the rectangle is 300cm². Find its perimeter?

2 The sum of a number and its square is 30. Find the number.

3 Simplify the followings.

a. $\frac{4a + 2}{3} + \frac{49}{3}$

b. $\frac{a}{(a + 2)} - \frac{(a + 2)}{(a - 2)}$

c. $7 - \frac{3a}{(4 + 3b)} + \frac{2b}{a}$

4 The sum of the ages of a mother and her daughter is 42. If the product of their ages is 243, three years ago. Find their ages.

5 Expand the following

(i) $(a + 3)(a - 4)$

(ii) $(a + 9)^2$

(iii) $(a + b)^2 - 2ab$

8.0 QUADRATIC EXPRESSION AND EQUATION

8.1 Quadratic Expression:

They are mostly of the form $ax^2 + bx + c$ where a, b and c can be any integer but the highest power of x should be 2. Examples of quadratic expression include $x^2 - 4x - 45$, $2x^2 + 3x + 1$. etc.

These types of expressions are usually factorized and their product written as two linear expressions, taking into consideration:

i The distributive Property(ies) of the constituent.
ii Difference of two squares
iii Scissors method of factorization.
iv And/or if they could form perfect squares.

8.1.1 Solving quadratic expressions using their distributive properties.

a. Factorise the following expressions
(i) $x^2 + 2x$
(ii) $ab^2 - 4b$
(iii) $ac^2 + 4ac$

Solution

(i) $x^2 + 2x$

$= x\left[\frac{x^2}{x} + \frac{2x}{x}\right]$

$= x(x + 2)$

(ii) $ab^2 - 4b$

$= b\left[\frac{ab^2}{b} - \frac{4b}{b}\right]$

$= b(ab - 4)$

From $x^2 + 2x$
Now, we divide both of th
expression by their commo
factor which is x and plac
same outside an ope
bracket. It was explained
the previous chapter.

(iii) $ac^2 + 4ac$

$= ac\left[\frac{ac^2}{ac} + \frac{4ac}{ac}\right]$

$= ac(c + 4)$

Factorise the following expressions:
(i) $2x^2 + 6x$ (ii) $3ab^2 - 9ba^2$
(iii) $9ac^2 + 18bc$

Solution

i. $2x^2 + 6x$

$= 2x\left[\frac{2x^2}{2x} + \frac{6x}{2x}\right]$ (3)

$= 2x[x + 3]$

i. The common factor is 2x

ii. $3ab^2 - 9ba^2$

$= 3ab\left[\frac{3ab^2}{3ab} - \frac{9ba^2}{3ab}\right]$ (3)

$= 3ab[b - 3a]$

ii. The common factor is 3ab

iii $9ac^2 + 18bc$

$= 9c\left[\frac{9ac^2}{9c} + \frac{18bc}{9c}\right]$

$= 9c[ac + 2b]$

iii. The common factor is 9c

c. Factorise the expression
$9x - 81x^2$

Solution

$9x - 81x^2$

$= 9x\left[\frac{9x}{9x} - \frac{81x^2}{9x}\right]$

$= 9x[1 - 9x]$

Common factor is 9x

8.1.2 Solving quadratic expressions using difference of two squares and perfects square

a. Factorize the following:

(i) $a^2 - 4$ (ii) $9x^2 - 16$

(iii) $25a^2 - 144$

Solution

i. $a^2 - 4$

$= a^2 - (2)^2$

$= [a - 2][a + 2]$

ii. $9x^2 - 16$

$= (3x)^2 - (4)^2$

$= (3x - 4)(3x + 4)$

iii. $25a^2 - 144$

$= (5a)^2 - (12)^2$

$= (5a - 12)(5a + 12)$

b. Factorize the following

(i) $x^2 - 8x + 16$

(ii) $9x^2 - 24x + 16$

(iii) $4a^2 + 28a + 49$

Solution

i. $x^2 - 8x + 16$

$= x^2 - 4x - 4x + 16$

$= (x^2 - 4x) - (4x - 16)$

$= x(x - 4) - 4(x - 4)$

$= (x - 4)(x - 4)$

$= (x - 4)^2$

This will be treated a difference of two square where

$a^2 - b^2 = (a - b)(a + b)$

Now, A = a B = 2

Check chapter 7 for detail

$9x^2 = (3x)^2$

$16 = (4)^2$

$a \Rightarrow 9x^2 = (3x)^2$

$b \Rightarrow 16 = (4)^2$ Substitute above

$A \Rightarrow 25a2 = (5a)^2$

$B \Rightarrow 144 = (12)^2$ treat above.

The factors of 16, that will g 8 on adding are 4 and 4 (4)

On introducing the bracket sign changes.

ii. $9x^2 - 24x + 16$

$= 9x^2 - 12x - 12x + 16$

$= [9x^2 - 12x] - [12x - 16]$

$= 3x[3x - 4] - 4[3x - 4]$

$= [3x - 4][3x - 4]$

$= [3x - 4]^2$

iii. $4a^2 + 28a + 49$

$= 4a^2 + 14a + 14a + 49$

$= [4a^2 + 14a] + [14a + 49]$

$= 2a[2a + 7] + 7[2a + 7]$

$= [2a + 7][2a + 7]$

$= [2a + 7]^2$

Factorize the expression $49a - a^3$

Solution

$49a - a^3$

$= a[49 - a^2]$

$= a[7 - a][7 + a]$

Calculations involving the use of scissors method of factorization.

a. Factorize the following expressions

(i) $x^2 + 8x + 15$

(ii) $a^2 + 12a + 27$

(iii) $b^2 + 15b + 56$

Solution

i. $x^2 + 8x + 15$

$= x^2 + 3x + 5x + 15$

$8x$

$= (x^2 + 3x) + (5x + 15)$

$= x(x + 3) + 5(x + 3)$

$= (x + 3)(x + 5)$

The coefficient of x^2, which is 9 and the constant 16 are both perfect squares

$9x = (3x)^2$

$16 = 4^2$

$4a^2 = (2a)^2$

$49 = 7^2$

Common factor is a.

Then, treat as difference of two square

$A = 49 = 7^2$

$B = a^2$

In using the scissors method you look for the factors of the coefficient of the constant which when added together or subtracted will give the coefficient of x.

x 3 $5 \times 3 = 15$

x 5 $5 + 3 = 8$

ii. $a^2 + 12a + 27$

$= a^2 + 3a + 9a + 27$ (12a)

$= a(a+3) + 9(a+3)$

$= \underline{\mathbf{(a+3)(a+9)}}$

iii $b^2 + 15b + 56$

$= b^2 + 7b + 8b + 56$

$= (b^2 + 7b) + (8b + 56)$

$= b(b+7) + 8(b+7)$

$= \underline{\mathbf{(b+7)(b+8)}}$.

b. Factorize the following

i. $a^2 - 2a - 15$

ii. $a^2 - 3a - 18$

iii. $x^2 + 4x - 5$

iv. $x^2 + x - 20$

v. $b^2 - 11b + 30$

vi. $b^2 - 7b + 12$

Solution

i $a^2 - 2a - 15$

$= a^2 + 3a - 5a - 15$

$= [a^2 + 3a] - [5a + 15]$

$= a[a+3] - 5[a+3]$

$= \underline{\mathbf{[a+3][a-5]}}$.

ii. $a^2 - 3a - 18$

$= a^2 + 3a - 6a - 18$ (-3a)

$= (a^2 + 3a) - (6a + 18)$

As above. The factors of 27 are 1, 3, 9 and 27 but, the factors whose sum will add-up to give 12 are 3 and 9

a × 3 ; a × 9 $\quad 3 \times 9 = 27$, $3 + 9 = 12$

$(x - 3)(x + 9)$

Factor of are 1,2,4,8,7,14,28 and56, but the 8 and 7 which add-up to give

x × 7 ; x × 8 $\quad 7 \times 8 = 56$, $7 + 8 = 15$

$(x + 7)(x + 8)$

Factor of 15 are 1, 3, 5 and 15

Note: $-3x - 5 = +15$

$+3x - 5 = -15$

$+3 - 5 = -2$

a × +3 ; a × -5

$(a + 3)(a - 5)$

Factor of 18 are 1,2,3,6,9 and 18.

$+2x - 9 = -18$

$+2 - 9 = -7$

$= a(a+3) - 6(a+3)$

$= \underline{\mathbf{(a+3)(a-6)}}$

iii $x^2 + 4x - 5$

$= [x^2 - x + 5x - 5]$ (4x)

$= [x^2 - x] + [5x - 5]$

$= x[x - 1] + 5[x - 1]$

$= \underline{\mathbf{[x-1][x+5]}}$

iv $x^2 + x - 20$

$= x^2 - 4x + 5x - 20$ (x)

$= [x^2 - 4x] + [5x - 20]$

$= x[x - 4] + 5[x - 4]$

$= [x - 4][x + 5]$

$= \underline{\mathbf{(x+5)(x-1)}}$

v $b^2 - 11b + 30$

$= b^2 - 5b - 6b + 30$

$= [b^2 - 5b] - [6b - 30]$

$= b[b - 5] - 6[b - 5]$

$= [b - 5][b - 6]$

$3 \times 6 = 18$

$3 + 6 = 9$

$-3 \times 6 = -18$

$-3 + 6 = +3$

$+3 \times -6 = -18$

$+3 - 6 = -3$

The last two set of numbers satisfy the condition a × +3 ; a × -6

$\underline{\mathbf{(a+3)(a-6)}}$

Factor of 5 are 1 and 5

$-5 \times 1 = -5$

$5 \times -1 = -5$

$-5 + 1 = -4$

$5 - 1 = +4$

x × +5 ; x × -1 $\quad (x + 5)(x - 1)$.

Following the same procedures above

$4 \times 5 = 20$

$-4 + 5 = 1$

x × -4 ; x × +5

$[x - 4][x + 5]$

$-5 \times -6 = 30$

$-5 + -6 = -11$

$[b - 5][b - 6]$

vi. $b^2 - 7b + 12$

$= [b^2 - 3b] - [4b - 12]$

$= b[b - 3] - 4[b - 3]$

$= [b - 3][b - 4]$

-3 X -4 = 12

-3 + -4 = -7

as above

8.2 Quadratic equation

The main distinction between quadratic expression and equation is the presence of the equal signs in quadratic equation.

The presence of the equal signs in quadratic equations lead to definite conclusion unlike what was obtained in quadratic expressions above.

The general quadratic equation can be represented thus: $ax^2 + bx + c = 0$, "a" being the coefficient of x^2, b the coefficient of x and c is the constant.

However, the methods used above for solving quadratic expressions can as well be applicable in the case of quadratic equation. But, other methods that are only peculiar to quadratic equation are the graphical method [which will be handled in chapter ten] and the completing the square method, which normally lead to the quadratic formula (erroneously called the almighty formula)

1 Solving quadratic equation using the methods of factorization

a) Solve the following quadratic equation

i. $a^2 - 7a + 6 = 0$

ii. $x^2 - 8x + 16 = 0$

iii. $9x^2 - 24x + 16 = 0$

Solution

i. $a^2 - 7a + 6 = 0$

$<>$ $a^2 - a - 6a + 6 = 0$

$<>$ $[a^2 - a] - [6a - 6] = 0$

$<>$ $a[a - 1] - 6[a - 1] = 0$

$<>$ $[a - 1][a - 6] = 0$

$\therefore$ $a - 1 = 0$ OR $a - 6 = 0$

$\therefore$ **$a = 1$ OR 6**

ii. $x^2 - 8x + 16 = 0$

$<>$ $x^2 - 4x - 4x + 16 = 0$

$<>$ $x^2 - 4x - 4x + 16 = 0$

$<>$ $[x^2 - 4x] - [4x - 16] = 0$

$<>$ $x[x - 4] - 4[x - 4] = 0$

$<>$ $[x - 4][x - 4] = 0$

$x - 4 = 0$

$x = 4$

iii. $9x^2 - 24x + 16 = 0$

$<>$ $9x^2 - 12x - 12x + 16 = 0$

$<>$ $[9x^2 - 12x] - [12x - 16] = 0$

$<>$ $3x[3x - 4] - 4[3x - 4] = 0$

$<>$ $[3x - 4][3x - 4] = 0$

$3x - 4 = 0$

$3x = 4$

$x = \frac{4}{3}$

$x = 1\frac{1}{3}$

Following the same patterns above. <> Equivalance sign. Because $[a - 1][a - 6] = 0$ therefore either of the brackets or both has to be zero.

$a - 1 = 0$

$\therefore a = 1$

$a - 6 = 0$

$\therefore a = 6$

so, $a =$ **1 OR 6**

See example b1 above, note that the main difference is the presence of the equal sign(=0) The two brackets are the same i.e $[x - 4][x - 4] = 0$ so, anyone can be use to obtain the same answer.

See example 8.1.2 (bii) for detail as above.

b. Factorise the following quadratic equations

i. $x^2 + 8x + 15 = 0$

ii. $a^2 + 16a + 64 = 0$

iii. $a^2 + 12a + 27 = 0$

Solution

i. $x^2 + 8x + 15 = 0$

$\Leftrightarrow$ $x^2 + 5x + 3x + 15 = 0$

$\Leftrightarrow$ $[x^2 + 5x] + [3x + 15] = 0$

$\Leftrightarrow$ $x[x + 5] + 3[x + 15] = 0$

$\Leftrightarrow$ $[x + 5][x + 3] = 0$

$\therefore$ $x + 5 = 0$ OR $x + 3 = 0$

$\therefore$ **$x = -5$ OR -3**

ii. $a^2 + 16a + 64 = 0$

$\Leftrightarrow$ $a^2 + 8a + 8a + 64 = 0$ (16a)

$\Leftrightarrow$ $[a^2 + 8a] + [8a + 64] = 0$

$\Leftrightarrow$ $a[a + 8] + 8[a + 8] = 0$

$\Leftrightarrow$ $[a + 8][a + 8] = 0$

$\therefore$ $a + 8 = 0$ OR $a + 8 = 0$

$\therefore$ $a = \underline{-8}$

iii. $a^2 + 12a + 27 = 0$

$\Leftrightarrow$ $a^2 + 3a + 9a + 27 = 0$

$\Leftrightarrow$ $[a^2 + 3a] + [9a + 27] = 0$

$\Leftrightarrow$ $a[a + 3] + 9[a + 3] = 10$

$\Leftrightarrow$ $[a + 3][a + 9] = 0$

$\therefore$ $a + 3 = 0$ or $a + 9 = 0$

$\therefore$ **$a = -3$ or -9**

same as above

$x + 5 = 0$

$\therefore$ $x = -5$

$x + 3 = 0$

$\therefore$ $x = \underline{-3}$

$8 \times 8 = 64$

$8 + 8 = 16$

$[a + 8][a + 8] = 0$

Check

$a = -8$, substitute for a as shown

$a^2 + 16a + 64 = 0$

$(-8)^2 + 16(-8) + 64 = 0$

$64 - 128 + 64 = 0$

$128 - 128 = 0$.

Using the same procedures as above

$a + 3 = 0$

$\therefore$ $a = -3$

$a + 9 = 0$

$\therefore$ $a = -9$

All the above examples hav repeated roots.

equation.

i $a^2 - 3a - 18 = 0$

ii $a^2 - 4a - 60 = 0$

iii $a^2 = 5a$

Solution

i $a^2 - 3a - 18 = 0$

$\Leftrightarrow$ $a^2 + 3a - 6a - 18 = 0$

$\Leftrightarrow$ $[a^2 + 3a] - [6a + 18] = 0$

$\Leftrightarrow$ $a[a + 3] - 6[a + 3] = 0$

$\Leftrightarrow$ $[a + 3][a - 6] = 0$

$a + 3 = 0$ OR $a - 6 = 0$

$a = -3$ or 6

ii $a^2 - 4a - 60 = 0$

$\Leftrightarrow$ $a^2 + 6a - 10a - 60 = 0$

$\Leftrightarrow$ $[a^2 + 6a] - [10a + 60] = 0$

$\Leftrightarrow$ $a[a + 6] - 10[a + 6] = 0$

$\Leftrightarrow$ $[a + 6][a - 10] = 0$

$a + 6 = 0$ OR $a - 10 = 0$

$a = -6$ OR 10

iii. $a^2 = 5a$

$\Leftrightarrow$ $a^2 - 5a = 5a - 5a$

$\Leftrightarrow$ $a^2 - 5a = 0$

$\Leftrightarrow$ $a[a - 5] = 0$

$\therefore$ $a = 0$ OR $a - 5 = 0$

$\therefore$ **$a = 0$ OR 5**

Subtract 5a from both sides of the equation

Here $a = 0$ OR $(a - 5) = 0$

$\therefore$ $a = 0$ OR 5

Note

when $a = 0$; $a^2 = 5a$

$0 = 5 \times 0 = 0$

when $a = 5$

$a^2 = 5a$

$5^2 = 5 \times 5$

$\therefore$ $\underline{25 = 25}$

8.2.2 Deriving the quadratic formula using the method of completing the square.

Starting from the general quadratic equation: $ax^2 + bx + c = 0$

Step 1: Re-write the equation:

$$ax^2 + bx + c = 0$$

By subtracting c from both sides of the equation

$$ax^2 + bx + c - c = 0 - c$$

$$\therefore \quad ax^2 + bx = -c \quad \text{.....i}$$

Step 2: Divide equation i by the coefficient of x^2, which is a

$$ax^2 + bx = -c \quad \text{..... i}$$

$$\frac{ax^2}{a} + \frac{bx}{a} = -\frac{c}{a}$$

$$\therefore \quad x^2 + \frac{b}{a}x = -\frac{c}{a} \quad \text{..... ii}$$

Step 3: Add [half the coefficient of x] square to bothsides of equation ii

$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

$$x^2 + \frac{b}{a}x + \left[\frac{b}{2a}\right]^2 = -\frac{c}{a} + \left[\frac{b}{2a}\right]^2$$

$$\therefore \quad \left[x + \frac{b}{2a}\right]^2 = -\frac{c}{a} + \frac{b^2}{4a^2}$$

$$= \frac{b^2}{4a^2} - \frac{c}{a}$$

$$= \frac{b^2 - 4ac}{4a^2}$$

$$\therefore \quad \left[x + \frac{b}{2a}\right] = \frac{b^2 - 4ac}{4a^2} \quad \text{.....iii}$$

Note

(i) The coefficient of x^2 is a

(ii) The coefficient of x equation (ii) is $^b/_a$

(iii) Half of $\frac{b}{a} = \frac{b}{a} \times \frac{1}{2} =$

(iv) $\left[\frac{x+b}{2a}\right]^2$

$$= x^2 + \frac{bx}{a} + \frac{b^2}{4a^2}$$

$$= x^2 + \frac{bx}{a} + \left(\frac{b}{2a}\right)^2$$

bothsides of the equation iii

$$\sqrt{\left[x + \frac{b}{2a}\right]^2} = \sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$\therefore \left[x + \frac{b}{2a}\right] = \pm\frac{\sqrt{b^2 - 4ac}}{2a}$$

$$\therefore \quad x + \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{2a} \quad \text{...(iv)}$$

Step 5: Subtract $^b/_{2a}$ from both sides of equation (iv)

$$x + \frac{b}{2a} - \frac{b}{2a} = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$\therefore \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The above equation is the quadratic formula, for, it is use in solving many types of quadratic equation.

Sum of roots of quadratic equation from the above.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Let the first root of any quadratic equation in x be represented by α and the second β

$$\therefore \quad \alpha = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

$$\beta = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

$$\alpha + \beta = \frac{-b + \sqrt{b^2 - 4ac}}{2a} + \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{-b + \sqrt{b^2 - 4ac} + -b - \sqrt{b^2 - 4ac}}{2a}$$

(v) $\sqrt{\left[x + \frac{b}{2a}\right]^2} = \left[x + \frac{b}{2a}\right]$

$$\sqrt{\frac{b^2 - 4ac}{4a^2}} = \frac{\pm\sqrt{b^2 - 4ac}}{\sqrt{2a \times 2a}}$$

$$= \frac{\pm\sqrt{b^2 - 4ac}}{2a}$$

$+\sqrt{b^2 - 4ac}$

$-\sqrt{b^2 - 4ac}$

0

Because the signs cancels each other

$= \frac{-b - b}{2a}$

$= \frac{-2b}{2a}$

$= -\frac{b}{a}$

Sum of roots $= -\frac{b}{a}$

8.2.2.2 Product of roots

The products of the roots of quadratic equation from above

$\Rightarrow \alpha \times \beta \quad \alpha\beta$

$= \frac{-b + \sqrt{b^2 - 4ac}}{2a} \times \frac{-b - \sqrt{b^2 - 4ac}}{2a}$

$= \frac{4ac}{4a^2} = \frac{4ac}{4 \times a \times a} = \frac{c}{a}$

$\therefore$ Product of root $= \frac{c}{a}$

8.2.2.3 Forming quadratic equations

The general formula for forming any type of quadratic equation implies.

x^2 - [Sum of roots] x + Product of root= 0

But from above:

Sum of roots $= \alpha + \beta = \frac{-b}{a}$

Product of roots $= \alpha\beta = \frac{c}{a}$

$\therefore x^2 - [\alpha + \beta]x + [\alpha\beta] = 0$

$\Leftrightarrow x^2 - \left[\frac{-b}{a}\right]x + \left[\frac{c}{a}\right] = 0$

$x + \frac{bx}{a} + \frac{c}{a} = 0$

Numerator: $-b + \sqrt{b^2 - 4}$

$x \quad -b - \sqrt{b^2 - 4}$

$+b\sqrt{[b^2 - 4ac]} - [b^2 - 4a$

$b^2 - b\sqrt{[b^2 - 4ac]}$

$b^2 \quad - [b^2 - 4$

$\therefore b^2 - [b^2 - 4ac] = b^2 - b^2 + 4$

$= 4ac$

Denominator: $2a \times 2a = 4a$

Multiply through by a

$\therefore ax^2 + bx + c = 0$ which is the general quadratic equation

Examples: Find the respective equations with the following roots:

(i) 4 and 5
(ii) - 3 and 7
(iii) - 4 and -5

Solution

i. Given roots 4 and 5

$\therefore \alpha + \beta$ = Sum of roots

$= 4 + 5$

$= 9$

$\therefore \alpha\beta$ = Product of roots

$= 4 \times 5$

20

$\therefore$ The general equation

x^2 - [Sum of roots] x + Product of roots = 0

$\therefore$ [9]x + 20 = 0

$\Leftrightarrow x^2 - 9x + 20 = 0$ is the required equation with the given roots.

ii Given roots: -3 and 7

Sum of roots $= \alpha + \beta = -3 + 7 = 4$

Product of roots $= \alpha\beta = -3 \times 7 = -21$

The general equation

$\therefore x^2$ - [Sum of roots]x + Product of roots = 0

$x^2 - [4]x + [-21] = 0$

$x^2 - 4x - 21 = 0$ [Is the required equation].

Always assume the first number to be α and the second β, then find $\alpha + \beta$ and $\alpha\beta$. And slot it into the general equation as shown.

$\alpha + \beta$ = Sum of roots

$= -4 + (-5) = -4 - 5 = -9$

$\alpha\beta$ = Product of roots $= -4 \times -5 = 20$

x^2 - [Sum of roots]x + Product of roots $= 0$

x^2 - [-9]x + 20 $= 0$

$\therefore$ $x^2 + 9x + 20 = 0$ is the required equation.

8.2.3 Solving quadratic equation with the quadratic formula

Solve the following quadratic equation

i. $12x^2 + x - 35 = 0$

ii. $4x^2 - 11x = 3$

iii. $a^2 - 4a - 60 = 0$

Solution

i. $12x^2 + x - 35 = 0$

$a = 12$

$b = 1$

$c = -35$

$\therefore$ $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \dfrac{-[1] \pm \sqrt{[1]^2 - 4(12)(-35)}}{2(12)}$

$= \dfrac{-1 \pm \sqrt{1 + 1680}}{24}$

$= \dfrac{-1 \pm \sqrt{681}}{24}$

$= \dfrac{-1 + 41}{24}$

$= \dfrac{-1 + 41}{24}$ OR $\dfrac{-1 - 41}{24}$

$= \dfrac{40}{24}$ OR $\dfrac{-42}{24}$

$= 1.67$ OR -1.75

Note

The coefficient of x^2 is 12.

The coefficient of x is 1 wh
that of the constant is -
which are a,b and
respectively.

Substitute for the values in
$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

ii $4x^2 - 11x = 3$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$a = 4 \quad b = -11 \quad c = -3$

$x = \dfrac{-[-11] \pm \sqrt{(-11)^2 - 4(4)(-3)}}{2(4)}$

$= \dfrac{11 \pm \sqrt{121 + 48}}{8}$

$= \dfrac{11 \pm \sqrt{169}}{8}$

$= \dfrac{11 \pm 13}{8}$

$= \dfrac{24}{8}$ OR $\dfrac{-2}{8}$

$= 3$ OR $-1/4$

iii $a^2 - 4a - 60 = 0$

$a = 1$

$b = -4$

$c = -60$

$a = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \dfrac{-(-4) + \sqrt{(-4)^2 - 4(1)(-60)}}{2(1)}$

$= \dfrac{4 \pm \sqrt{16 + 240}}{2}$

$= \dfrac{4 \pm \sqrt{256}}{2}$

$= \dfrac{4 \pm 16}{2}$

$= \dfrac{20}{2}$ OR $\dfrac{-12}{2}$

$= 10$ OR -6

$4x^2 - 11x = 3$

$\Leftrightarrow 4x^2 - 11x - 3 = 0$

$a = 4$

$b = -11$

$c = -3$

Substitute into quadratic formula as above

$\dfrac{11 + 13}{8}$ OR $\dfrac{11 - 13}{8}$

$= \dfrac{24}{8}$ OR $\dfrac{-2}{8}$

Same as above
x = a in this case

1 Factorize the following

a $b^2 - 7b - 18$

b $x^2 - 4x - 5$

c $b^2 - 13b - 30$

d $b^2 + 7b + 12$

e $a^2 + 18a + 81$

2 Solving the following equations

a. $x^2 + 17x + 60 = 0$

b. $a^2 - 12a + 27 = 0$

c. $b^2 + 7b - 18 = 0$

3. Find the equation whose roots are

a. $^3/_4$ and 4

b. $-^1/_2$ and $-^2/_3$

c. -6 and $^1/_6$

SIMULTANEOUS EQUATION AND INEQUALITIES

SIMULTANEOUS EQUATIONS

Simultaneous linear equations are equations with two variables, usually x and y, in which the highest powers of the variable is one e.g.

$x + y = 3$.....i

$2x + y = 4$.....ii

These pairs of equations can be resolved by using the following methods

i Graphical method (which will be discuss in chapter 10)

ii Elimination method

iii Substitution method.

Using elimination methods in solving simultaneous equation.

In using elimination method, the pair of equation is reduced through basic arithmetic process (es) into one simple equation which can easily be solved. Below are examples

Solve the following pairs of equation using elimination method

i $x + y = 3$.....i

$2x + y = 4$.....ii

ii $2x + 2y = 10$.....i

$2x + 4y = 16$.....ii

iii $2a + 3b = 19$.....i

$a + 2b = 11$.....ii

Solution

i $x + y = 3$ i

$2x + y = 4$ ii

Subtract equation i from ii

$[2x + y] - [x + y] = 4 - 3$

$2x + y - x - y = 1$

$\therefore$ **$x = \underline{1}$**

Using equation i solve for y, when x = 1

$x + y = 3$ i

$\therefore$ $1 + y = 3$

$\therefore$ $y = 3 - 1$

$= \underline{2}$

ii. $2x + 2y = 10$ i

$2x + 4y = 16$ ii

Subtract equation i from ii

$[2x + 4y] - [2x + 2y] = 16 - 10$

$\therefore$ $2x + 4y - 2x\ 2y = 6$

$\therefore$ $2x - 2x + 4y - 2y = 6$

$\therefore$ $2y = 6$

$\therefore$ $y = {}^{6}/_{2}$

$= \underline{3}$

Using equation ii, solve for x when y = 3

$2x + 4y = 16$ ii

$\therefore$ $2x + 4(3) = 16$

$\therefore$ $2x + 12 = 16$

$\therefore$ $2x = 16 - 12$

$= 4$

$\therefore$ $x = \underline{4}$

$\underline{2}$

$= \underline{2}$

In the two equations give
you will notice that y is t
same so, the value of y c
be eliminated by subtractio
Now use any of the equat
to solve for y, after obtain
the value of x

OR

$x + y = 3$.....i

(-) $2x + y = 4$.....ii

$-x = -1$

$\therefore$ $x = {}^{-1}/_{-1}$

$= 1$

The negative sign multi
everything in equation ii

$- x + = -$

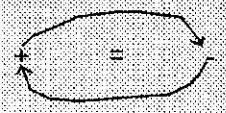
$+ x - = -$

$+ x + = +$

$- x - = +$

Always subtract the sm
from the big, as shown.
collect like terms.

When +12 crosses the
sign it becomes –12.



iii $2a + 3b = 19$ i

$a + 2b = 11$ ii

Multiply equation i by 2

$2[2a + 3b = 19]$ i

$4a + 6b = 38$ iii

Multiply equation ii by 3

$3[a + 2b = 11]$ ii

$3a + 6b = 33$ iv

Now subtract equation iv from iii

$[4a + 6b] - [3a + 6b] = 38 - 33$

$4a + 6b - 3a - 6b = 5$

$\therefore$ **$a = \underline{5}$**

Solve for b in equation ii when

$a = 5$

$a + 2b = 11$

$5 + 2b = 11$

$2b = 11 - 5$

$= 6$

$b = \underline{6}$

2

$= \underline{3}$

Solve the following equations, using the elimination method.

$x + y = 6$ i

$2x - y = 3$ ii

$2x + 3y = 3$ i

$3x - 3y = 12$.....ii

$3x + 2y = 3$.....i

$4x - 3y = 4$.....ii

There is none of the values of a and b that are equal as in the above cases.

So, to eliminate b multiply equation i by 2 and equation ii by 3, to obtain equations iii and iv respectively i.e. just interchange the coefficient of b in the two equations

Solution

i. $x + y = 6$ i

$2x - y = 3$ ii

Addition of equations i and ii

$(x + y) + (2x - y) = 6 + 3$

$x + y + 2x - y = 9$

$3x = 9$

$x = \frac{9}{3}$

$= 3$

Solve for y in equation i when x = 3

$x + y = 6$ i

$\therefore \quad 3 + y = 6$

$y = 6 - 3$

$= 3$

ii. $2x + 3y = 3$ i

$3x - 3y = 12$ ii

Addition of equations i and ii

$[2x + 3y] + [3x - 3y] = 3 + 12$

$2x + 3y + 3x - 3y = 15$

$\therefore \quad 5x = 15$

$x = \frac{15}{5}$

$\therefore \quad x = 3$

Solve for y in equation i when x = 3

$2x + 3y = 3$

$2(3) + 3y = 3$

$3y = 3 - 6$

$= -3$

$\therefore \quad y = \frac{-3}{3}$

$= -1$

Note: The value of y cancels out with simple addition. It can as well be handle in this way:

$x + y = 6$

$(+) \quad 2x - y = 3$

$3x = 9$

$x = \frac{9}{3}$

$= 3$

Here too, the values of y in equations I and ii cancels out

Recall: $3y - 3y = 0$

It can as well be handle in this way

$2x + 3y = 3$

$(+) \quad 3x - 3y = 12$

$5x = 15$

$\therefore \quad x = \frac{15}{5}$

$= 3$

iii. $3x + 2y = 3$ i

$4x - 3y = 4$ ii

Multiply equation i by 3

$3[3x + 2y = 3]$ ii

$9x + 6y = 9$ iii

Multiply equation ii by 2

$2[4x - 3y = 4]$ ii

$8x - 6y = 8$ iv

Now, add both equation iii and iv together

$[9x + 6y] + [8x - 6y] = 9 + 8$

$9x + 6y + 8x - 6y = 17$

$9x + 6y + 8x - 6y = 17$

$\therefore \quad 17x = 17$

$x = \frac{17}{17}$

$= 1$

Solve for y in equation ii when x = 1

$4x - 3y = 4$ ii

$4(1) - 3y = 4$

$4 - 3y = 4$

$\therefore \quad -3y = 4 - 4$

$\therefore \quad y = \frac{0}{-3}$

$= 0$

Solve the following equations using elimination method.

i $a - b = 10$ i

$2a - b = 25$ ii

ii $a - 2b = -3$ i

$a - b = 2$ ii

iii $3x - y = 5$ i

$x - y = -1$ ii

Because none of the values of y or x are equal you multiply equation i by 3 [which is the coefficient of y in equation ii] to obtain equation iii and equation ii by 2 to obtain equation iv respectively.

Solution

i. $a - b = 10$ i

$2a - b = 25$ ...ii

subtract equation (i) from (ii)

$[2a - b] - [a - b] = 25 - 10$

$\therefore \quad 2a - b - a + b = 15$

$\therefore \quad a = 15$

Solve for b in equation i when a = 15

$a - b = 10$ i

$15 - b = 10$

$-b = 10 - 15$

$-b = -5$

$b = \frac{+5}{+1}$

$\therefore \quad \mathbf{b = 5}$

ii. $a - 2b = -3$ i

$a - b = 2$ ii

subtract equation (i) from (ii)

$[a - b] - [a - 2b] = 2 - (-3)$

$a - b - a + 2b = 2 + 3$

$\therefore \quad b = 5$

solve for a when b = 5 in equation ii

$a - b = 2$ ii

$a - (5) = 2$

$\therefore \quad a = 2 + 5$

$= 7$

iii. $3x - y = 5$ i

(-) $x - y = -1$

$2x = 6$

$\therefore \quad x = \frac{6}{2}$

$= 3$

The values of 'b' in the two equations are the same, so they cancel by mere. Subtraction of equation i and ii. Then open the bracket as shown.

OR

$a - b = 10$.....i

(-) $2a - b = 25$ ii

$-a = -15$

$\therefore \quad a = 15$

The negative sign multiplies through equation ii

Can be treated as above because the values of a are the same.

Open the brackets as usual andcollect like terms.

It can as well be solve thus:

$a - 2b = -3$

(-) $a - b = 2$

$-b = \frac{-5}{-1}$

$b = 5$

Now, solve for a as shown.

This will be solve directly as shown $-[x - y = -1]$ becomes $-x + y = +1$

Solve for y in equation i when x = 3.

$3x - y = 5$

$3(3) - y = 5$

$9 - y = 5$

$-y = 5 - 9$

$-y = -4$

$\therefore \quad y = \frac{-4}{-1}$

$\mathbf{y = 4}$

Using substitution method in solving simultaneous equations

Use substitution method to solve the simutaneous equation

i $2x + 3y = 9$ i

$x + y = 4$ ii

ii $x + y = 3$ i

$2x + y = 4$ ii

iii $3x + y = 10$ i

$3x + 2y = 14$ ii

Solution

i $2x + 3y = 9$ i

$x + y = 4$ ii

Substitute for x in equation i

$2x + 3y = 9$

$2(4 - y) + 3y = 9$

Open the bracket

$8 - 2y + 3y = 9$

$\therefore \quad y = 9 - 8$

$= 1$

From equation ii

$x + y = 4$

$\therefore x = 4 - y$

Substitute this in equation i

From equation ii

$x + y = 4$ ii

$\therefore x = 4 - y$

$= 4 - 1$

$= \underline{3}$

ii $x + y = 3$ i

$2x + y = 4$ ii

From equation i

$x + y = 3$

$\therefore x = 3 - y$

Substitute $(3 - y)$ for x in equation ii

$2x + y = 4$ ii

$2(3 - y) + y = 4$

$6 - 2y + y = 4$

$6 - y = 4$

$-y = 4 - 6$

$-y = \frac{-2}{-1}$

$= \underline{2}$

$\therefore$ **$x = 3 - 2 = 1$**

iii $3x + y = 10$ i

$3x + 2y = 14$ ii

from equation i

$3x + y = 10$ i

$\therefore y = 10 - 3x$

$3x + y = 10$

Substitute $(10 - 3x)$ for y in equation ii

$3x + 2y = 14$ ii

$3x + 2(10 - 3x) = 14$

$3x + 20 - 6x = 14$

$-3x = 14 - 20$

$-3x = -6$

$\therefore x = \frac{-6}{-3}$

$= \underline{2}$

$x + y = 3$

Subtract y from bothside

$x + y - y = 3 - y$

$\therefore x = 3 - y$

Divide through by -1 to get

From equation i

$x = 3 - y$

$= 3 - 2$

$= 1$

$3x + y = 10$

subtract 3x from both sides

$3x + y - 3x = 10 - 3x$

$\therefore y = 10 - 3x$

After substitution:

1. Open the bracket
2. Collect like terms togeth

$-3x = -6$

$\therefore x = \frac{+6}{+3}$

$3x + y = 10$

$y = 10 - 3x$

$= 10 - 3(2)$

$= 10 - 6$

$= \underline{4}$

Solve the following pairs of equation using substitution method.

i $x - y = 2$ i

$3x + 2y = 16$ ii

ii $x - y = 4$ i

$3x + 2y = 2$ ii

iii $x - 3y = 1$ i

$2x + y = 16$ ii

Solution

i $x - y = 2$ i

$3x + 2y = 16$ ii

From equation i

$x - y = 2$ i

$\therefore x = 2 + y$

Substitute $(2 + y)$ for x in equation ii

$3x + 2y = 16$ i

$3(2 + y) + 2y = 16$

$6 + 3y + 2y = 16$

$\therefore 5y = 16 - 6$

$= 10$

$y = \frac{10}{5}$

$= \underline{2}$

After substitution:

1. Open the bracket
2. Collect like terms together.

from equation i

$x - y = 2$ i

$\therefore x = 2 + y$

$= 2 + 2$

$= \underline{4}$

ii $x - y = 4$ i

$3x + 2y = 2$ ii

from equation i

$x - y = 4$

$\therefore x = 4 + y$

Add y to both sides of equation i

$x - y + y = 4 + y$

$\therefore x = 4 + y$

Substitute $(4 + y)$ for x in equation ii

$3x + 2y = 2$ ii

$3(4 + y) + 2y = 2$

$12 + 3y + 2y = 2$

$12 + 5y = 2$

$\therefore 5y = 2 - 12$

$y = \frac{-10}{5}$

$= \underline{-2}$

solve for x in equation i

$x - y = 4$

$\therefore x = 4 + y$

$= 4 + (-2)$

$= 4 - 2$

$= \underline{2}$

iii $x - 3y = 1$ i

$2x + y = 16$ ii

from equation i

$x - 3y = 1$ i

$\therefore x = 1 + 3y$ .....i*

Substitute $(1 + 3y)$ for x in equation ii

$2x + y = 16$ ii

$2[1 + 3y] + y = 16$

$2 + 6y + y = 16$

$2 + 7y = 16$

$7y = 16 - 2$

$y = \frac{14}{7}$

$= \underline{2}$

Solve for x in equation i*

$x = 1 + 3y$ .....i*

$\therefore x = 1 + 3(2)$

$= 1 + 6$

$= \underline{7}$

C Solve the following pairs of simultaneous equation using substitution method.

i $3x - 2y = 1$ i

$x - y = 1$ ii

ii $5a - b = 8$ .....(i)

$4a - 2b = -2$ .....(ii)

iii $x - y = -2$ .....(i)

$6x - 5y = -6$ .....(ii)

Solution

i $3x - 2y = 1$ i

$x - y = 1$ ii

from equation ii

$x - y = 1$ ii

$x - y = 1$

$x - y + y = 1 + y$

$\therefore x = 1 + y$

Pick the simple equation ie $x - y = 1$

Add y to bothsides of the equation as above

Substitute (1 + y) for x in equation i

$3x - 2y = 1$ i

$3[1+y] - 2y = 1$

$3 + 3y - 2y = 1$

$3 + y = 1$

$\therefore y = 1 - 3$

$= \underline{-2}$

solve for x when y = -2

$x = 1 + y$

$= 1 + (-2)$

$= 1 - 2$

$= \underline{-1}$

ii $5a - b = 8$ i

$4a - 2b = -2$ ii

from equation i

$5a - b = 8$

$-b = 8 - 5a$

$\therefore b = 5a - 8$

Substitute (5a - 8) for b in equation ii

$4a - 2b = -2$

$4a - 2[5a - 8] = -2$

$4a - 10a + 16 = -2$

$4a - 10a = -2 - 16$

$-6a = -18$

$\therefore a = \frac{-18}{-6}$

$= \underline{3}$

Solve for b in equation i when a = 3

$5a - b = 8$

$5(3) - b = 8$

$15 - b = 8$

After substitution,
Open the bracket and
Collect like terms together.

Multiply bothsides by -1 cancel the negative sign

Open the bracket and collect like terms together.

$-b = -7$

$b = \frac{-7}{-1}$

$\mathbf{b = \underline{7}}$

iii $x - y = -2$ i

$6x - 5y = -6$ ii

from equation i

$x - y = -2$ i

$\therefore x = -2 + y$

$= y - 2$

Substitute (y - 2) for x in equation ii

$6x - 5y = -6$ ii

$6(y - 2) - 5y = -6$

$6y - 12 - 5y = -6$

$6y - 5y = -6 + 12$

$\therefore y = 6$

from equation i

$x = y - 2$

$= 6 - 2$

$= \underline{4}$

Collect like terms

3 Simutaneous linear and quadratic equations

a Solve the following pairs of equation

i $x + y = 5$ i

$x^2 + y^2 = 13$ ii

ii. $2x + y = 17$ i

$xy = 35$ ii

iii. $3ab + a^2 = 76$ i

$a + b = 9$ ii

i. $x + y = 5$ i

$x^2 + y^2 = 13$ ...ii

from equation i

$x + y = 5$ i

$\therefore \quad x = 5 - y$

Substitute $(5 - y)$ for x in equation ii

$x^2 + y^2 = 13$ ...i

$(5 - y)^2 + y^2 = 13$

$25 - 10y + y^2 + y^2 = 13$

$25 - 10y + 2y^2 = 13$

re-arrange as below

$2y^2 - 10y + 25 - 13 = 0$

$\therefore \; 2y^2 - 10y + 12 = 0$

Divide through by 2: $2y^2 - 10y + 12 = 0$

$\therefore \quad y^2 - 5y + 6 = 0$

$\Rightarrow \quad y^2 - 2y - 3y + 6 = 0$

$\Rightarrow \quad [y^2 - 2y] - [3y - 6] = 0$

$\Rightarrow \quad y[y - 2] - 3[y - 2] = 0$

$\Rightarrow \quad [y - 2] \quad [y - 3] = 0$

$\therefore \quad y - 2 = 0$ OR $y - 3 = 0$

$\therefore \quad \mathbf{y = \underline{2 \text{ OR } 3}}$

solve for x when y = 2 from equation

$x + y = 5$

$\therefore \; x + 2 = 5$

$\therefore \quad x = 5 - 2$

$= 3$

when $y = 3$

$x = 5 - y$

$= 5 - 3$

$\therefore \quad \mathbf{x = \underline{2}}$

The same principle as
will be useful in tackling
sets of equation.

$5 - y$

$5 - y$

$-5 + y^2$

$25 - 5y$

$25 - 10y + y^2$

Remember
Factorization of quadr
chapter 8.

when x = 2, y = 3,
when x = 3, y =2

Note
Because of the presen
quadratic each of
parameter will have
solution except when the
the same

$xy = 35$ ii

from equation i

$2x + y = 17$ i

$y = 17 - 2x$ ....i*

substitute for y in equation ii

$xy = 35$ ii

$x[17 - 2x] = 35$

$17x - 2x^2 = 35$

$0 = 35 - 17x + 2x^2$

$2x^2 - 17x + 35 = 0$

$x = \dfrac{-(-17) \pm \sqrt{(17)^2 - 4(2)(-35)}}{2(2)}$

$= \dfrac{17 \pm \sqrt{289 + 280}}{4}$

$= \dfrac{17 \pm \sqrt{9}}{4}$

$= \dfrac{17 + 3}{4}$ OR $\dfrac{17 - 3}{4}$

$= \dfrac{20}{4}$ $\qquad$ $\dfrac{-14}{4}$

$= \mathbf{5 \text{ OR } 3.5}$

when x = 5 from equation i* above

$\therefore \; y = 17 - 2x$

$= 17 - 2(5)$

$= 17 - 10$

$= \underline{7}$

when x = 3.5

$\therefore \; y = 17 - 2(3.5)$

$= 17 - 7$

$= \underline{10}$

$y = 17 - 2x$ from equation i
Substitute and open bracket

Then re-arrange as shown use quadratic formula

$x = \dfrac{-b \pm \sqrt{b2 - 4ac}}{2a}$

a = 2
b = 17
c = -35
Substitute a,b and c respectively.

ii

ii Same procedure as above

= b) for a in equation i

= 76 i

- b) + (9 - b)² = 76

27b - 9b + 81 - 18b + b² = 76.

Collect the like terms

3b² + b² + 27b - 18b + 81 = 76

Collect like terms

-3b² + b² + 27b - 18b + 81 = 76

-2b² + 9b + 81 = 76

∴ 0 = 2b² - 9b - 81 + 76

∴ 0 = 2b² - 9b - 5

A = 2 B = -9 C = -5

$$\therefore b = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$\therefore b = \frac{-(-9) \pm \sqrt{(-9)^2 - 4(2)(-5)}}{2(2)}$$

$$= \frac{9 \pm \sqrt{81 + 40}}{4}$$

$$= \frac{9 + 11}{4}$$

$$= \frac{20}{4} \text{ or } \frac{-2}{4}$$

$$= \underline{5 \text{ or } -\tfrac{1}{2}}$$

when b = 5

from a = 9 - b

= 9 - 5

= **4**

We shall use equation ii because it is more simpler than equation i.

Note: 3ab = 3ba

Substitute for a in equation i

Open bracket and expand $(9 - b)^2$ to give $81 - 18b + b^2$ having obtained.

$2b^2 - 9b - 5 = 0$ by collecting like terms use quadratic formula as above.

when b = $-\frac{1}{2}$

from a = $9 - (-\frac{1}{2})$

= $9 + \frac{1}{2}$

= $\underline{9\frac{1}{2}}$.

Solve the following pairs of equation

i $x - y = 1$ i

$x^2 + y^2 = 13$ ii

ii $2a - b = 5$ i

$4a^2 - b^2 = 15$ ii

iii $2x^2 - y^2 = 2$ i

$x + y = 6$ ii

Solution

i $x - y = 1$ i

$x^2 + y^2 = 13$ ii

from equation i

$x - y = 1$ i

$\therefore x = 1 + y$

Substitute $(1 + y)$ for x in equation ii : $x^2 + y^2 = 13$ ii

$(1 + y)^2 + y^2 = 13$

$1 + 2y + y^2 + y^2 = 13$

$2y^2 + 2y + 1 = 13$

$2y^2 + 2y + 1 - 13 = 0$

$2y^2 + 2y - 12 = 0$

$y^2 + y - 6 = 0$

factorise

$y^2 - 2y + 3y - 6 = 0$

$(y^2 - 2y) + (3y - 6) = 0$

$y(y - 2) + 3(y - 2) = 0$

$(y - 2)\ (y + 3) = 0$

$y - 2 = 0$ or $y + 3 = 0$

$y = \underline{2 \text{ or } -3}$

Divide through $2y^2 + 2y - 12 = 0$ by 2 to get $y^2 + y - 6 = 0$ and factorise as shown.

when y = 2

$\therefore$ x = 1 + y

= 1 + 2

= **3**

when y = -3

$\therefore$ x = 1 + y

= 1 - 3

= **-2**

ii. $2a - b = 5$ i

$4a^2 - b^2 = 15$ ii

from equation i

$2a - b = 5$

$\therefore 2a = 5 + b$

$a = \frac{5 + b}{2}$

Substitute $\left(\frac{5 + b}{2}\right)$ for a in equation ii

$4a^2 - b^2 = 15$ ii

$4\left[\frac{5 + b}{2}\right]^2 - b^2 = 15$

$\therefore 4\left[\frac{25 + 10b + b^2}{4}\right] - b^2 = 15$

$\therefore [25 + 10b + b^2] - b^2 = 15$

$\therefore 25 + 10b + b^2 - b^2 = 15$

$25 + 10b = 15$

$\therefore 10b = 15 - 25$

$= -10$

$\therefore b = \frac{-10}{10}$

$\therefore b = -1$

Substitute -1 for b in equation i

$2a - b = 5$

$\therefore a = \frac{5 + b}{2}$

$= \frac{5 + (-1)}{2}$

$2 \times 2 = 4$

$[5 + b][5 + b] = 25 + 10b +$

$4 \times \left[\frac{25 + 10b + b^2}{4}\right]$

Replace b with -1 as usu

$a = \frac{4}{2}$

$= \mathbf{2}$

$= \frac{5 - 1}{2}$

$= \mathbf{2}$

iii $2x^2 - y^2 = 2$ i

$x + y = 6$ ii

from equation ii

$\therefore x + y = 6$

$\therefore x = 6 - y$

Substitute (6 - y) for x in equation i

$2x^2 - y^2 = 2$ i

$2(6 - y)^2 - y^2 = 2$

$2(36 - 12y + y^2) - y^2 = 2$

$72 - 24y + 2y^2 - y^2 = 2$

$72 - 24y + y^2 = 2$

$y^2 - 24y + 72 - 2 = 0$

$y^2 - 24y + 70 = 0$

Using the quadratic formula. Substitute a = 1 b -24 c = 70 into the formula.

$\therefore y = \frac{-(-24) \pm \sqrt{(-24)^2 - 4(1)(70)}}{2(1)}$

$= \frac{24 \pm \sqrt{576 - 280}}{2}$

$= \frac{24 \pm \sqrt{296}}{2}$

$= \frac{24 \pm 17.20}{2}$

$= \frac{41.2}{2}$ or $\frac{6.8}{2}$

$= 20.6$ or 3.4

When y = 20.6

$\therefore x = 6 - y$

$= 6 - 20.6$

$= \mathbf{-14.6}$

When y = 3.4

$\therefore x = 6 - y$

$= 6 - 3.4$

$= \mathbf{2.6}$

Equation ii is simpler.

$(6 - y)^2 = 36 - 12y + y^2$

Re-arrange the resultant equation

$72 - 24y + 2y^2 = 2$ to give

$y^2 - 24y + 72 - 2 = 0$.

Now, make use of quadratic formula

$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

From the equation when

x = 2.6

y = 3.4 and when x = -14.6

y = 20.6

9.2 INEQUALITIES

Inequalities are numerical open statements that make use of the following signs

- $<$ is less than
- $>$ is greater than
- $\leq$ is less than or equal to
- $\geq$ is greater than or equal to

Inequalities may be solved by carrying out the same operation on both sides of an inequality.

However, multiplying or dividing both sides of such an inequality by a negative sign or number reverses the inequality sign between them. Examples of different types will be shown below. But, its important to note that the solution of an inequality is its truth set, which may be found by finding the simplest inequality. The truth set of relation which involves only one variable may be illustrated on a number line - the picture often depends on the domain of the variable.

9.2.1 Solving simple linear inequalities.

(a) Solve the following inequalities

i. $x + 4 \geq 5$

ii. $2x > x - 3$

iii. $x - 5 < -2$

Solution

i. $x + 4 \geq 5$

Subtract 4 from both sides

$\therefore x + 4 - 4 \geq 5 - 4$

$\therefore x \geq 1$

To handle this ineq
carryout the same op
on both side of the ine

So, to obtain x subtract
both sides of the inequ

ii. $2x > x - 3$

Subtract x from both sides

$\therefore 2x - x > x - 3 - x$

$\therefore$ **$x > -3$**

iii. $x - 5 < -2$

Add 5 to both sides

$\therefore x - 5 + 5 < -2 + 5$

$\therefore$ **$x < 3$**

b. Solve the following inequalities

i. $4x - 3 \leq 3x + 3$

ii. $5 - 3x \leq 8$

iii. $a + 2 < 36 - a$

Solution

i. $4x - 3 \leq 3x + 3$

Subtract 3x from both sides

$4x - 3x - 3 \leq 3x + 3 - 3x$

$\therefore x - 3 \leq 3$

Add 3 to both sides

$x - 3 + 3 \leq 3 + 3$

$\therefore$ **$x \leq 6$**

ii. $5 - 3x \leq 8$

Subtract 5 from both sides

$5 - 5 - 3x \leq 8 - 5$

$\therefore -3x \leq 3$

Divide both sides by -3 and change the inequality sign from $\leq$ to $\geq$.

$\frac{-3x}{-3} \geq \frac{3}{-3}$

$\therefore$ **$x \geq -1$**

Since there are x on both sides, try and bring them together or eliminate the less by subracting it from the more as shown.

To obtain x add 5 to both sides of the inequalities.

On the left hand side (LHS) there are 4x while on the Right hand side (RHS) there are 3x so we just subtract the small from the big, as a way of collecting like terms.

We repeat same for the numbers too.

Remember, whenever you are dividing or multiply inequalities by negative sign the inequality sign also change as shown.

iii $a + 2 < 36 - a$

Add a to both sides

$a + a + 2 < 36 - a + a$

$\therefore\ 2a + 2 < 36$

Subtract 2 from both sides

$2a + 2 - 2 < 36 - 2$

$\therefore\ 2a < 34$

Divides both sides by 2.

$$\frac{2a}{2} < \frac{34}{2}$$

$\therefore$ **$a < 17$**

9.2.2 Solving harder types of inequalities

(a) Solve the following inequalities.

i. $\frac{1}{3}(x + 2) > \frac{1}{2} + \frac{1}{4}(x - 1)$

ii. $\frac{y + 8}{3} - \frac{2y - 4}{7} < 1$

iii. $x + \frac{3}{2} \leq \frac{5x}{3} - \frac{7}{6}$

Solution

i. $\frac{1}{3}(x + 2) > \frac{1}{2} + \frac{1}{4}(x - 1)$

Multiply through by 12.

$12 \times \frac{1}{3}[x + 2] > 12 \times \frac{1}{2} + 12 \times \frac{1}{4}[x - 1]$

$4[x + 2] > 6 + 3[x - 1]$

Open the bracket

$4x + 8 > 6 + 3x - 3$

$\therefore\ 4x + 8 > 3 + 3x$

Collect the terms starting with a as shown.

The easiest way to resolve this, is to first find the L.C.M of the denominator (i.e. 2, 3 and 4) which is 12.

Multiply through by 12 as shown and open the resulting brackets.

Subtract 3x from both sides

$4x - 3x + 8 > 3 + 3x - 3x$

$\therefore\ x + 8 > 3$

Again, subtract 8 from both sides

$x + 8 - 8 > 3 - 8$

$\therefore$ **$x > -5$.**

ii $\frac{y + 8}{3} - \frac{2y - 4}{7} < 1$

Multiply through by 21

$21 \times \frac{[y + 8]}{3} - 21 \times \frac{[2y - 4]}{7} < 1 \times 21$

$7[y + 8] - 3[2y - 4] < 21$

Collect like terms

$7y + 56 - 6y + 12 < 21$

Subtract 68 from both sides

$y + 68 - 68 < 21 - 68$

$\therefore$ **$y < -47$**

iii $x + \frac{3}{2} \leq \frac{5x}{3} - \frac{7}{6}$

Multiply through by 6

$6(x) + 6\left(\frac{3}{2}\right) \leq 6\left[\frac{5x}{3}\right] - 6\left(\frac{7}{6}\right)$

$\therefore\ 6x + 9 \leq 10x - 7$

Subtract 10x from both sides.

$6x + 9 - 10x \leq 10x - 7 - 10x$

$\therefore\ -4x + 9 \leq -7$

Subtract 9 from both sides

$-4x + 9 - 9 \leq -9 - 7$

$\therefore\ -4x \leq -16$

The L. C. M. of 3 and 7 is 21.

Open brackets and continue as shown.

Following the same steps as above.

NOTE:

As we divide by a negative sign the inequality sign changes.

Divide both sides by -4

$$\frac{-4x}{-4} \geq \frac{-16}{-4}$$

$\therefore x \geq 4.$

b. Solve the inequalities

i. $(x + 3)(x + 1) > 9 + x^2$

ii. $(x + 3)(x - 2) \leq 12 + x - x^2$

Solution

$(x + 3)(x + 1) > 9 + x^2$

Expand L. H. S

$\therefore x^2 + 4x + 3 > 9 + x^2$

Subtract x^2 from both sides

$x^2 - x^2 + 4x + 3 > 9 + x^2 - x^2$

$\therefore 4x + 3 > 9$

Subtract 3 from both sides

$4x + 3 - 3 > 9 - 3$

$\therefore 4x > 6$

$\therefore x > {}^{6}/_{4}$

$\therefore \mathbf{x > 1\,{}^{1}/_{2}.}$

ii. $(x + 3)(x - 2) \leq 12 + x - x^2$

Expand the L. H. S. of the inequality

$\therefore x^2 + x - 6 \leq 12 + x - x^2$

Add x^2 to both sides

$x^2 + x^2 + x - 6 \leq 12 + x - x^2 + x^2$

$\therefore 2x^2 + x - 6 \leq 12 + x$

Subtract x from both sides

$2x^2 + x - x - 6 \leq 12 + x - x$

$\therefore 2x^2 - 6 \leq 12$

Add 6 to both sides

$2x^2 - 6 + 6 \leq 12 + 6$

$\therefore 2x^2 \leq 18$

Divide both sides by 2

First, expand the LH
Hand Side).
i.e. $(x + 3)(x + 1)$
$= x^2 + 4x + 3.$

Treat as above
$\therefore (x + 3)(x - 2)$
$= x^2 + x - 6.$

This is one of the e
in handling ine
because the square
two possible resu

$$\frac{2x^2}{2} \leq \frac{18}{2}$$

$\therefore x^2 \leq 9.$

Take the square root of both sides

$\sqrt{x^2} \leq \sqrt{9}$

$\therefore \mathbf{x \leq \pm 3}$

PRACTICE EXERCISE

Solve the following pairs of simultaneous equations.

2. $4a - 2.2b = 15.8$ i
$3.6a + 1.4b = 0.2$ ii

$6x + 4y = 14$ i
$14x - 6y = 2$ ii

$2^{x+2y} = 1$ i
$3^{x-y} = 27$ ii
[WAEC]

Four knives and six forks cost N1.36 ; Six knives and five forks cost N1.64. Find the cost of (a) a knife (b) a fork WAEC.

$x - 3y = 1$; $x^2 - 2xy - y^2 = 7.$

Solve the following inequalities

$2(x + 3) \geq 3(2 - x)$

$3x - 5 < 5x - 3$

$\frac{5x}{3} - \frac{1}{3} - \frac{1 - 2x}{5} \leq 8 + x$

What is the truth set of $(3 + x)(1 - x) > 9 - x$?

positive and the other negative.

10.1 Graphs are diagramatical illustrations, showing the relationship between quantities. These quantities are basically the x - axis and the y - axis which stand for the horizontal and vertical axes respectively.

The type of the graph often depends on the domain of the x and y variables. The value of x and y are called the co-ordinates of a point. And, the co-ordinate of any point is written as (x, y), where x comes before y as in the counting of a, b, c - and they are separated by a coma in a bracket. So, the origin of any type of graph has the co-ordinate (0, 0).

The gradient of a graph is a fraction of the

$$\frac{\text{distance risen vertically}}{\text{distance moved horizontaly}}$$

or tan θ i.e. the tangent of the particular point required.

10.2 Types of graphs

In this text graphs will be grouped under three main sub-headings namely.

i. Linear or straight line graphs. Most of the graphs under this category satisfy the condition $y = mx + c$. where m is the gradient and c is the intercept of

also be made in this text to treat some inequality graphs and simultaneous equation graphically under this heading.

ii. Quadratic graphs. The set of graphs under this category satisfies the condition $y = ax^2 + bx + c$ where a, b and c could be any integer.

iii. Cubic curve and others.

10.2.1. Linear or straight line graphs

10.2.1.1 We will start examples of straight line graphs with the solving of some inequalities and representing the solution on number line. After which, we shall take some other types and end it with simultaneous linear relations graph.

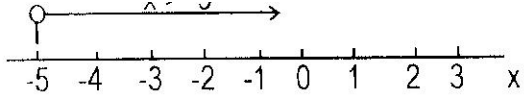
a. Show the following relations on the number line.
   i. $x > -5$
   ii. $x < 3$
   iii. $x \geq 1$

Solution

i. $x > -5$

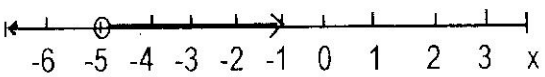
a. The graph of the relation can be drawn as

We are ask to draw the line graph of $x > -5$ so, we need only the x axis or the horinzontal axis.



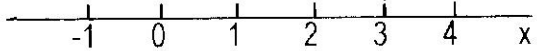
Note: The small ring or circle on -5 is not shaded or close, because x is not equal to -5.

b. The number line can also be drawn as



Note: The parts that illustrate the inequality are made darker than the rest, but in this book we shall stick to the first option (i.e a above) for the purpose of clearity.

ii. $x < 3$



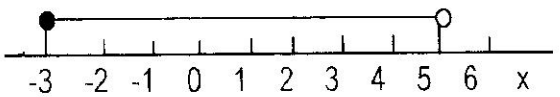
iii. $x \geq 1$

b. Draw the line graphs to show the following ranges of values of x.

i. $-3 \leq x < 5$
ii. $0 < x < 7$
iii. $-4 < x < 8$

Solution

i. $-3 \leq x < 5$



Since, x is > -5 means x is greater than -5 so x can be , -3,-4, o, 10 etc.

The relationship can be shown in the two ways a and b.

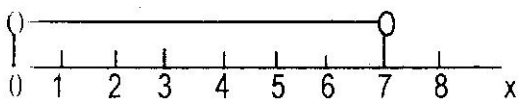
$x < 3$ means x is less than 3, which implies that x can be 2, 1 or 0 etc.

$x \geq 1$ means x is greater or equal to 1. So x can be 1, 2, 3 etc.
Now on the number line the circle will be closed because 1 is included.

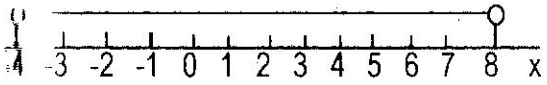
$-3 \leq x < 5$ means that -3 is less or equal to x while x is at the same time less than 5. Therefore x lies between -2 and 4.

$-3 \leq x$ and at the same time x is less than 5 that is while the second circle is not shaded.

ii $0 < x < 7$

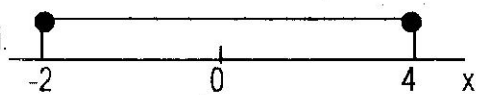


iii $-4 < x < 8$



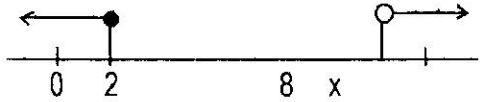
c. State the range of values of x represented by each of the number line below

i.



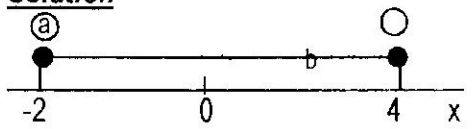
ii.

iii.



Solution

i.



a. $-2 \geq x$
b. $x \leq 4$

Combining a and b for x
$\therefore -2 \geq x \leq 4$ is the required range of x is the above number line.

Here $0 < x < 7$ means zero is less than x and x is less than 7.

So, x cannot be zero, but 1, 2, 3, etc.

Note:
The circles are shaded which means that the corresponding values are included.

Therefore -2 is equal or greater than x (because the line start from -2) and x is equal or less than 4 (becuase 4 is shaded and the line did not extend after 4.

ii

a. -1 is greater than x
b. x is less than 6 i.e -1 < x.

Combining a and b

$\therefore$ **$-1 > x < 6$**

iii.

a. $x \leq 2$
b. $x > 8$

$\therefore$ The solution to the above number line is either $x \leq 2$ or $x > 8$ and not both because they repel each other unlike the above examples which show complete attraction (see i and ii above.)

10.2.1.2 **Cartesian Graphs**

Before taking examples on the cartesian graphs, we would like to show the cartesian plane itself, having explained above that the coordinates of any point is written as (x, y).

We also need to emphasize that regions not wanted on the cartesian plane are always shaded.

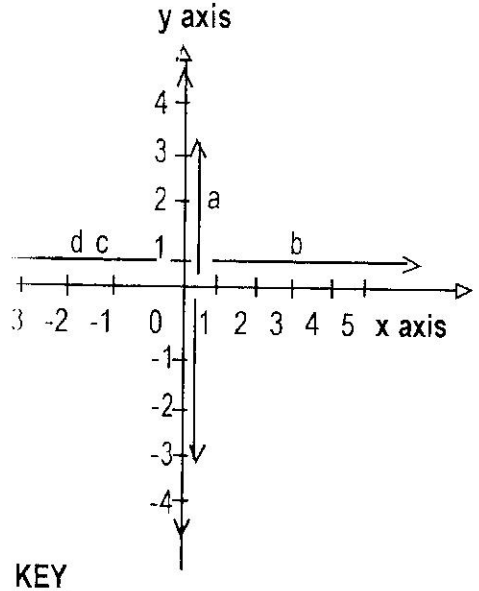
Here

The two circles are not close
or shaded, this means they ar
not included.

So, -1 is greater than x and
is also less than 6.

Here: The two arrows poin
towards different directions i.e
a points towards the LHS t
show that 2 is less or equal t
x while (b) point towards the
right to show that x is greater
than 8.

These two set of inequalities
cannot be combined together.



KEY

a $\Rightarrow$ The arrow shows the positive parts of the y - axis from the origin. i.e. upwards from the x - axis.

b $\Rightarrow$ Shows the negative parts of y-axis from the origin i.e. downwards from the x-axis.

c $\Rightarrow$ This arrow shows the positive side of the x-axis which right hand side of y-axis.

d $\Rightarrow$ Shows the negative side of x-axis which is on the left hand side of y-axis.

0 $\Rightarrow$ Is the origin i.e. the point at which the x and y axis meet. It has a known coordinate of (0, 0).

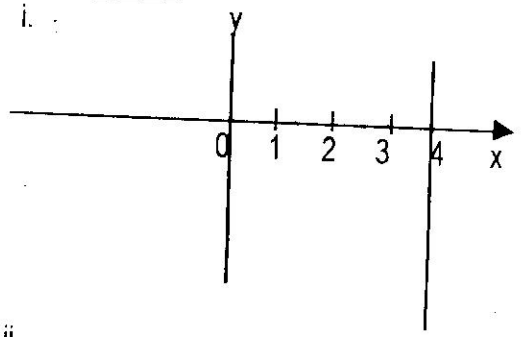
Moreover, its important to note that on the cartesian plane, equal to is shown by a thick straight line while less than

a On the cartesian plane, sketch the regions show by the following:

i. $x = 4$
ii. $x \geq 4$
iii. $y < 5$
iv. $y \geq -6$
v. $y = 3$
vi. $x \leq -1$

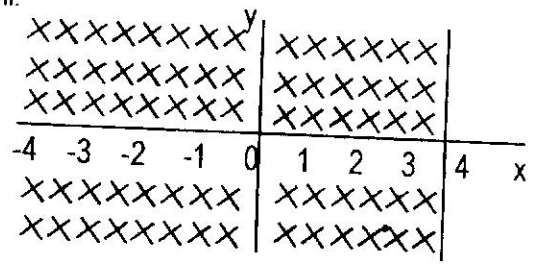
Solution

i.



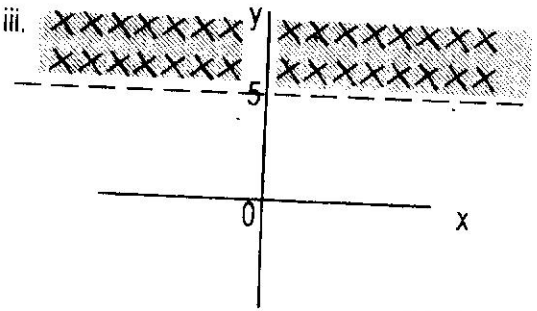
Note: x is equal to 4 on the thick line which is parallel to the y axis at any point.

ii.



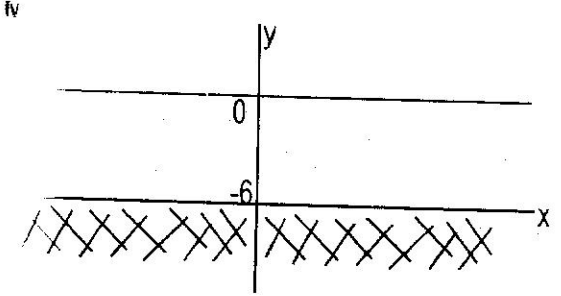
Note: Because x is equal or greater than 4, the shaded parts are not required, since they are less than 4.

iii.



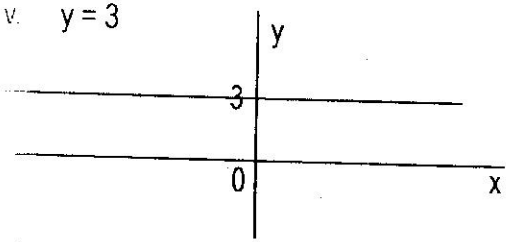
Note: $y < 5$ means y is less than 5, which implies its not 5, that necessitates the use of the broken line or dotted line. Which is parallel to the x axis. The shaded parts are not required.

iv



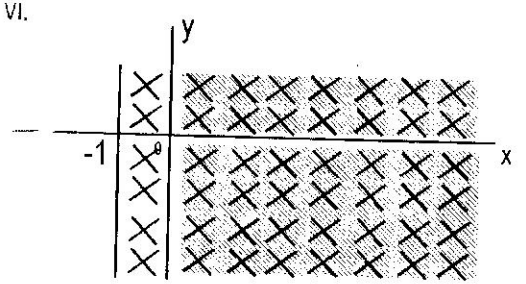
Note: The thick line indicate that its equal to -6 while the shaded part show the unwanted part

v. $y = 3$



Note: Y can only be equal to 3 on the thick straight line parallel to the x axis.

vi.



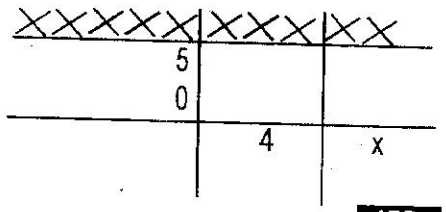
Note: Since $x \leq -1$, the shaded parts are not required.

b. Combine the following pairs from the above on a single graph

i. i and iii : $x = 4$ and $y < 5$
ii. ii and iv : $x \geq 4$ and $y \geq 6$???
iii. v and vi : $y = 3$ and $x \leq -1$

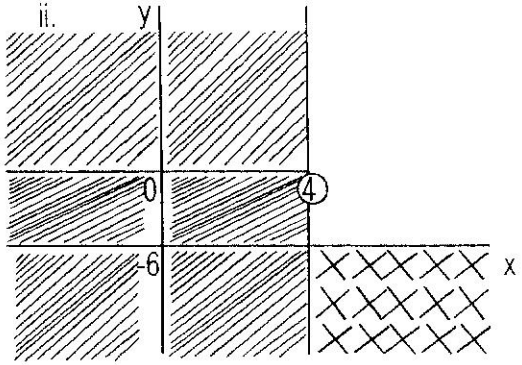
Solution

i.

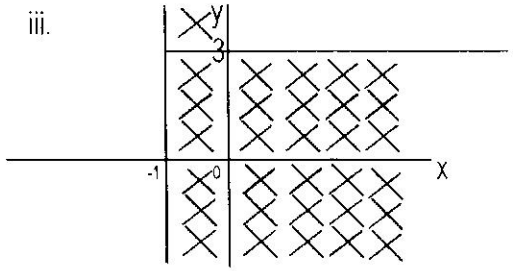


Note: You get the right pick by just bringing them together individually. So, the visible region is represented by the inequality. $x = 4$; $y < 5$.

ii.



iii.



c. Show the region bounded by the following on a graph.

i. $3x + 2y < 6$

ii. $2x + y \geq 6$

iii. $y \geq 0, x \geq 0$
$5x + 3y \leq 15$

Solution

i. $3x + 2y < 6$

first obtain the border line:

$\therefore\ 3x - 3x + 2y < 6 - 3x$

$\therefore\ 2y < 6 - 3x$

$\therefore\ \frac{2y}{2} < \frac{1}{2}[6 - 3x]$

$\therefore\ y < \frac{1}{2}[6 - 3x]$

$\therefore$ The assumed boundary line will be

Note: The visible region o
the right hand side satisfy th
inequalities $x \geq 4$ and $y \geq -6$

Note: The visible region o
the left hand side satisfy th
inequalities $x \leq -1$ and $y = 3$

This is an inequality, $3x + 2$
$< y$ may be re-arranged a
$2y < 6 - 3x$ by subtracting $3x$
from both sides. Then divid
through by 2 as shown.

$y = \frac{1}{2}[6 - 3x]$

When $x = 0$

$y = \frac{1}{2}[6 - 3(0)]$

$= \frac{1}{2}(6)$

$= 3$

The first set of coordinate of the border line is [0, 3]

When $y = 0$

$y = \frac{1}{2}[6 - 3x]$

$0 = \frac{1}{2}[6 - 3x]$

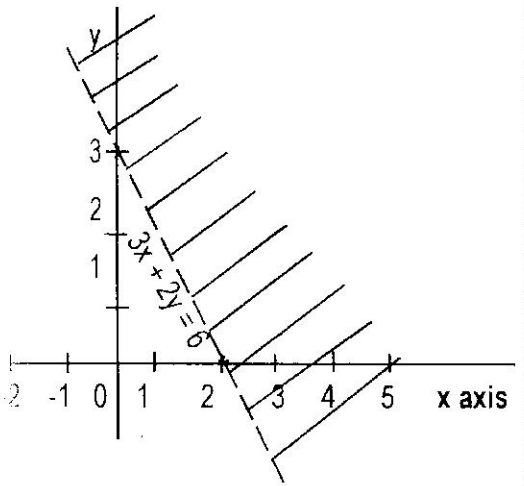
$0 = 3 - 1.5x$

$-3 = -1.5x$

$x = \frac{-3}{-1.5}$

$= \underline{2}$

So, the second coordinite of the border line is [2, 0]



ii $2x + y \geq 6$

To obtain the border line equation, subtract 2x from both sides.

$\therefore\ 2x + y - 2x \geq 6 - 2x$

$\therefore\ y \geq 6 - 2x$

The border or boundary line is the straight line that defines the inequality graph. So, the coodinates of the border line can be obtain as shown form the equation of the assumed border line.

The first and second coordinates of the border line are now plotted as show.

The shaded part are not wanted, since the inequality is $3x + 2y < 6$.

The border line is also a dotted or broken line be cause it not equal to.

Follow the same procedures listed above.

∴ The assumed border line equation is $y = 6 - 2x$.

To determine the coordinates when $x = 0$

$y = 6 - 2x$

∴ $y = 6 - 2(0)$

$= 6$

∴ The first coordinate is (0, 6) when

$y = 0$

$y = 6 - 2x$

∴ $0 = 6 - 2x$

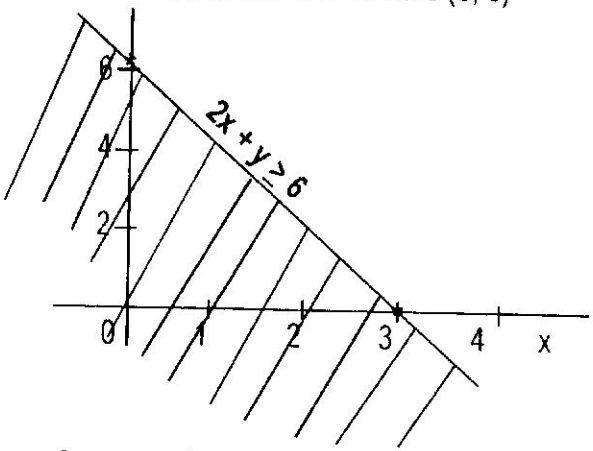
$-6 = -2x$

$^{-6}/_{-2} = x$

$3 = x$

$x = \underline{3}$

∴ The Second coordinate is (3, 0)



3. $y \geq 0$, $x \geq 0$ and $5x + 3y \leq 15$.

To obtain the border line equation of $5x + 3y \leq 15$ subtract $5x$ from both sides $5x - 5x + 3y \leq 15 - 5x$

∴ $3y \leq 15 - 5x$

Divide both side by 3.

$$\frac{3y}{3} \leq \frac{[15 - 5x]}{3}$$

The border line is thick and not broken because the inequality is $2x + y \geq 6$

$5x + 3y \leq 15$ of the three simultaneous inequalities $y \geq 0$ and $x \geq 0$ are straight forward because y will be greater than zero on the positive sides of x while x will be greater than zero on the positive side are two.

$y \leq {}^{1}/_{3} [15 - 5x]$

∴ The border line equation is $y = {}^{1}/_{3} [15 - 5x]$

When $x = 0$

∴ $y = {}^{1}/_{3} [15 - 5x]$

$= {}^{1}/_{3} [15 - (5 \times 0)]$

$= {}^{1}/_{3} \times 15$

$= 5$

The first coordinate (0, 5)

When $y = 0$

$y = {}^{1}/_{3} [15 - 5x]$

∴ $3y = 15 - 5x$

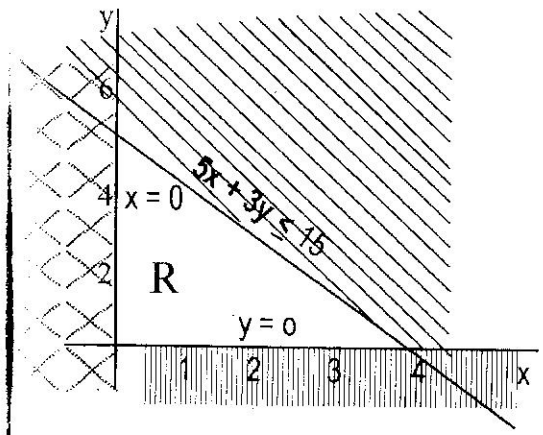
$3 \times 0 = 15 - 5x$

∴ $-15 = -5x$

∴ $x = {}^{-15}/_{-5}$

$= \underline{3}$

The second coordinate (3, 0)



We now solve $5x + 3y \geq 15$ as in the examples above.

For the two inequalities $y \geq 0$ and $x \geq 0$ their border lines are $y = 0$ and $x = 0$ respectively

The unshaded region R is the Required region

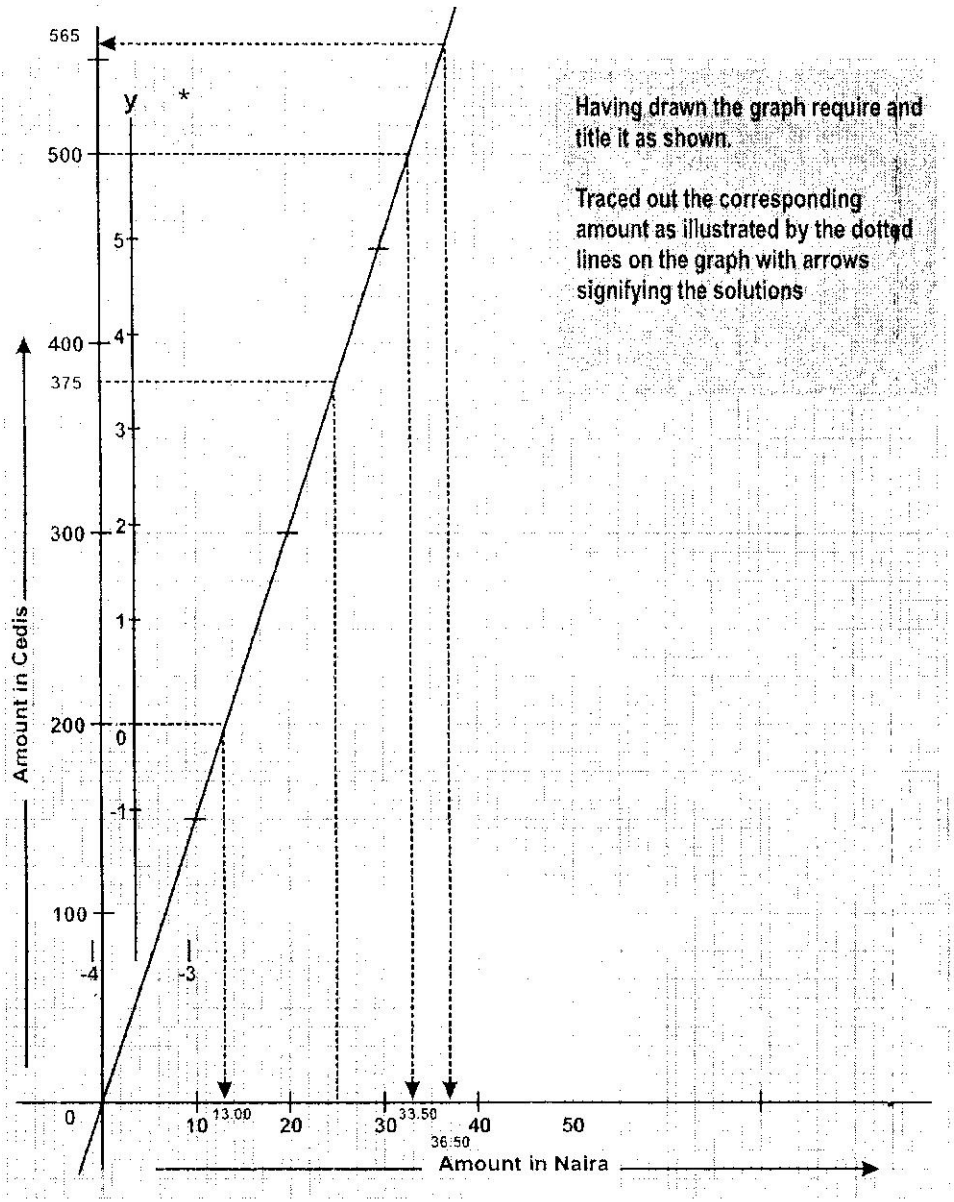
10.2.1.3 Conversion Graphs.

These are graph which are used to depict either direct or inverse relationship between two quantities, which are connected by a formula.

The graph drawn is use to read the corresponding value of the other quantity.

The table below shows the exchange of Naira to Cedis:

1. Currency conversion graph of naira to cedis



Having drawn the graph require and title it as shown.

Traced out the corresponding amount as illustrated by the dotted lines on the graph with arrows signifying the solutions

Naira	10	20	30
Cedis	150	300	450

i Draw a straight line graph through the points using a suitable scale.

ii From the graph, find the coresponding cedis to the following Naira (a) N 25 (b) N 36 .

iii Frome the graph find the corresponding Nira to the following Cedis (a) ¢ 200 (b) ¢ 500 (c) ¢ 550

Solution

i The graph is title: Currency conversion graph of Naira to Cedis above.

ii From the graph,
(a) N 25.00 ⇔ ¢ 375.00
(b) N 36.00 ⇔ ¢ 530.00

iii From the graph,
(a) ¢ 200 ⇔ N 13.00
(b) ¢ 500 ⇔ N 33.50
(c) ¢ 550 ⇔ N 36.50

10.2.1.4 Graphical solution of simultaneous equations.

(a) Solve graphically, the equations

$x + y = 3$ ---------- (i)

$2x + y = 4$ ---------- (ii)

(b) Solve graphically the equations

$2x + 2y = 10$ (i)

$2x + 4y = 16$ (ii)

The said graph is on page 174.

The arrows on the dotted lines show the various solution to the conversions.

* It is important to note that the true set of each of the pair of the simultaneous equations above is the set of values of (x, y) as previously explained, which make both equations true at the same time. The sets of the solution of each pair of equation will be given by the intersection of the straight line graphs of the relation as shown on graphs 2 and 3.

(a) $x + y = 3$ ---------- i

$2x + y = 4$ ---------- ii

From equation i

$x + y = 3$

$y = 3 - x$

∴ When $x = 0$

$y = 3 - x$

$= 3 - 0$

$= 3$

First coordinate is (0, 3)

when $y = 0$

$y = 3 - x$

∴ $0 = 3 - x$

∴ $-3 = -x$

∴ $x = 3$

Second coordinate is (3, 0)

Therefore, the line $x + y = 3$ pass through the following coordinates (0, 3) and (3, 0).

From equation (ii)

$2x + y = 4$ ---------- (ii)

∴ $y = 4 - 2x$

Re-write equation i as

$y = 3 - x$

Then look for the value of y when x = 0 and x when y = 0

when, $x = 0$

$y = 4 - 2x$

$y = 4 - 2(0)$

$\therefore \quad y = 4$

when $y = 0$

$y = 4 - 2x$

$0 = 4 - 2x$

$\therefore \quad -4 = -2x$

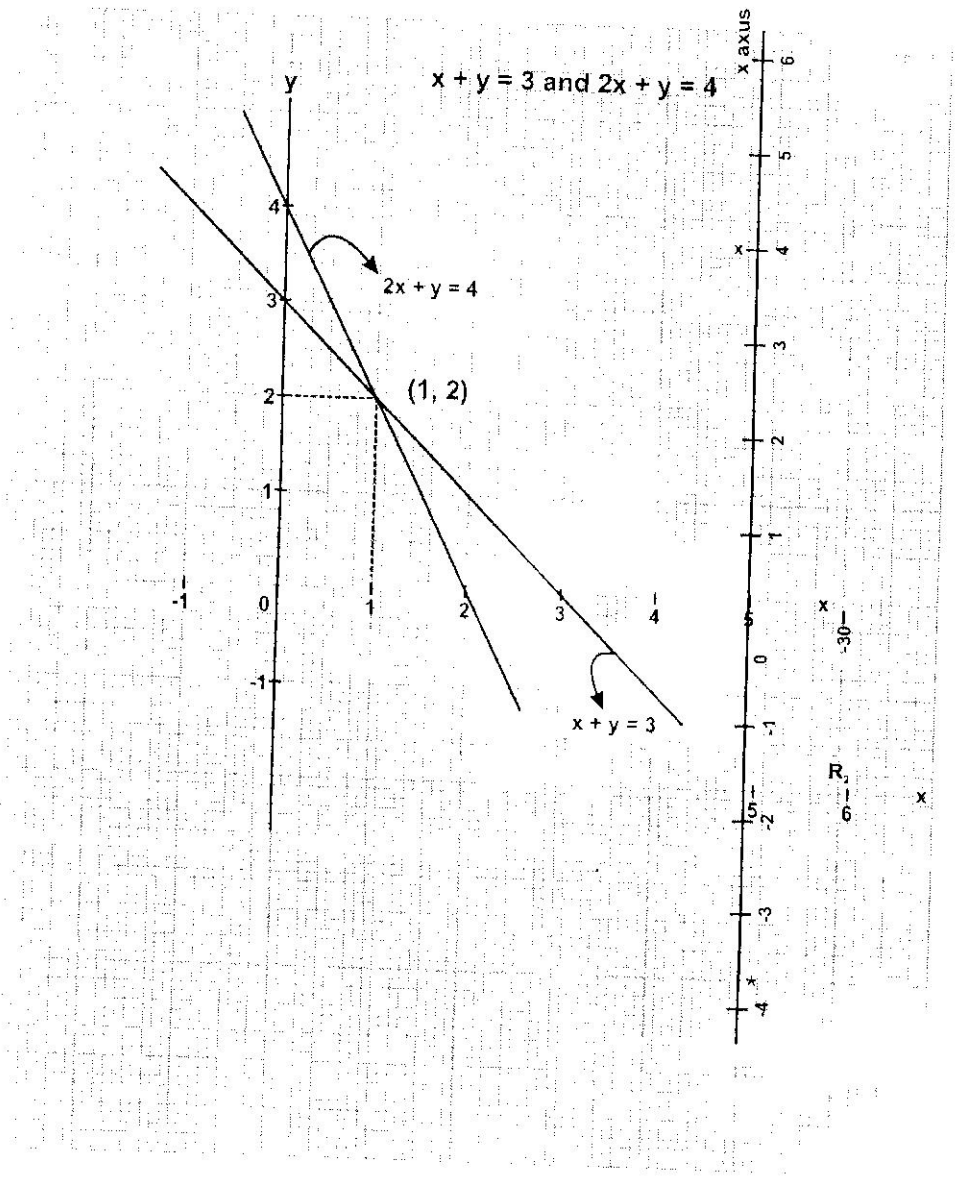
$\therefore \quad x = 2$

Therefore, the line $2x + y = 4$ pass through the following coordinates (0, 4) and (2, 0).

See the graph on page 178.

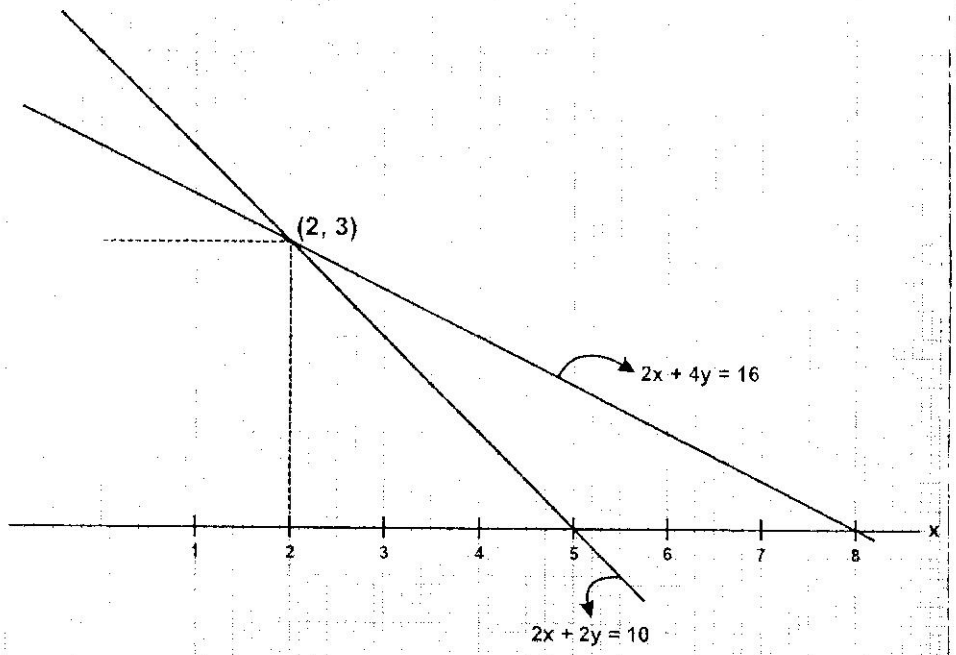
The intersection of the two straight lines gives the solution to the equation.

2. The graph of the simultaneous equations



3. The graph of the simultaneous equations

$2x + 2y = 10$ and $2x + 4y = 16$



10.2.2. Quadratic Graphs

10.2.2.1 Graphical solutions of quadratic equations.

a. Draw the graph of $x^2 - 7x + 6 = y$ for the range of $-1 \leq x \leq 6$. Does y has a maximum or minimum value. Find the roots of the equation and draw a line of symentry to the graph.

Solution:

$y = x^2 - 7x + 6$

Since, we are given that $-1 \leq x \leq 6$ it implies that x lies between -1 and +6. So we will obtain the values of y for each of the values of x from -1 to + 6 as shown in the table below.

The table of values for $y = x^2 - 7x + 6$

$y = x^2 - 7x + 6$

x	-1	0	1	2	3	4	5	6
x^2	1	0	1	4	9	16	25	36
-7x	7	0	-7	-14	-21	-28	-35	-42
+6	6	6	6	6	6	6	6	6
y	14	6	0	-4	-6	-6	-4	0

The above values of y can also be obtain from

GRAPH

$f(x) = x^2 - 7x + 6$

$f(-1) = (-1)^2 - 7(-1) + 6 = 1 + 7 + 6 = 14$

$F(0) = (0)^2 - 7(0) + 6 = 6$

$F(1) = (1)^2 - 7(1) + 6 = 1 - 7 + 6 = 0$

$F(2) = (2)^2 - 7(2) + 6 = 4 - 14 + 6 = -4$

The first step in plotting any curve or graph is to have a good table of values. This we've been doing since by looking for the co-ordinates of points.

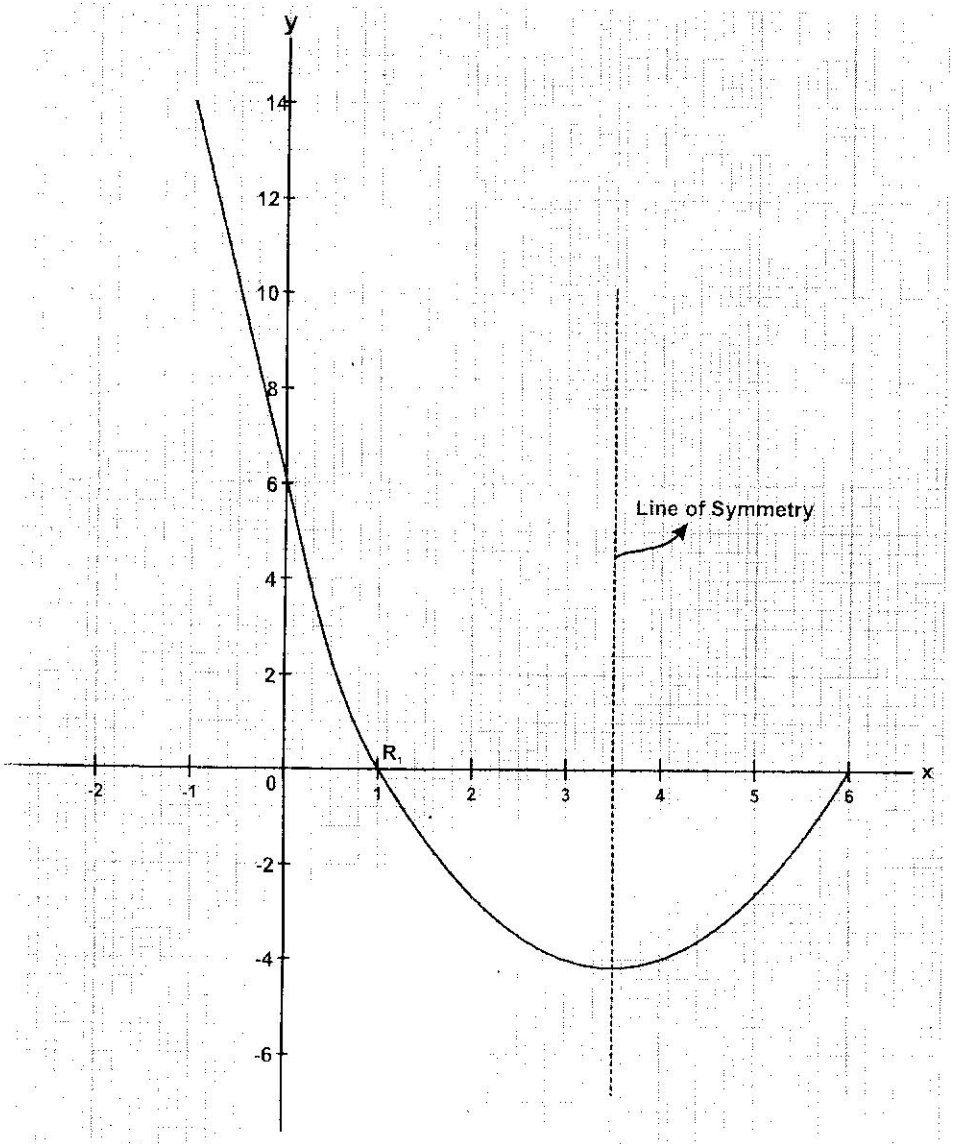
But, here we are going to show how to produce a good table of values, from which the graph will be drawn, bearing in mind that the graph of any quadratic is always parabola in nature.

$F(3) = (3)^2 - 7(3)^2 - 7(3) + 6 = 9 - 21 + 6 = -6$
$F(4) = (4)^2 - 7(4) + 6 = 16 - 28 + 6 = -6$
$F(5) = (5)^2 - 7(5) + 6 = (5)^2 - 7(5) + 6 = -4$
$F(6) = (6)^2 - 7(6) + 6 = 36 - 42 + 6 = 0$

Note: The number 6 is constant in the both cases and the values of y are the same. Graph 4 shows the plotting of the curve of $y = x^2 - 7x + 6$.

See graph 4 on page 184 for the details.

4. The graph of $y = x^2 - 7x + 6$



From the graph

i It was found out that the graph has a minimum value of about -6

ii The roots of the equation $y = x^2 - 7x + 6$ are 1 and 6 respectivelyand are shown on the graph with R_1 and R_2 where the curve cuts the x axis.

iii The line of symmetry,which is the line which passes through the mid-point of the graph is drawn with broken lines.

b. Draw the graph of $y = x^2 - 3x$ for the range of $-3 \leq x \leq 4$, using a scale of 2cm to 1 unit on y axis (ii) Deduce the root of $x^2 - 3x = 0$ from the graph.

Solution

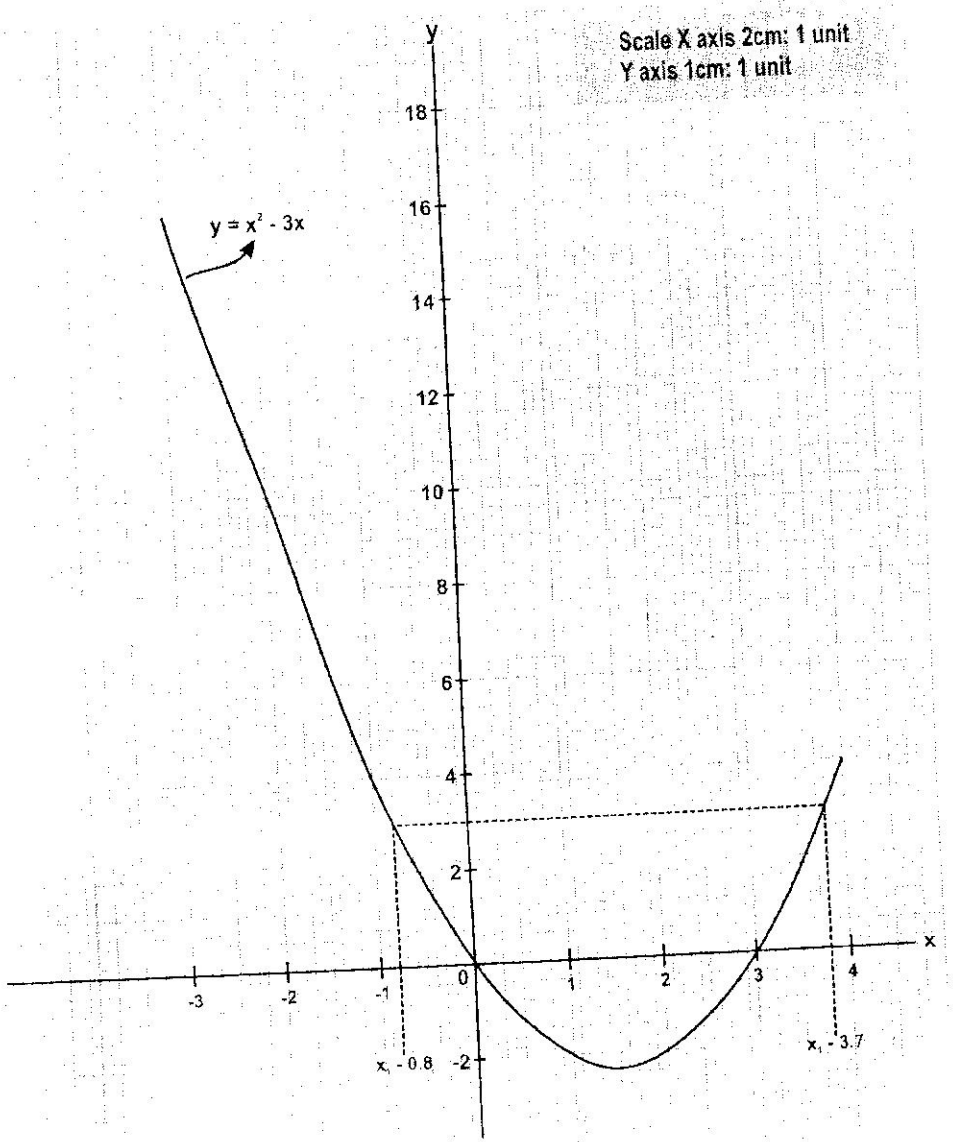
The table of values: $y = x^2 - 3x$

x	-3	-2	-1	0	1	2	3	4
x^2	9	4	1	0	1	4	9	16
-3x	9	6	3	0	-3	-6	-9	-12
y	18	10	4	0	-2	-2	0	4

Having obtained for $y = x^2 - 3x$, Graph 5 showns the curve.

The first step is to have a good table of value. Then, you plot the graph as stipulated based on the scale given.

5. The graph of $y = x^2 - 3x$



ii. We are asked to deduce the roots of the equation
$x^2 - 3x - 3 = 0$ but the equation use in drawing the graph is
$y = x^2 - 3x$.

So, if you add 3 to bothsides of the equation $x^2 - 3x - 3 = 0$ the result will be:
$x^2 - 3x - 3 + 3 = 0 + 3$
$\therefore \quad x^2 - 3x \quad = 3$
Now, look at the equations again:
$x^2 - 3x = 3$
$x^2 - 3x = y$

What do you observe? It shows that $y = 3$, because their L.H.S are now the same.

To obtain the roots of the equation $x^2 - 3x - 3 = 0$ from the graph of $y = x^2 - 3x$ go back to graph 5 and draw a line from where $y = 3$ and read the values of x as shown on the graph.

So, the roots of the equation $x^2 - 3x - 3 = 0$ are -0.8 and 3.7 respectively.

c. Draw the graph of $y = 3x [4 - x]$ for values of x ranging from
-2 to 6. On the same graph draw the line $y = 5(x - 2)$. Use a scale of 1cm to 1 unit on x axis and 1cm to 5 unit on y axis. From your graph

ii. Write down the maximum value of $y = 3x (4 - x)$

iii. Deduce the root of the equation $3x (4 - x) = 5(x - 2)$

(WAEC)

Solution

i $y = 3x(4 - x)$
$= 12x - 3x^2$

ii $y = 5(x - 2)$
$= 5x - 10.$

iii. Because you are to draw two graphs on the same sheet you are expected have two tables of values, but for that of the straight line you may just use two or few points to get your curve.

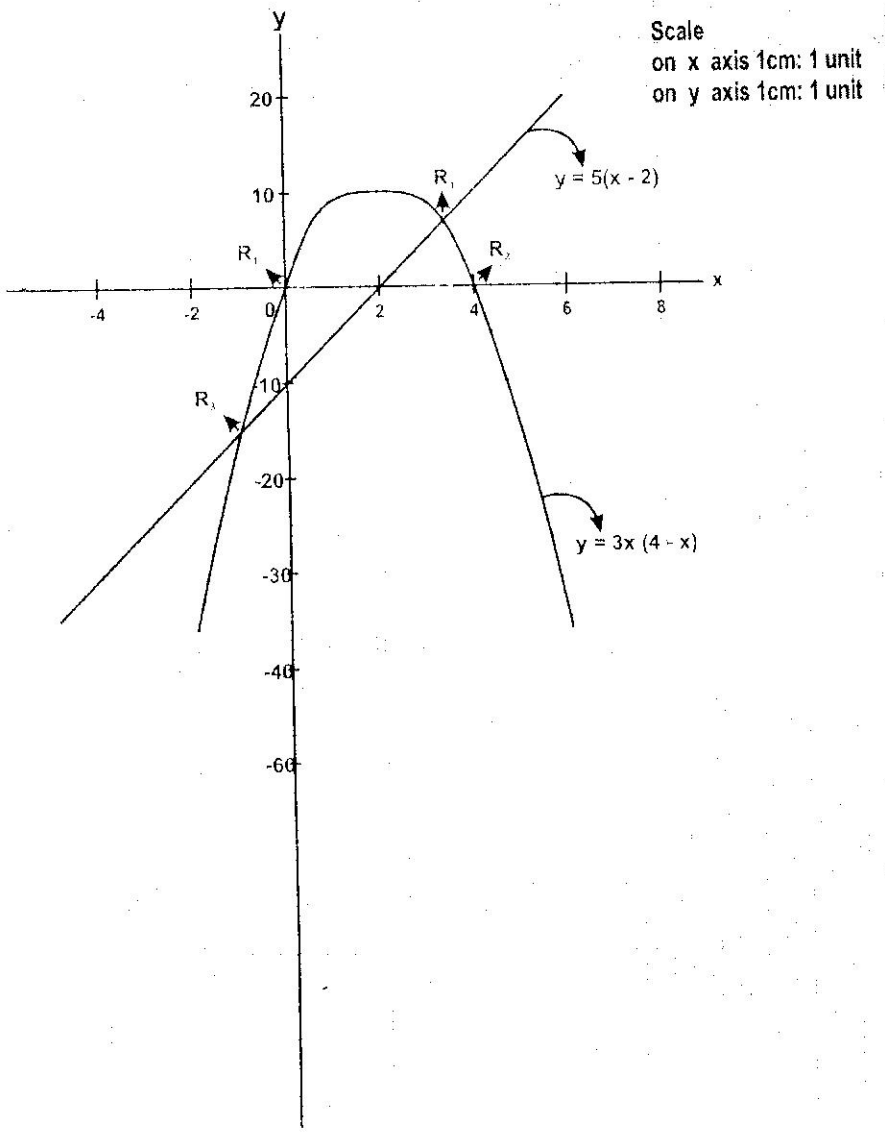
TABLE 1 : $y = 12x - 3x^2 = 3x[4 - x]$

x	-2	-1	0	1	2	3	4	5	6
12x	-24	-12	0	12	24	36	48	60	72
-3x	-12	-3	0	-3	-12	-27	-48	-75	-108
y	-36	-15	0	9	12	9	0	-15	-36

TABLE 2 : $y = 5x - 10 = 5(X - 2)$

x	-2	-1	0	1	2	3	4	5	6
5x	-10	-5	0	5	10	15	20	25	30
-10	-10	-10	-10	-10	-10	-10	-10	-10	-10
y	-20	-15	-10	-5	0	5	10	15	20

6. The graph of $y = 3^x (4 - x)$ and $y = 5 (x - 2)$



i. The roots of the equation $3x[4 - x] = 0$ are 0 and 4 respectively (i.e where $y = 0$ on the x axis where the graph cut through which are shown by R_1 and R_2)

ii. The maximum value of $3x[4 - x] = 0$ is 12 (i.e the peak of the parabolic curve)

iii. The roots of the equation $3x[4 - x] = 5[x - 2]$ are shown with R_3 and R_4 which are approximately -1 and 3.4 respectively from the graph.

10.2.3 Cubic curves and others

10.2.3.1 Cubic curves

Draw the graph of $y = x^3 + 3x^2 - x - 3$ for the range $-3 \leq x \leq 5$. which type of symmetry does the graph have ? use the graph to solve the equation $x^3 + 3x^2 - x - 3 = 0$

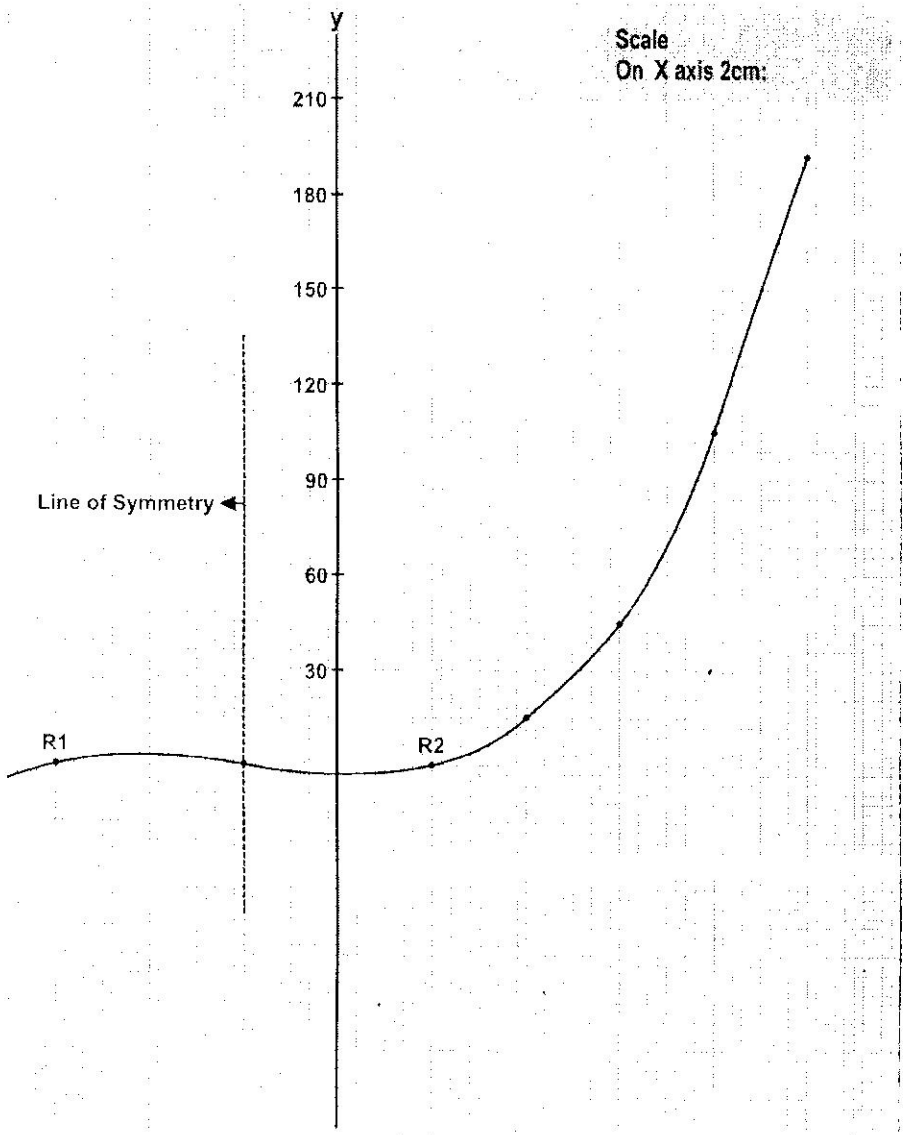
Solution

Table of values of $y = x^3 + 3x^2 - x - 3$

x	-3	-2	-1	0	1	2	3	4	5
x^3	-27	-8	-1	0	1	8	27	64	125
$+3x^2$	27	12	3	0	3	12	27	48	75
-x	3	2	1	0	-1	-2	-3	-4	-5
-3	-3	-3	-3	-3	-3	-3	-3	-3	-3
y	0	3	0	-3	0	15	48	105	192

The is shown on graph 7,which is on page 191. It is observed that it turn twice.

7. The cubic graph of $y = x^3 + 3x^2 - x - 3$



From the graph

i. The graph has a centrally located line of symmetry which passes through the point x = -1 . shown on the graph with dotted lines

ii. The values of the equation $x^3 + 3x^2 - x - 3 = 0$ are -3, -1 and 1 [which are the values of R_1, R_2 and R_3 respectively]

10.2.3.2 OTHER TYPES OF GRAPHS

Specifically by other types of graphs, we are refering to trigonometrical curves (i.e graphs of sin θ cosθ and tan θ) or the others which could not be classified as cubic or quadratic curves.

For detail understanding of these types of curves reference should be made to further mathematics texts or statistical works.

NOTE: It is important to note that the curve turn twice when x was cubed, but once when x is squared.

PRACTICE EXERCISE

1. Draw the line graph to show the following ranges of values of x
   i. $-3 \leq x < 7$
   ii. $-5 \leq x \leq 10$
   iii. $-1 < < 3$

2. Show the region bounded by the following
   i. $3x + 2y < 6$
   ii. $2x + y \geq 6$

3. Solve graphically the equations
   $x - y = 1$ i
   $x + y = 3$ ii

4. Draw the graph of $y = \frac{12}{x + 1}$ for the range of $-5 \leq x \leq 3$.

5. Draw the graph of $y = x^2 - 4x + 2$ for the range of $-2 \leq x \leq 5$. What are the roots of the equation $x^2 - 4x + 2 = 0$. what is the least value of y?

6. Draw the graphs of
   $x + 2y = 10$ i
   $x - y = 10$ ii
   What is the co-ordinate of the intersection of the two curves

7. Draw the graphs of $y = 5 + 3x - x^2$ and $y = 2x - 3$ for the range of $-3 \leq x \leq 5$. Hence, find the roots of $5 + 3x - x^2 = 2x - 3$

11.0 SET THEORY

11.1 SET

A set is a well defined collection of data or things. examples of which include the set of the failed banks i.e (Trade Bank; Gulf Bank; Savanah Bank .....) or that of the big banks in the new dispensation i.e. [Spring Bnak; Skye Bank; Bank PHB; .....]

It can also be the set of old or even numbers less than 20

Even numbers = [2,4,6,8..... 18]

Old numbers = [1,3,5,7..... 19]

The members of a given set are endosed by brankets [....] and seperated by a common.

Even numbers are numbers divisible by two without any remainder whole

Odd number cannot be divisible by two.

11.2. ELEMENTS OF A SET

The members of a particular set are refer to as the elements of the set for example, the elements of the set of failed banks mentioned above are tradebank, Gulf Bank etc.

11.3. THE NUMBER OF ELEMENTS IN A SET.

The short notation for the number of elements or member of a given set A is n{A}

Examples

If B = [whole numbers between 7 and 17], fine n(B) since B = [whole numbers between 7 and 17

Note: Between 7 and 17 dose not includes7 and 17 so, the expected numbers are 8,9,10,11,12,13, 14, 15 and the result is 9 by counting

∴ B = [8,9, ...16]
∴ **n(B) = 9**

ii. If B = (prince numbers less than 20)
find n(B).
since: B = (prince numbers less than 20)
∴ B = [2,3,5 ..... 19]
∴ n(B) = 8

11.4. NULL OR EMPTY SET

The null or empty set is a set without any member. its represesnted by 0 or { }, since theres nothing inside.

11.5 SUBSET

If there are two sets A and B such that all the elements at in B can be found in A, then B is said to be a subset of A, this is written as BCA.

Example
A = {1,2,3,6,7,8}
B = {3,6,8}
The relationship that exists between them

Solution
A = [1,2,3,6,7,8]
B = [3,6,8]
∴ BCA means B is a subsset of A.

Note: Prime numbers that have no other factors, apart from 1 and its e.g. 2,3,5,7,11,13,17 and 19. But, 1 is not a prime number

A contains all the elements B, so B is a subset of A

The intersection of two or more sets is represented by the symbol ∩, which implies that the sets have some element in common. Example.

If A and B are two sets, such that A = [2,3,4,5] and B = [3,4,6,8], find the intersect of the sets.

Solution
A = [2,3,4,5]
B = [3,4,6,8]
∴ A∩B = [3,4]

The common elements in the two sets are 3 and 4 so they are the intersect.

11.7 UNION OF SETS

The union of two sets is denoted by the symbol ∪. The union of set A and B is thus written as A∪B which means all the elements in A and B combine together.

Example
If A and B are two sets, A = (1,3,5) and B = (2,4,6). Fine A∪B.

Solution
A = [1,3,5]
B = [2,4,6]
A∪B = [1,2,3,4,5,6]

The union of two sets is the combination of the two sets such that each of the set will now be a subset of the union
∴ A∪B =[1,2,.....,6]

11.8. EQUAL AND EQUIVALENT SET

Two sets A and B are said to be equal, if they are subsets of each

element is repeated or not.

While, two sets are said to be equivalent if they contain the same number of elements regardless of their values or size

Example

i. Equal sets

A = [10,15,20,225]

B = [0,5,10,15,20,25]

$\therefore A \subset B$ and $B \supset A$

ii. Equivalent set

A = [3,5,7]

B = [e,f,d]

11.9. COMPLEMENT OF SET

The complement of a particular set are those element which could not be found in the set but are present in the universal set (ε). It is denoted by A'

Example

Given that ε = [1,2,3,-----,20] and A = [5,8,9,14,16]. Find A^1

Solution

A^1 = [1,2,3,4,6,7,10,11,12,13,15,17,18,19,20]

Note

$A \subset B$ = [10,15,20,25]

$B \supset A$ = [10,15,20,25]

n (A) = 3

n (B) = 3

$\therefore A = B$

They correspond to each other.

Note that n(A) = 5

n (A) = 5

n (ε) = 20

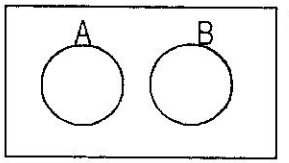
$\therefore n[A]^1 = 20 - 5$

$= \underline{15}$

So, there will be 15 elements in A^1

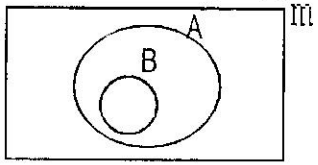
Venn diagram is the diagrammatical representation of various relationships that exist between two or more sets.

11.10.1. REPRESENTING NULL OR EMPTY SET ON THE VENN DIAGRAM.



the above diagram illustrates null sets { }

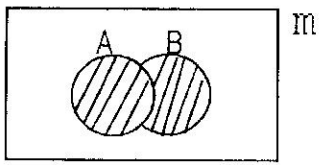
11.10.2. REPRESENTING SUBSETS ON THE VENN DIAGRAM.



$B \subset A$

The above diagram illustrate B as a subset of A, where m is the universal set.

11.10.3. REPRESENTING UNION ON THE VENN DIAGRAM



$A \cup B$

Note:

i. m or ε be use to denote the universal set.

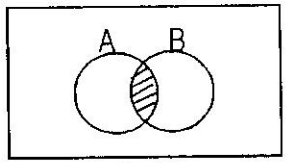
ii. The venn diagram did not intersect because it denotes an empty or null set.

The diagram shows that every in B is present in A

The element in the two sets are shaded, to illustrate the combination

The above diagram illustrates A union B

11.10.4. REPRESENTING INTERSECT ON THE VENN DIAGRAM.

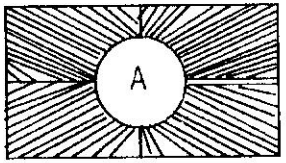


A∩B

The diagram above illustrates A intersect B

The shaded part show the point of intersect or the element (s)present in the intersect of both A and B

11.10.5. REPRESENTATION OF COMPLEMENT ON THE VENN DIAGRAM



A' = The shaded part

The shaded part are the complement to A since, they are outside A but present in m.

11.11.0. WORKED EXAMPLES

11.11.1. GIVEN THAT ε = {1,2,3,..... 30}

A = [Even numbers less than 30]
B = [odd numbers divisible by 3 but les than 30]
C = [Multiplys of 5 less than 30]

a. List all the elements in each of the sets.
b. Find (i). A ∩ C
(ii). A ∪ B
(iii). B ∩ C

Solution

ε = [1,2,3,.....,30]
A = [Even numbers less than 30]
B = [Odd numbers dividble by 3 but less than 30]
C = [Multiples of 5 less than 30]

List of the element

i. A = [2,4,6,8,10,12,14,16,18,20, 22,24,26,28]
ii. B = [3,9,15,21,27]
iii. C = [5,10,15,20,25]

b i. A∩C = [2,4,6,.....,28] [5,10....25]
= [10,20]
ii. A∪B = [2,4,6,....28]∪ [3,9,15,21,27]
= [2,3,4,6,8,9,10,12,14,15,16, 18,20,21,22,24,26,27,28]
iii. B∩C = [3,9,15,21,27]∩ [5,10,15,20,25]
= [15]

Even number less than 30 means numbers divisibly by 2 which are less than 30.
A = [2,4,6,.....28]
B = [3,9,.....29]
note that 1,5,7 etc. are not included because they are not divisible by 3 though they are odd numbers.
C = [5,10,15,.....25]
multiples of 5 are 5 x 1 = 5
5 x 2 = 10
The common element (3) to A and C are 10 and 20

A∪B will produced mixed of the two together.
B∩C = [15] because the only common element to the two sets is 15

11.11.2. USE THE VENN DIAGRAM TO ILLUSTRATED THE INFORMATION BELOW

ε = {1,2,.....50}
A = {Multiples of 10 including 50}
B = {Even numbers divisible by 4 but less than 50}
C = {Multiples of 5 less than 50}

Solution

ε = {1,2,.....50}

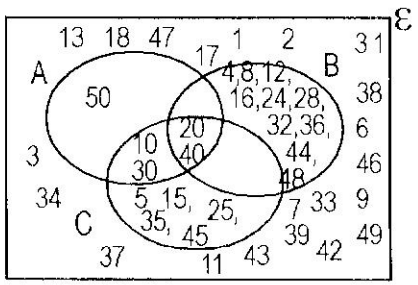
A = {10,20,30,40,50}

B = {4,8,12,16,20,24,28,32, 36,40,44,48}

C = {5,10,15,20,25,30,35,40,45}

$A \cap B \cap C$ = [20,40]

$A \cap C$ = [10,20,30,40]



Hint

First list all the members of each set before drawing the venn diagram.

Multiples of 10 including 50 are 10,20,30,40,50.

Remember, for even numbers divisible by 4, some numbers will not be included in B, even if they are Even numbers except they are divisibly by 4.

The numbers outside the brackets are the complements.

11.11.3. DURING ANALYSIS

The following information were obtained by a company producing three brands P, Q and R of sugar 250

customers liked Brand P, 240 customers liked Brand Q and 100 Customers liked Brand P and 50 Customers liked Brand P and Q but not R. 250 customers like Brand P or Q but not R. 128 customers liked R but disliked P or Q. 70 custmoers liked P and R but not Q and 17 customer like none of the Brands

a. Determine the number of customers that liked all the three Brands

b. Determine the number pf customers that like Brands Q and R but not P

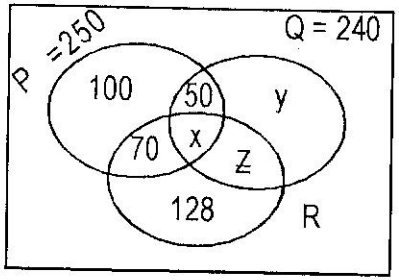
c. How many were involved in the analysis

d. What percentage of the customers interviewed liked at less the of the Brands.

e. How many customers liked Brands P or R but disliked Brand Q

f. If the company is willing to introduce only two Brand into the market which Brands will you recommend

Solution



n(P) = 250 n(Q) = 240

$n(PnQnR^1)$ = 50

$n(PuQnR^1)$ = 250

$n(RnP^1nQ^1)$ = 128

$n(PuQuR)^1$ = 70

$n(PuQuR)^1$ = 17

Since n(A) = 250

100 + 50 + 70 x = 250

220 + x = 250

∴ x = 250 -220

= 30

n (P) = number of elements/consumers who like P

n (Q) = number of element/consumers who like Q

$n (PnQnR^1)$ = customers who liked P and Q but not R etc.

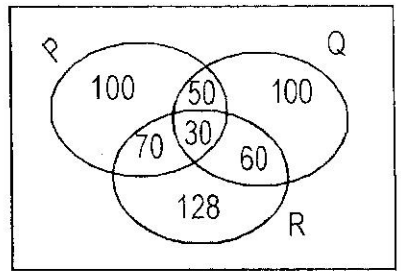
Also:
$n(P \cup Q \cap R^1) = 250$
$100 + 50 + y = 250$
$\therefore y = 250 - 150$
$= 100$
further more
$n(Q) = 240$
$50 + 30 + 100 = 240$
$= 240 - 100$
$= 60.$

From the above:

a. 30 customers like all the Brands.
b. 60 customers like P and Q but not R.
c. Total number of customers involved with analysis
= 100 + 50 +128 +17 = 555 customers
d. The total numbers of customers that like at lasst two of the Brands are 210 [50 + 70 + 60 + 30] customers.
Perantage $= \frac{210}{555} \times 100$
$= \underline{\mathbf{37.8\%}}$
e. The customers that liked P or R but dislike Brand Q are (100 + 70 + 128) 298 customers
+ P and Q would be recommended becuse the sale more

See the new venn diagrams below

Percentage =
$\frac{\text{(Number involved)}}{\text{Total customers}} \times \frac{100}{1}$
$n(P) = 250$
$n(Q) = 240$
$n(R) = 180$
so, since the every business man is to make mize profit P and Q will be recommended.
Note for all types of set De morgan's rule states that
$(A \cup B)^1 = A^1 \cap B^1$ and



$(A \cap B)^1 = A^1 \cup B^1$
$A \cap \mathcal{E} = A$ where $\mathcal{E}$ is the universay set $\therefore (A \cup E) = \mathcal{E}$

PRACTICE EXERCISE

1. List the elements in the following sets
   a. [Prime numbers between 10 and 20]
   b. [Even numbers between 10 and 20]
   e. [Odd numbers between 10 and 20]
   d. [Numbers divisible by 3 between 10 and 20]

2. Given that the universal set $\mathcal{E}$ = [1,2,3,.....,25]
   A = [Numbers divisible by 3 but less than 25]
   B = [Multiple of 6, less than 25]
   C = [Multiple of 5 including 25]
   Find (i) $A \cap B$ (ii) $B \cap C$ (iii) $B \cup C$ (iv) $(A \cup B)^1$

3. Illustrate the information in 2 (above) with a Venn diagram.
4. Out of the 1000 undergraduate studenst in the faculty of management sciences 650 are

offering Business maths and 500 are offering Economics

(a). How many students are offering both Business maths and Economics, if only 210 students are offering neither Business maths or Economics.

(b). How many students are offering at least one of Business maths or Economics.

5. A company provides three Brands A,B and C of powder milk in order to determine the marketability of this product, some samples were sent out. The result of this exercise shows that 188 customers liked Brand A, 98 customers liked brand B and C, 20 customers liked brand A and C, the same number of those customers liked all the three brands also disliked all the three brands. In a sample of 275 customers, how many of them liked

i. Brands A, B and C
ii. Brands A or B but not C
iii. Brands B and C but not A
iv. Single brand only.

12.0 MATRICES:

12.1 **Definition of terms and facts on matrices:** Matrix (plural matrices) is a rectangular array of numbers, which can be identified in linear transformation between two vector spaces. The arrangement of the numbers in the space, is such that the number(s) on the row(s) is (are) read before the number(s) on the column(s).

Order of Matrix: This simplify implies the number of row(s) by the number of column(s) in a given matrix. A matrix having M rows and n columns is called an m x n (i.e 'm by n') matrix; and its said to be of order m x n.

Types of Matrices

(i) Row Matrix: This occurs when a matrix has only one row.

(ii) Column Matric: This occurs when a given matrix has only one column.

The general form of a matrix is thus

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{13} \\ a_{21} & a_{22} & \cdots & a_{23} \\ \vdots & \vdots & & \vdots \\ \vdots & \vdots & \cdots & \vdots \\ a_{n1} & a_{n2} & & a_{n3} \end{bmatrix}$$

where

a_{11} means a_{ij} on the first row and first column.

a_{n2} = a is on the nth row and second column etc.

Note: for matrices, the row always comes before the column.

$$A = [2\ 3\ 5\ 7]$$

$$B = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$$

(iii) Square Matrix: This occcurs when given matrix has the same number of rows and column. i.e. the number of rows equal the number of column.

Note: Both row and column matrices are vectors

$$A = \begin{bmatrix} 5 & 8 \\ 7 & 9 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$$

Both A and B are examples of square matrices.

Note: A square matrix (a_{ij}) is skew - symetric if $a_{ij} = -a_{ji}$

(iv) Identity Matrix: This takes place in a square matrix, when the main diagonal elements are one (unit) while the other elements are 0 (Zero). It is denoted by I. It is also called unit matrix.

Identity Matrices

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(v) Diagonal Matrix: This is a square matrix

The elements are either 1 or 0 (Zero). It must satisfies the condition that

$A = (a_{ij})$ with $i = j = 1$ (i.e first item) must be 1 and not zero).

An example of diagonal matrix is A when $A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix}$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix} = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix}$$

$\therefore a_{11} = 1, a_{12} = 3$

$a_{13} = 5$ $a_{21} = 2$ $a_{22} = 4$

[redacted] elements are not [redacted] are zero. It is a

[redacted] matrices are said [redacted] elements in the m [redacted]

[redacted]

(vii) Transpose of a Matrix: The transpose of a matrix is the interchanging of the rows of the given matrix to form columns. It is denoted by A^T.

$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

i.e the first row becomes the first column, the second row become the second column, and other rows and column follow suit.

(viii) Symmetric Matrix: This occurs in the case of a square matrix, when the matrix A is the same as its transpose A^T.

$$A = \begin{bmatrix} 4 & 2 & 5 \\ 2 & 0 & 4 \\ 5 & 4 & 6 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 4 & 2 & 5 \\ 2 & 0 & 4 \\ 5 & 4 & 6 \end{bmatrix}$$

$A = A^T$ (by all standard). It is a symmetric matrix.

(ix) Triangular Matrix: This is a special case of square matrix, where the elements below and above the main diagonals are zeros.

(x) Zero Matrix: This is a matrix whose elements or members are all zero.

$$A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

MATRICES

The addition and/or substraction of matrices can only takes place,if the matrices are of the same order. Or more precisely, they are equal.

12.2.1 Addition of Matrices

(i) $\begin{bmatrix} 2 & 6 \\ 4 & 7 \end{bmatrix} + \begin{bmatrix} 3 & 8 \\ 0 & 9 \end{bmatrix}$

$= \begin{bmatrix} 5 & 14 \\ 4 & 16 \end{bmatrix}$

(ii) $\begin{bmatrix} 6 & 4 & 10 \\ 7 & 6 & 5 \end{bmatrix} + \begin{bmatrix} 3 & 10 & 11 \\ 7 & 8 & 6 \end{bmatrix}$

$= \begin{bmatrix} 9 & 14 & 21 \\ 14 & 14 & 11 \end{bmatrix}$

(iii) $\begin{bmatrix} 9 & 8 & 3 \\ 10 & 5 & -4 \\ 11 & 6 & 7 \end{bmatrix} + \begin{bmatrix} 10 & 14 & 9 \\ 6 & 8 & 4 \\ 9 & 10 & 8 \end{bmatrix}$

$= \begin{bmatrix} 19 & 22 & 12 \\ 16 & 13 & 0 \\ 20 & 16 & 15 \end{bmatrix}$

The addition is done by adding corresponding elements, that is why they have to be equal. i.e.

$a_{11} + a'_{11} = 2 + 3 = 5$
$a_{12} + a'_{12} = 6 + 8 = 14$
$a_{21} + a'_{21} = 4 + 0 = 4$
$a_{22} + a'_{22} = 7 + 9 = 16$

Following the same step as above:

$a_{11} + a'_{11} = 9 + 10 = 19$
$a_{12} + a'_{12} = 8 + 14 = 22$
$a_{13} + a'_{13} = 3 + 9 = 12$
$a_{21} + a'_{21} = 10 + 6 = 16$

$a_{11} + a'_{11} = 6 + 3 = 9$
$a_{12} + a'_{12} = 4 + 10 = 14$
$a_{13} + a'_{13} = 10 + 11 = 21$
$a_{21} + a'_{21} = 7 + 7 = 14$
$a_{22} + a'_{22} = 6 + 8 = 14$
$a_{23} + a'_{23} = 5 + 6 = 11$

a_1 = first matrix
a'_1 = second matrix

(i) $\begin{bmatrix} 9 & 10 \\ 6 & 11 \end{bmatrix} - \begin{bmatrix} 3 & 6 \\ 4 & 8 \end{bmatrix}$

$= \begin{bmatrix} 6 & 4 \\ 2 & 3 \end{bmatrix}$

(ii) $\begin{bmatrix} 12 & 15 & 18 \\ 13 & 16 & 19 \\ 14 & 17 & 20 \end{bmatrix} - \begin{bmatrix} 9 & 17 & 16 \\ 10 & 20 & 9 \\ 8 & 13 & 25 \end{bmatrix}$

$= \begin{bmatrix} 3 & -2 & 2 \\ 3 & -4 & 10 \\ 6 & 4 & -5 \end{bmatrix}$

12.3.0 Multiplication of Matrices

The multiplication of matrices is divided into two broad headings, namely;

(i) Multiplication of a matrix by a scalar e.g. 2, $^1/_2$, 10 etc.

(ii) Multiplication of two or more matrices by each other.

12.3.1 Multiplication of a Matrix by a Scalar

1. If $A = \begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix}$

find the values of 2A, 5A and 10A.

Solution

$A = \begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix}$

$2A = \begin{bmatrix} 2 \times 3 & 2 \times 5 \\ 2 \times 4 & 2 \times 6 \end{bmatrix}$

The subtraction is done, the same way as the addition by subtracting corresponding value as below:

$a_{11} - a_{11} = 9 - 3 = 6$
$a_{12} - a_{12} = 10 - 6 = 4$
$a_{21} - a_{21} = 6 - 4 = 2$
$a_{22} - a_{22} = 11 - 8 = 3$

From the above

$a_{11} - a'_{11} = 12 - 9 = 3$
$a_{12} - a_{12} = 15 - 17 = -2$
$a_{13} - a_{13} = 18 - 16 = 2$ etc.

Multiplication of a matrix by a scalar is done by multiplying each elements of the matrix by the given scalar as shown.

$2 \times a_{11} = 2 \times 3 = 6$
$2 \times a_{12} = 2 \times 5 = 10$
$2 \times a_{12} = 2 \times 4 = 8$
$2 \times a_{22} = 2 \times 6 = 12$

$\begin{bmatrix} & \\ 8 & 12 \end{bmatrix}$

$5A = \begin{bmatrix} 5 \times 3 & 5 \times 5 \\ 5 \times 4 & 5 \times 6 \end{bmatrix}$

same as above

$= \begin{bmatrix} 15 & 25 \\ 20 & 30 \end{bmatrix}$

$10A = \begin{bmatrix} 10 \times 3 & 10 \times 5 \\ 10 \times 4 & 10 \times 6 \end{bmatrix}$

same as above

$= \begin{bmatrix} 30 & 50 \\ 40 & 60 \end{bmatrix}$

2. Find the values of $^1/_2$ B and $^1/_5$B given that B = $10 \begin{bmatrix} 30 & \\ 20 & 40 \end{bmatrix}$

Solution

(i) $B = \begin{bmatrix} 10 & 30 \\ 20 & 40 \end{bmatrix}$

Just divide through the matrix by the given values as shown in the example

$^1/_2B = \begin{bmatrix} ^1/_2 \times 10 & ^1/_2 \times 30 \\ ^1/_2 \times 20 & ^1/_2 \times 40 \end{bmatrix}$

$^{10}/_2 = 5 = a_n$

$^{30}/_2 = 15\ a_{12}$ etc.

$= \begin{bmatrix} 5 & 15 \\ 10 & 20 \end{bmatrix}$

(ii) $^1/_5B = \begin{bmatrix} ^1/_5 \times 10 & ^1/_5 \times 30 \\ ^1/_5 \times 20 & ^1/_5 \times 40 \end{bmatrix}$

$^1/_5 \times 10 = {}^{10}/_5 = a_{11} = 2$ etc.

Note: You can also express your answers in fraction, if the given divisor cannot divide through.

$= \begin{bmatrix} 2 & 6 \\ 4 & 8 \end{bmatrix}$

12.3.2 Multiplication of Two Matrices

This is possible when the number of rows equals the number of columns on both sides

(i) Find the value of A.B, given that

$A = \begin{bmatrix} 2 & 4 & 6 \\ 3 & 5 & 7 \end{bmatrix}$ and

$B = \begin{bmatrix} 3 \\ 4 \\ 8 \end{bmatrix}$

The most important fact or point to always bear in mind in this case is that two matrices can be multiplied together ONLY when the first equals the number of rows in the second.

Solution

$A = \begin{bmatrix} 2 & 4 & 6 \\ 3 & 5 & 7 \end{bmatrix}$ $B = \begin{bmatrix} 3 \\ 4 \\ 8 \end{bmatrix}$

Multiply each of the rows in A by the column of B as shown and added up the result per row.

$A.B = \begin{bmatrix} 2 \times 3 + 4 \times 4 + 6 \times 8 \\ 3 \times 3 + 5 \times 4 + 7 \times 8 \end{bmatrix}$

$= \begin{bmatrix} 6 + 16 + 48 \\ 9 + 20 + 56 \end{bmatrix}$

$= \begin{bmatrix} 70 \\ 85 \end{bmatrix}$

(ii) Given that $A = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$

and $B = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \end{bmatrix}$

find A, B

Note that multiplying a (3 x 2) matrix gives a product order of 3 x 4.

So, the general accepted fact is that (a x m) matrix and an (m x n) matrix gives a matrix of the order (b x n). See this example.

Solution

$$A = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \end{bmatrix}$$

$$A.B \begin{bmatrix} 1x2+4 & 5\ 1x2+4x6 & 1x3+4x\ 7 & 1x4+4x8 \\ 2x1+5x5 & 2x2+5x6 & 2x3+5x7 & 2x4+5x8 \\ 3x1+6x5 & 3x2+6x6 & 3x3+6x7 & 3x4+6x8 \end{bmatrix}$$

$$\begin{bmatrix} 1+20 & 2+24 & 3+28 & 4+32 \\ 2+25 & 4+30 & 6+35 & 8+40 \\ 3+30 & 6+36 & 9+42 & 12+48 \end{bmatrix}$$

$$\begin{bmatrix} 21 & 26 & 31 & 36 \\ 27 & 34 & 41 & 48 \\ 33 & 42 & 51 & 60 \end{bmatrix}$$

12.3.3 Multiplication of other matrices with the identify or unit matrix

Find I. A given that $I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

and $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ (ii) Also find A.I

Read it over several times to get the principle

In multiplication the orders of the factors is not communicative i.e in matrices multiplication.
$A.B \neq B.A$

Solution:

$$I.A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

ii) $$A.I = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

12.4.0 The Determinant of matrices

12.4.1 The Determinant of a square matrix of 2 by 2

Find the determinant of A,B and C given that

$$A = \begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix}$$

$$B = \begin{bmatrix} 5 & 7 \\ 6 & 9 \end{bmatrix}$$

$$C = \begin{bmatrix} 10 & -1 \\ 3 & 3 \end{bmatrix}$$

Note:
I = identify or unit matrix

$I.A = A$

$A.I = A$

$\therefore A.I = I.A = A$
So, the unit matrix is the unit factor in algebra.

The determinant of a matrix A is a scalar represented by /A/ or det A

olution:

A $\begin{vmatrix} 3 & 5 \\ 4 & 6 \end{vmatrix}$

/A/ = [3 x6 - (4 x 5)
= 18 - 20
= -2

/B/ $\begin{vmatrix} 5 & 7 \\ 6 & 9 \end{vmatrix}$
= 45 - 42
= 3

/C/ $\begin{vmatrix} 10 & -1 \\ 3 & 3 \end{vmatrix}$
= 30 - (-3)
= 33

12.4.2 Determinant of square matrix (3by3)

Given that A = $\begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$

and B = $\begin{vmatrix} 1 & 1 & 7 \\ 0 & -2 & 2 \\ 2 & 8 & 4 \end{vmatrix}$

Find the determinant of A and B.

Solution:-

det A = $\begin{vmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{vmatrix}$

det A

= $\begin{vmatrix} 3 & 5 \\ 4 & 6 \end{vmatrix}$

= 3 X 6 - 4 x 5
= -2

$\begin{bmatrix} 5 & 7 \\ 6 & 9 \end{bmatrix}$
= 45 - 42 =3

Note:-
-x- =+
∴ 30 + 3 =33

Fellow the solution thoroughly
-4 (18-24)
= -4(-6)
= 24

= $1\begin{vmatrix} 5 & 8 \\ 6 & 9 \end{vmatrix} - 4\begin{vmatrix} 2 & 8 \\ 3 & 9 \end{vmatrix} + 7\begin{vmatrix} 2 & 5 \\ 3 & 6 \end{vmatrix}$
= 1(45 -48)-4 (18-24) +7 (12-15)
= -3 + 24 -31
= 24 - 24 =0

ii) det B = $\begin{vmatrix} 1 & 1 & 9 \\ 0 & -2 & 2 \\ 2 & 8 & 4 \end{vmatrix}$

= $1\begin{vmatrix} -2 & 2 \\ 8 & 4 \end{vmatrix} - 1\begin{vmatrix} 0 & 2 \\ 2 & 4 \end{vmatrix} + 9\begin{vmatrix} 0 & -2 \\ 2 & 8 \end{vmatrix}$

= 1(-8 -16) -1(0 -4) +9 (0-4)
= 1(-24) +4 +9 (-4)
= -24 +4 -36
= -56

12.5.0 Minors of a Matrix

With a matrix above for instance the minor of each elements in the matrix are usually obtained by eliminating the row and column of the particular element, then find the determinant of the elements left. The following are the examples showing the minors of a_{11}, a_{12} and a_{13}.

A = $\begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$

a11 = $\begin{bmatrix} 5 & 8 \\ 6 & 9 \end{bmatrix}$ = 45-48 = -3

-3 +24 -21
= -24 - 21= 0

The removal of a_{11} will leave
$\begin{bmatrix} ① & -④- & ⑦ \\ ② & 5 & 8 \\ ③ & 6 & 9 \end{bmatrix}$

a_{12} = $\begin{bmatrix} ① & ④ & ⑦ \\ 2 & ⑤ & 8 \\ 3 & ⑥ & 9 \end{bmatrix}$

a_{12} $\begin{bmatrix} 2 & 8 \\ 3 & 9 \end{bmatrix}$ = 18 - 24 = -6 $a_{13} = \begin{bmatrix} ① & ④ & ⑦ \\ 2 & 5 & ⑧ \\ 3 & 6 & ⑨ \end{bmatrix}$

a_{13} $\begin{bmatrix} 2 & 5 \\ 3 & 6 \end{bmatrix}$ = 12 -15 = -3

12.6.0 Cofactors of a matrix

The cofactors of a matrix usually represented by Cij = $(-1)^{i+j}$ Mij is obtained by multiplying the minor of each element by $(-1)^{i+j}$.

The minor can represented Mij

12.7.0 Adjoint of a matrix

The adjoint of a matrix is the transpose of the cofactor of the matrix. Suppose the cofactor of

13.8.0 $A = \begin{bmatrix} 3 & 6 & 9 \\ -4 & 7 & 10 \\ 5 & 8 & 11 \end{bmatrix}$

Adj A = Adjoint A
Co F [A] = Cofactor A.

[Co F [A]]^T = Adjoint A

= $\begin{bmatrix} 3 & -4 & 5 \\ 6 & 7 & 8 \\ 9 & 10 & 11 \end{bmatrix}$

The rows of the cofactor becomes column.

12.9.0 Matrix Inversion

The inversion of a matrix A is repesented by A^{-1} which is the same as the adjoint of matrix A divided by its determinant.

i.e $A^{-1} = \frac{\text{adj } A}{|A|}$

12.10 Solving system of linear equations

We may also wish to use idea of adjoint to solve sysytem of linear equations.

Example: Suppose A is a matrix as shown, solve the system below:
$2x + 3y + 4z = 3$
$5x + 7y + 8z = 9$
$x + y - 2z = 3$

Solution: The system can be put as

$\begin{bmatrix} 2 & 3 & 4 \\ 5 & 7 & 8 \\ 1 & -1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \begin{bmatrix} 3 \\ 9 \\ 3 \end{bmatrix}$

$x = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{6} \begin{bmatrix} 6 & -2 & 4 \\ -18 & 8 & -4 \\ 12 & -5 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 9 \\ 3 \end{bmatrix}$

$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} \frac{-1}{6} \begin{bmatrix} 12 \\ 6 \\ -6 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$

$\therefore x = 2,\ y = 1$ and $z = 1$

12.11 Cramer's Rule

We can also solve a system of linear equation by using determinant method usually called cramer's rule.
Using cramer's rule to solve the above.

Note that:
AX = B Where A is a matrix, x the columns vector of the unknowns and B a column vector of solution to the sytem.
$\therefore X = A^{-1}B$

$2x + 3y + 4z = 3$
$5x + 7y + 8z = 9$
$x - y - 2z = 3$

$\begin{bmatrix} 2 & 3 & 4 \\ 5 & 7 & 8 \\ 1 & -1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \begin{bmatrix} 3 \\ 9 \\ 3 \end{bmatrix}$

$A^{-1} = \frac{\text{adj } (A)}{|A|} = \frac{1}{6}$

since $A^{-1} = \frac{1}{6} \begin{bmatrix} 6 & -2 & 4 \\ -18 & 8 & -4 \\ 12 & -5 & 1 \end{bmatrix}$

From above
$A = \begin{bmatrix} 2 & 3 & 4 \\ 5 & 7 & 8 \\ 1 & -1 & -2 \end{bmatrix}$

$\therefore |A| = -6$

Solution:

$2x + 3y + 4z = 3$
$5x + 7y + 8z = 9$
$x - y - 2z = 3$

1) The coefficient matrix is

$$\begin{bmatrix} 2 & 3 & 4 \\ 5 & 7 & 8 \\ 1 & -1 & -2 \end{bmatrix} \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 9 \\ 3 \end{bmatrix}$$

$$|A_1| = \begin{bmatrix} 3 & 3 & 4 \\ 9 & 7 & 8 \\ 3 & -1 & -2 \end{bmatrix}$$

$= 3(-14 + 8) - 3(-18 - 24) + 4(-9 - 21)$
$= 3(-6) - 3(-42) + 4(-30)$
$= -18 + 126 - 120 = -12$

$$|A_2| = \begin{bmatrix} 2 & 3 & 4 \\ 5 & 9 & 8 \\ 1 & 3 & -2 \end{bmatrix}$$

$2(-18 - 24) - 3(-10 - 8) + 4(15 - 9)$
$2(-42) - 3(-18) + 4(6)$
$84 + 54 + 24 + 24 = -6$

$$|A_3| = \begin{bmatrix} 2 & 3 & 3 \\ 5 & 7 & 9 \\ 1 & -7 & 3 \end{bmatrix}$$

$2(21 + 9) - 3(15 - 9) + 3(-5 - 7)$
$2(30) - 3(6) + 3(-12)$
$60 - 18 - 36$
$60 - 54 = 6$

Hence: $x = \frac{|A_1|}{|A|} = \frac{-12}{-6} = 2$

$y = \frac{|A_2|}{|A|} = \frac{-6}{-6} = 1$

$z = \frac{|A_3|}{|A|} = \frac{6}{-6} = -1$

$x = 2, y = 1, z = -1$

12.12.0 Application of Matrices into Business and Management

Example: Josaj Plc. produces for products x,y,z, and w. Each product constitute of three materials A, B,C.
A unit of x requires 3 units of A,2 of B and 4 of C.
A unit of y requires 4 units of A, 3 of B and 5 of C.
A unit of z requires 1 unit of A,1 of B and 1 of C.
A unit of W requires 1 unit of A,2 of B and 3 of C.

The cost of each unit of materials are A N200, B N300 and C N400 while the direct production costs per unit are X N500, Y N600, Z N300 and W N400.

The selling prices are X ₦4,000, Y ₦5,000, Z ₦6,000, W ₦5,000.

The weekly demands for each product are X ₦40,000; Y ₦40,000; Z ₦50,000; W ₦30,0000.

Prepare

(a) a 4 x 3 matrix showing he component requirements.
(b) a 1 x 4 matrix showing the weekly demands for each product.
(c) a 3 x 1 matrix showing the unit materials cost.
(d) A 4 x 1 matrix showing he direct production costs per unit for each product.

Solution:

a) $\begin{matrix} x \\ y \\ z \\ w \end{matrix}\begin{bmatrix} 3 & 2 & 4 \\ 4 & 2 & 5 \\ 1 & 1 & 1 \\ 1 & 2 & 3 \end{bmatrix}$

b) The weekly demands for each product is given by [40,000, 40,000, 50,000,30,000]

c) Unit material costs is shown as

$\begin{matrix} A \\ B \\ C \end{matrix}\begin{bmatrix} 200 \\ 300 \\ 400 \end{bmatrix}$

d) Direct production cost per unit is

$\begin{bmatrix} 500 \\ 600 \\ 800 \\ 400 \end{bmatrix}$

Example II: A block making industry produces slabs whose contents are sand, cement and gravel. The table shown below gives the quantity of materials required for each tpye of concrete slab and the amount of the materials available

Materials

Type	Sand (m^3)	Cement (m^3)	Gravel (m^3)
A	5	7	8
B	2	1	3
C	25	3	4
Total	1500	1040	2100

Calculate the number of each type of concrete slabs to be made out of the marterials available (to the nearest whole number)

Pick the numbers representing cost and materials

Solution:

This can be solved using the crammer's rule in matix form, we have

$$\begin{bmatrix} 5 & 2 & 25 \\ 7 & 1 & 3 \\ 8 & 3 & 4 \end{bmatrix}\begin{bmatrix} A \\ B \\ C \end{bmatrix}\begin{bmatrix} 1500 \\ 1040 \\ 2100 \end{bmatrix}$$

Let $F = \begin{bmatrix} 5 & 2 & 25 \\ 7 & 1 & 3 \\ 8 & 3 & 4 \end{bmatrix}$

$|F| = 5(4-9)-2(28-24)+25(21-8)$

$= 5(-5)-2(4)+25(13)$

$= -25-8+325 = 292$

$|F_1| = \begin{vmatrix} 1500 & 2 & 25 \\ 1040 & 1 & 3 \\ 2100 & 3 & 4 \end{vmatrix}$

$= 1500\,(4-9)-2(4160-6300)+25(3120-2100)$

$= 1500(-5)-2(2140)+25(1020)$

$= -7500-4280+25500$

$= 27500-11780$

$= 13{,}720$

$|F_2| = \begin{vmatrix} 5 & 1500 & 25 \\ 7 & 1040 & 3 \\ 8 & 2100 & 4 \end{vmatrix}$

$= 5(4160-6300)-1500(28-24)+25(14700-8320)$

$= 5(-2140)-1500(4)+25(6380)$

$= -10700-600+159500$

$= 142800$

$|F_3| = \begin{vmatrix} 5 & 2 & 1500 \\ 7 & 1 & 1040 \\ 8 & 3 & 2100 \end{vmatrix}$

Putting this in matrix form we have

$5A + 2B + 25c = 1500$

$7A + B + 3C = 1040$

$8A + 3B + 4C = 2100$

5(2100-3100)-2(14700-8320)
+1500(21-8)
5(-1020)-2(6380)+1500(13)
-5100-12760+19500
1640

A 46.99 B = 489.0 C = 5.62

Question

1 Define following terms/type of matrices
   a. Unit Matrix
   b. 'mby n' Matrix
   c. Transpose MAtrix

2 Add up the following martices

a. $\begin{bmatrix} 3 & 3 & 6 \\ 6 & 4 & 7 \\ 10 & 5 & 8 \end{bmatrix}$ and $\begin{bmatrix} 3 & 0 & 5 \\ 13 & -4 & 6 \\ 4 & 10 & 7 \end{bmatrix}$

b $\begin{bmatrix} 3 & -8 & -4 \\ 4 & -4 & 1 \end{bmatrix}$ and $\begin{bmatrix} 9 & -8 & 20 \\ 3 & -6 & 21 \end{bmatrix}$

3 If A = $\begin{bmatrix} 6 & 8 & 12 \\ 3 & 9 & 13 \\ 4 & 10 & 14 \end{bmatrix}$ B = $\begin{bmatrix} 6 & 1 & 7 \\ 8 & 5 & 16 \\ 9 & 0 & 5 \end{bmatrix}$

Find B - A, 2A + B, A + 3B

4 If A = $\begin{bmatrix} 5 & 2 \\ 7 & 4 \\ 3 & 1 \end{bmatrix}$ and B = $\begin{bmatrix} 9 & 2 & 4 \\ -2 & 3 & 6 \end{bmatrix}$

Find A, B and BA

5 Ques. Example 3.14.
The table below show the cost in N1000 of each packet of three different types of buscuits sold in four different branches of

∴ A = $\frac{/F_1/}{/F/}$ = $\frac{13720}{292}$
= 46.99

B = $\frac{/F_2/}{/F/}$ = $\frac{142800}{292}$
= 489.0

C = $\frac{/F_3/}{/F/}$ = $\frac{1640}{292}$
= 5.62

Ojolow Ventures Ltd., Lagos.
Costs <in Now> per packet

Branch	Digestive	Cracker	Kings
Agege	50	45	20
Obalende	45	39	25
Dopemu	38	51	30
Ijaye	61	48	24
Mile 2	48	46	30

If Aderome purchases to packets of Digestive, 8 packets of Cracker and 12 packets of Kings, Find the total cost of Aderome's purchases at each branch of Ojolowo Venture Ltd. Using matrix multiplication.

Hence advise Aderome on which of the branches he should buy from.

ANSWERS

CHAPTER 1

EX 1.2

(1) -2
(2) $1/10^4$
(3) 1/125
(4) 1/9x
(5) 2
(6) 1
(7) 1
(8) $1\frac{1}{4}$
(9) 64
(10) x = 4
(11) $x = {}^2/5 = 0.4$
(12) $x = {}^2/_7$
(13) $x = {}^2/3$
(14) x = 2
(15) x = 1 or 2
(16) x = 1

EX 1.2.6

1 (a) 497.7
(b) 0.4612
(c) 2956.0
(d) 0.0483
(e) 1. 444

2 a 10
b 4

3 a 0.7781
b 1.3222
c 1. 6232

4. a $a = 10^b$
b $a = 1000/b^3 \therefore ab^3 = 1000$
c $a = 0$

5. x = 0 or 1
6 a. 2. 09
b. 3.37
c. 1.89

EX 2.6 2

1. a $10\sqrt{3}$
b $5\sqrt{3}$
c $5\sqrt{5}$
2. a $\sqrt{162}$
b $\sqrt{605}$
c $\sqrt{722}$

3. a $13\sqrt{2} - \sqrt{10}$
[Alternatively $\sqrt{2}[13 - \sqrt{5}]$
b $3\sqrt{5}$
c $-\sqrt{5}$

4. (a) $\frac{4\sqrt{11}}{11}$ (b) $\frac{9+4\sqrt{2}}{7}$ (c) 2 + 3

5. (a) 11- 5 ; 116
(b) $20\sqrt{2} + 3\sqrt{3}$; 773
(c) $\sqrt{7} - 3$; -2

6. (a) 4.749
(b) 3.0312
(c) 0.7071

EX 3.3 3

1. (a) $\frac{v-u}{t} = a$

(b) $L = \frac{gT^2}{4\pi^2}$

(c) $b = \frac{a}{r} - d$

2. $k = \frac{b[a-s]}{\sqrt{t}}$

3. $r = \frac{2as}{3+s}$; 3. 75

4. $A = \frac{25d^2 - 150d + 225}{16}$; 1.56

5. (a) 34
(b) $a = \sqrt{\frac{b^2x^2}{b^2 - y^2}}$

EX 4 . 3 4

1. N18, 000
2. N8, 333 . 33
3. 12.5%
4. N9663. 06
5. N39, 366.00
6. N1700.00
7a. Ac = N637.97
b. Compound Interest = N137.97
8a. R = N557.72.
b. P.T.O for the table

Payment number	Periodic payment N	Interest at 0.025 N	Principal repaid N	Outstanding principal N
				4,000.00
1.	557.72	100.00	457.72	3,542.28
2.	557.72	88.56	469.16	3,073.12
3.	557.72	76.83	480.89	2,592.23
4.	557.72	64.81	492.91	2,099.32
5.	557.72	52.48	505.24	1,594.32
6.	557.72	39.85	517.87	1,096.21
7.	557.72	26.91	530.81	545.40
8.	557.72	13.64	544.08	0

EX 5. 4 5

1. 35
2. 8
3. $9^7 = 4782969$
4. 325
5. (a) 0.039
   (b) 3. 438
   (c) - 3.33
6. 2
7. i. $10^{10} - 1$ or 3333333333
   3 or $^1/3 [10^{10} - 1]$
   ii. 5555555555 or $0.56 [10^{10} - 1]$
   iii. $1.167[7^{11} - 1]$
8. 1.5, 3, 6, 12, and 24

EX 6.6

1 (i) 54
   ii. 81
   iii. 54
   iv. 54
   v. 81
2(i) $E = 1.2d + 1.5d^2$
   ii. 81.9
   iii. 6.44 [or - 6. 84] but the positive result is prefered.
3. 635.19,
4(a) $c = 0.75 d^2$, 75 , 18
   (b) a= 120 / b , 30
5. $173cm^3$

7.6

1. 70cm
2. 5
3. (a) 2/3 (4a + 1)
   (b) $\frac{2[2-a]}{a^2 - 4}$
4. 30 years and 12 years.
5. i. $a^2 - a - 12$.
   ii. $a^2 + 18a + 81$
   iii. $a^2 + b^2$

8.3

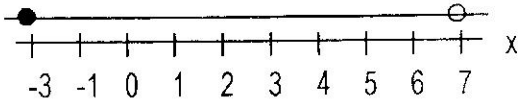
1 a (b +2)(b - 9)
   b (x + 1)(x - 5)
   c (b + 2)(b - 15)
   d. (b + 3)(b + 4)
   e. $(a + 9)^2$
2 a. -12 and -5
   b. 3 and 9
   c. - 9 and 2
3 a. $4x^2 - 19x + 12 = 0$
   b. $6x^2 + 5x + 1 = 0$
   c. $6x^2 + 35x - 6 = 0$

9.3 9

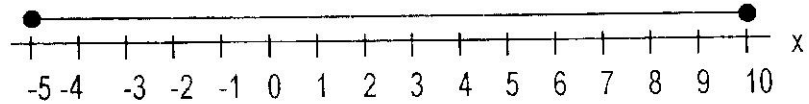
1. a = 2 b = - 5
2. x = 1 y = 2
3. x = 2 y = -1
4. F = 0.1 k = 0.19
5. When x = 4 ; y = 1
   x = -8 ; y = -3
6. $5x \geq 0$ or x = 0
7. x > -1
8. $x \leq 8$
9. x < -3

10

1 (i)



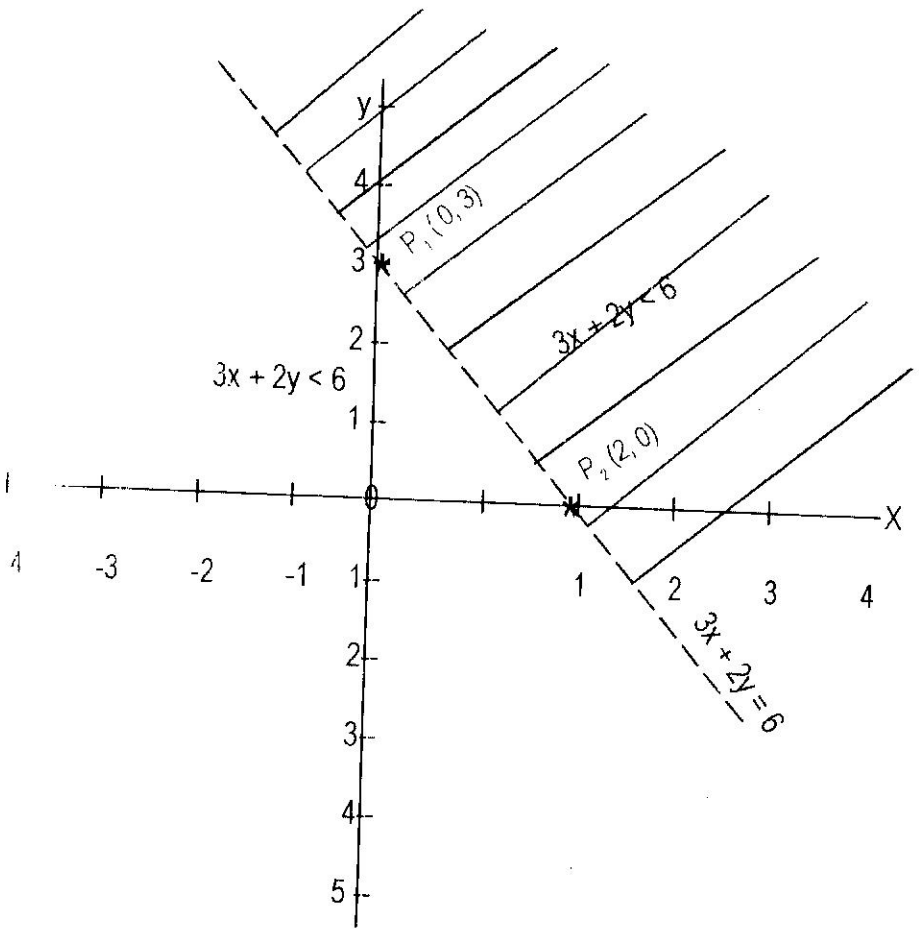
ii



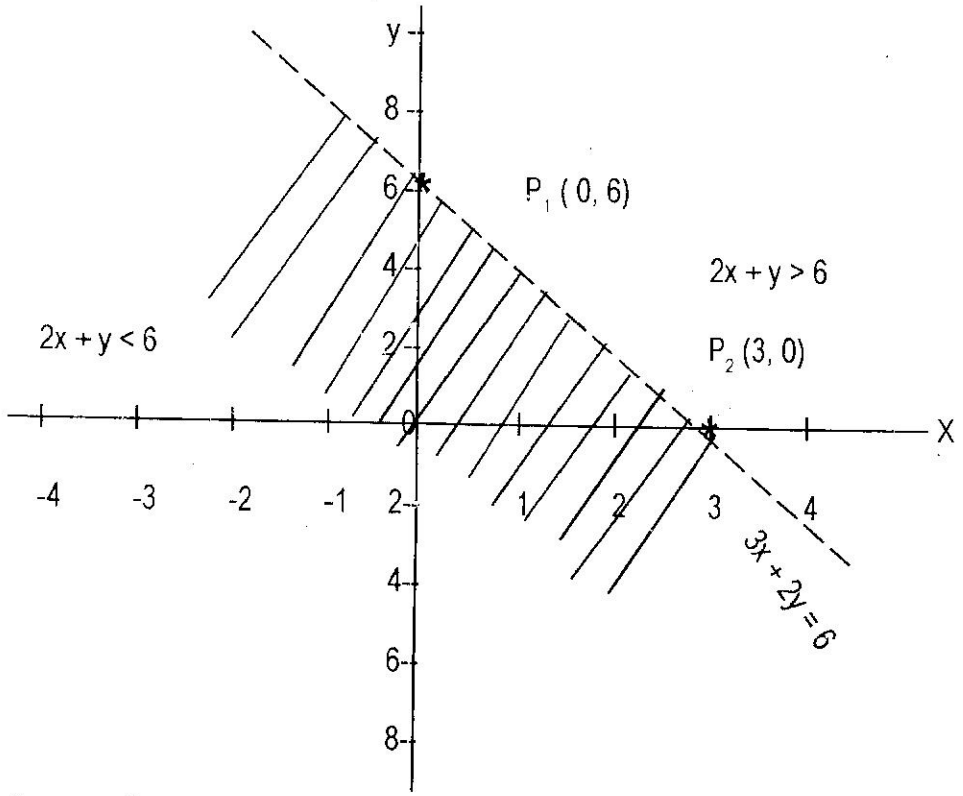
iii.

(i) when x = 0; y = 3, P_1 (0, 3)
when y =0, x =2, P_2 (2, 0)

11.3



ii. From 2x + y > 6
when x = 0, y = 6 (0, 6)
y = 0, x = 3 (3, 0)



3. x = 2 y = 1

Note: the graphs where not drawn because of space, but we hope to improve in future editions.

4 a. 360 students
b 790 students

5 i 13 customers
ii. 154 customers
iii. 12 customers
ii. 213 customers

Answers 12

4 $A \cdot B = \begin{bmatrix} 41 & 16 & 32 \\ 55 & 26 & 52 \\ 25 & 9 & 18 \end{bmatrix}$

$B \cdot A = \begin{bmatrix} 71 & 30 \\ 29 & 14 \end{bmatrix}$

5a. The martrix maltiplication for the total cost is

$$\begin{bmatrix} 50 & 45 & 20 \\ 45 & 39 & 29 \\ 38 & 51 & 30 \\ 61 & 48 & 24 \\ 48 & 46 & 30 \end{bmatrix} \quad \begin{bmatrix} 10 \\ 8 \\ 12 \end{bmatrix}$$

5 x 3 3 x 1

$$\begin{bmatrix} A \\ 0 \\ D \\ I \\ M \end{bmatrix} \quad \begin{bmatrix} 1100 \\ 1062 \\ 1148 \\ 1282 \\ 1208 \end{bmatrix}$$

b. Hence Aderome is advised to buy the biscuits from Obalende branch of Ojolowo Ventures Ltd., Lagos since if gives the minimum cost of N1,062,000.

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NOTE

The references cited are to serve as suggested texts for the students and to acknowledge the work of the authors whose work are cited.