

Algebraic Relationship of Lexical Syntactic Structures in Information Retrieval

Enikuomehin, A.O.

Department of Computer Science
Lagos State University,
Lagos, Nigeria

Owate, P.A
Department of Computer Science
Lagos State University,
Lagos, Nigeria

Ameen, A.O.

Department of Computer Science University of Horin,

Ilorin, Nigeria

OLAOYE, A.A
Department of Science Education
Lagos State University,
Lagos, Nigeria

Abstract— In this paper, we improve the formalism generated for the relationship that exists between information retrieval processes and techniques, embodied in the concept quantum theory (QT) using their lexical constituents. Measurement is used as a parameter for understanding observables in relation to lexical content of a document which is extended to the structural analysis of lexical sentences in a way that a guntactic atmoture can be precented as a m a way mat a syntactic structure can be bresented as a measure for retrieval relevance for any given user's query. Such framework can be based on the notion of information need vector spaces where an event, such as document relevance or observed user interactions. corresponds to vector subspaces. The paper discusses the algebraic transformation of the information need repository as factors contributing to the success rate of any search process. The retrieval problem is therefore represented within the quantum theory framework in terms of vector subspaces based on some algebraic manipulations. We show the existence of the relationship between these concepts and how they can

be used to improve search by using the lexical relationship between nodes. A premise is further provided for the compositionality of Natural Language Grammar as an observable which can be seen from the geometric point of view since geometry naturally creates relations between entities.

Keywords: Information Retrieval, Algebra, Lexical Structures, Quantum Theory, Syntactic Tree

I. INTRODUCTION

Quantum theory (QT) model represents a physical model that can be considered as a state (a state change model) such as the change of representation of a wave function[1] which based on some analogies can be extended to Information Retrieval (IR). This change occurs as the system acquires more knowledge about the current state. This is closely related to concepts used in ostensive retrieval [2]. In wave theory, state change occurs by the dictates of agents of reality — nature. The external agent dictates how and when the change happens while the

researcher cannot determine the certainty of occurrence of the change resulting to an uncertain assumption of the next state. This is a higher level ostensive action that corresponds to the quantum theoretical state change which, in the wave-function method of modeling, is called a state collapse or reduction of the wave packet. Thus, there is now a high-level epistemological relationship (a relationship that studies knowledge. It attempts to answer the basic question: what distinguishes true (adequate) knowledge from false (inadequate) (knowledge) between QT and our formulation of IR. In IR, the state of the user of information changes as the user acquires more information about its current state of knowledge.

Information need is the most general description of a process that is assumed to be relative to the uncertainty presented in IR. Document, repository and query are all instances of the information need. Quantum mechanics has had an enormous impact on our everyday lives. In quantum mechanics, the Heisenberg uncertainty principle states the fundamental limit on the accuracy with which certain pairs of physical properties of a particle, such as position and momentum, can be simultaneously known. In layman's terms, the more precisely on property is measured the less precisely the other can be controlled, determined, or known. Published by Werner Heisenberg in 1927, the uncertainty principle was a monumental discovery in the

early development of quantum theory. It implies that is impossible to simultaneously measure the present position while also determining the future emotion of a particle or any system small enough to require quantum mechanical treatment[3]. Quantum theory evolved as a new branch of theoretical physics during the first few decades of the 20th century in an endeavor to understand the fundamental properties of matter [4][5][6]. The concept of uncertainty

that exists in measurement of physical items and do also exist in the context of information retrieval. Often times, they are being represented by fuzzy logic systems.

Generally, the relationship between object (text, image, data), lexical objects, have been of great importance in attempt to formulate a relationship for them within a given natural language. IR process involves the concept of relevance such that objects become useful only if they are relevant. In the context of a state system, relevance will exist between the present system and the next state; this is often measured by a level of prediction using the concept

of probably. Probability exists only when a relevance of a state is observable. Relevance, which is most commonly refers to topical relevance or aboutness, i.e. to what extent the topic of a result matches the topic of the query or information need, can also be interpreted more broadly, referring to how "good" a retrieved result is, with regard to the information need. The latter definition of relevance, sometimes referred to as user relevance, encompasses topical relevance and possibly other concerns of the user such as timeliness, authority or novelty of the result. "Something (A) is relevant to a task (T) if it increases the likelihood of accomplishing the goal (G), which is implied

by T." [7]. Thus, any IR model must be based on object relevance that can adequately act on text, image, audio etc., IR models are not media specific but the language can suggest such. This is best observed in object measurement. Measure in this concept is both a physical and logical concept. Measure itself has been defined in many ways depending on the area of use, most of which are attributed to physical quantities. Measurement has been said to be the ratio of a physical quantity to the other. It is the act of finding a number that shows the amount of something. Measurement involves ratio of any entity to a unit. As example, we measure some distance as a ratio of the

quality to the unit. As well, we can measure the relationship that exists between word to generated a meaning for it. This is used as lexical measurement for objects. A person reads a newspaper by a process called scanning. This means that he will not read every word to understand the meaning of the context of such publication but will read through suggesting that the read words has a relationship with the ones not read in other to generate a meaning. Scanning is an early concept in library science[8]. This was followed by the concept of full text search which does not require human annotator. Every word has a relation to the next. This relation can be viewed as a lexical relationship such that the text could be

measured as a ratio to the other. A human sees meaning on every word presented. The use of measurement in retrieval is proposed due to the following assumptions:

a. For the user of an information system such as an online readler of a web page, it is assumed that the content of a word are related to each other since the meaning of a particular word is thought to have a relation with the next word. This is

covered more in the structural discussion of natural languages. Such exist when retrieving information from databases based on semantics. The concept of measurement can be seen in the

same regard. In the information retrieval concept, the user is expected to have a large base of document knowledge which he/she is not sure of its relevance until the search process is carried out[9]. It means that for any document search, different topics are presumed in mind. Since they are not all relevant, they can be said to be incompatible with each other, which is the fact that leads us into the quantum theory concept. The concept of QT allows the description of multiple set of measurement that are internally compatible but externally incompatible that is, measurement

that overlap with uncertainty in each other outcome [10]. The relationship between two words is essential to determine the process of information retrieval. The relationship of words in natural language assists users to retrieve relevant objects. Two words can be said to be related if they appear in the same context. Document co-occurrence gives a measure of word relatedness that has proved to be too rough to be useful.

- Quantum theory answers the question of how to represent relevance in structured spaces. It can be
 - represented by interacting measurement into IR for representation of relevance. Relevance can be thought of as an observable that measurement can be performed upon. For an operator R, we can assume that a binary case exist such that there will be two Eigen values λ_1^{-1} , λ_2^{-0} 0 corresponding to the result of measuring any document for the value of R. this could change slightly when we consider documents represented in a 3 dimensional vector, (3 dimensional Hilbert space) then if degenerate exist (having at least one Eigen

space corresponding to λ_1) then the relevance will be 3 valued and R will have their Eigen values, $\lambda_1, \lambda_2, \lambda_3$.

The assumption that the observables R can be represented by a Hermitian operator (an operator whose matrix is equal to its conjugate transpose) has also been considered. This representation is acceptable following the postulation of Gleason [11] which states that a relationship exist between Hilbert space and Hermitian operator. This fact justifies the representation of IR by geometry. Therefore we can assume that a subsnace a morotoro, we out assume and a supspace correspond to each document with a measure associated to it such that the probability measure of the Hermitian operator is given by an algorithm based on Gleason theorem. If the measure is a probability measure then the Hermitian operator can be represented as a density operator. This establishes the relationship as a self adjoint linear

operator on any given sentence space. Relevance can then be presented as a measure of probability. Probability has been crucial to the success of relevance operators in retrieval thus; it becomes reasonable to represents some observable relevance as a linear operator like those specified in Gleason theorem.

Theorem 1

Gleason's Theorem for R^3 says that if f is a non-negative function on the unit sphere with the property that f(x)+f(y)+f(z) is a fixed constant, the weight of f, for each tuple x,y,z of mutually orthogonal unit vectors, then f is a quadratic form. That is

$$f(x) = a_{11}x_1^2 + a_{22}x_2^2 + a_{33}x_3^2 + 2a_{12}x_1x_2 + 2a_{13}x_1x_3 + 2a_{23}x_2^2x_3^2 + 2a_{12}x_1x_2 + 2a_{13}x_1x_3 + 2a_{23}x_2^2x_3^2 + 2a_{12}x_1x_2 + 2a_{13}x_1x_3 + 2a_{23}x_2^2x_3^2 + 2a_{23}x_1x_3 + 2a_{23}x_1x$$

Every non-negative frame function on the unit sphere S in \mathbb{R}^3 is regular. Gleason, Andrew M., Measures on the closed subspaces of a Hilbert space, J. Math. Mech. 6(1957), 885-893): the principal axes theorem says that we can then find an orthonormal coordinate system in which f has the form

$$f(x) = a_{11}x_1^2 + a_{22}x_2^2 + a_{33}x_3^2$$

but Gleason did not express the theorem very expressively. Gleason first proves that f is uniformly continuous. a highly nontrivial task. To avoid this latter step, Cooke, Keane and Moran developed an "elementary proof" of the theorem [12]. This means that IR consist of observables that can be presented as two physical variables corresponding to commuting Hermitian operators which has common set of eigenstates. In these eigenstates, both variables have precise values at the same time, but if two operators do not commute, in general one cannot specify both values precisely. Of course such operators could still have some common eigenvectors, but the interesting case arises in attempting to measure A and B simultaneously for state $|\psi\rangle$ in which the commentator [A, B] has a nonzero

expectation value,
$$\langle \psi [A, B] | \psi \rangle \neq 0$$
. (3)

II. A MODEL FOR REPRESENTING DOCUMENTS AS VECTORS

The approach taken is to structure these developments firmly in terms of the mathematics of Hilbert spaces and linear operators. This is the approach used in quantum mechanics. It is remarkable that the application of Hilbert space mathematics to information retrieval is very similar to its application to quantum mechanics (Information

Retrieval via truncated Hilbert-space expansions[13]. A document in IR can be represented as a vector in Hilbert space, and an observable such as 'relevance' or 'aboutness' can be represented by a Hermitian operator. It turns out to be very convenient that quantum mechanics provides a ready-made interpretation of this language. It is as if in physics, we have an example semantics for the language, and as such it will be used extensively to motivate a similar but different interpretation for IR.. Gleason's Theorem, which specifies an algorithm for computing probabilities associated with subspaces in Hilbert space, is of critical

importance in quantum mechanics and will turn out to be central for the same reasons in information retrieval whereas quantum theory is about a theory of measurement for natural systems [14]. So far, the above stated conditions are applicable to any set of observables if and only if we can generate a probability for the uncertainty of some object in the information space. Since the eigenvector of an operator generate its orthonormal basis, it means that each operator generate its own basis. Outside relevance, another observable applicable to IR is aboutness [15]. Aboutness is a term used in library and information science (LIS), linguistics, philosophy of language, and philosophy of mind. In LIS, it is often considered synonymous with subject (documents). In philosophy it has been often

considered synonymous with intentionality[16]. Aboutness is a relation that maps a set of documents to a topic. Representation has been generated for aboutness using some extensional projection operator. It is therefore important to deep into the application of these operators in IR representation.

III. PROJECTING OPERATOR INTO INFORMATION SUBSPACES

An operator can be given in terms of a ket vector followed by a bra vector as example. An operator which project a vector into the i th eigenstates is given as |i>< i|. The

bra vector dot into the state given the coefficient of |j> in the state, then its multiplied by the unit vector |j> turning it back in to a vector. Since an operator maps one vector into another, this can be called an operator. The sum of the project is 1, if we sum over a complete set of slots, like the eignestates of the Hermitian operator. $\sum_i |i> < i| = 1$ If $1\psi > i$ can be j and composed as

$$1\psi>=\sum_{i}|i>< i\mid \mu>$$
 . Then, a projection operator can

be formed into a subspace. The Hermitian and idempotent attribute will exist and can be experimented by the following example;

$$\frac{1}{2} \begin{pmatrix} +1 & -i \\ +i & +1 \end{pmatrix}$$
 is both Hermitian and idempotent because
$$\frac{1}{2} \begin{pmatrix} +1 & -i \\ +i & +1 \end{pmatrix} X \frac{1}{2} \begin{pmatrix} +1 & -i \\ +i & +1 \end{pmatrix} = \frac{1}{4} \begin{pmatrix} 1+1 & -i-i \\ i+1 & +1+1 \end{pmatrix}$$

$$2(+i +1) 2(+i +1) 4(i+1 +1-1)$$
(4)

$$= \frac{1}{4} \begin{pmatrix} +2 & -2i \\ +2 & +2 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} +1 & -i \\ +1 & +i \end{pmatrix}$$

Definition 1.0: (Rank)

The rank of a project of two matrices is less or equal to the rank of either the one or the other matrix. If a square matrix has a full rank (rank equal to the number of rows or columns) we call this matrix not singular.

Furthermore, if we have a square non singular matrix then the inverse of a matrix A can be defined as A^{-1} iff $AA^{-a} = I \sin ceif (AB = BA = I \land AC = CA = I) \Rightarrow B = C \cdots$

transposition does not alter the rank of a matrix. The rank of AB and the rank of CA are equal to the rank of A if B and C are non singular.

If A and B are of the same order then (AB)⁻¹=B⁻¹A⁻¹ Definition 1.1: Pre probability

This is generally known as Trace [17]. A trace has many application format but the one linked with this work is as presented by [18] where the trace is a positive self operator.

The quantity $\sum_{j=1}^{n} \langle \underline{e}j | T | \underline{e}j \rangle$ summed over the vectors in the basis, for any T is known as the trace of T, it is equal to the sum of the diagram elements of the matrix w.r.t. orthonormal basis. A T- tr(T) mapping has the

- (1) $tr(\alpha T_i + \beta T_2) = \alpha Er(T_i) + \beta (r(t_2))$ linearity
- (2) tr(T)=sum of eigenvalues of T.
- (3) tr(T*)=tr(T), the trace of adjoint is the complex conjugale of the trace of T.
- (4) tr(T)>0. Where ever T>0

omee on me meor! or noorem mbeore.

following properties:

(5) (T₁, T₂)=tr, (T₁, T₂) is a Hilbert space of N² [19]. It is interesting to use mathematics to represent some feature sof search processes. The initial framework is based on the theory of Boolean algebra.

IV. THE FRAMEWORK OF BOOLEAN LOGIC

The basis for quantum computation is not Boolean logic, but quantum logic. To date, there is still no appropriate quantum-logical calculus comparable to the classical calculus based on Boolean algebra. There are at least two essential differences between quantum and Boolean logic. One is that any quantum gate has to be reversible, i.e., input and output must always correspond uniquely to one another. In particular, the number of input and output qubits has to be equal. This is different than in the Boolean case, where most gates have two input bits and only one

output bit. In fact, all basic binary operations of Boolean algebra $(\land, \lor, \lnot, XOR, NAND, NOR, ...)$ are 2-1 valued, which implies that they are not reversible: in fact, since $1\land 0 = 0\land 1 = 0 = 0$, you cannot deduce from the result "0" which values the input bits have had. Another difference between quantum and Boolean logic is that quantum gates can transform a qubit basis, say $\{|0\gt,|1\gt\}$, to another, for instance $\{|0\gt+|1\gt,|0\gt-|1\gt\}$, just as a vector basis can be changed by reflections or rotations. This property is impossible in Boolean logic, where any operation transforms to one of the two values 0 or 1. In other words, the basis is never changed in Boolean logic. Recently, relationship has been established between

ordinary language and mathematical physics. It is thought that the relation of a particular syntactic structure is predictable using the lexical relationship between them. This is a key are of research and it is well supported by literature. Physical systems have also been successfully related to natural observables [20],[21],[22]. These phenomena contributed to the successes in the discovery of further use of quantum theory such as the database search. We explore the context of natural syntactic and semantic structure of natural language.

Natural language processing is a core part of computational linguistic, a well established area of computer science which has been under close research since 1900's. The advancement will also be attributed to the use of computer and growing need for internet. Natural language processing (NLP) deals with automated generation and understanding of human languages. Parsing in natural language processing (NLP) is not an end-goal, but a means to an end. Syntax: provides rules to put together words to form components of sentence and to put together these components to form sentences. Lay-people rarely care about a parse as the final output of an NLP application. However, NLP practitioners may include parsing in the

middle of a text processing pipeline. Inferred syntactic information can be leveraged to improve the accuracy of higher-level textual analysis. Consider the following example: (an example of a parse tree augmented for information extraction. Example from [23],

This paper provides formalism for understanding syntactic structure of natural language by applying the relations generated form quantum theory. The aim is to develop a NLIDB based on meaning representational model achievable by the understanding of underlining syntactic

structure of natural languages. The implementation of the syntactic structure of NLIs is based on the compositional treatment of words within a NL which was contained in the framework of [24] and further enhanced [25] is used in the definition of the structure of any given NL.

V. BUILDING A VECTOR

The approach is to build vectors from words in a way that enhance meaning generation such that we can formulate a semantic similarity between these meanings. Traditional method of NL semantics has earlier been proposed. Semantic parsers automatically recover representation of meaning from natural language sentences semantic parsing

is an integrated approach that combines the fundamentals of logic, reasoning with word structures. Unlike traditional parsing approaches, semantic parsing is not time consuming and does not suffer from the problems associated with robustness and incompleteness. It integrates the best aspects of existing inductive logic programming methods into a coherent, novel framework that stands at the intersection of the fields of machine learning and logic programming.

Semantic parsing is tasked with mapping natural language (NL) sentence into a complete formal meaning representation (MR) in a meaning representation language

(MRL), that is unambiguous which allows for automated reasoning, such as first-order predicate logic. Recent work has focused on learning such parsers directly from corpora made up of sentences paired with logical meaning [26]. This research primarily focuses on MRL such that it can be executable over several domain as being used by a third

party application such as answering questions from a database or controlling some machine automated systems. Structure of a natural language has always been attributed to the meaning of it. Linguist has made attempts to show the claim. They want to reconstruct language organization such that the structural notions could be observed form it. [27] introduced the dichotomoy of the linguistic sign, as being formed through an indissoluble link between a significant, or phonetic signifier, and a signifi'e, or signified concept. A topical division in linguistics distinguishes therefore between the study of the language structure (syntax) and the study of the language meaning

(semantics). In the following years, pragmatics appeared as a reaction to Saussure's structuralist linguistics, expanding upon his idea that language has an analyzable structure, composed of parts that can be defined in relation to others. Semantic parsing, by identifying and classifying the semantic entities in context and the relations between them, has great potential on its downstream applications, such as text summarization, question answering, and machine translation. As a result, semantic parsing can be an important intermediate step for natural language understanding.

The constituent-structure theory of sentence analysis derives from the perception that the words of a sentence

seem to combine naturally into recognizable units. A simple active declarative sentence, for example, appears to consist of three components: a subject, a verb, and an object, where the subject is a unit which includes words that identify the author or agent of an action; the verb identifies this action, and the object consists of words that identify the target, result, or theme of the action. While the subject and object can each consist of a single word, that being a noun which names someone or something, both constituents often include other words, such as objectives, that describe, explain, or elaborate whatever the noun names. It is the resulting collection of words that is then

recognized as comprising the unit which functions as the subject or object of the sentence.

Furthermore, the object and the verb together are seen as making up another, more comprehensive sentence constituent that is identified as the predicate. A sentence is therefore perceived as being composed of two major functional units, namely, its subject and predicate.

The composition or structure of a sentence is consequently regarded as a hierarchy formed as successive collections of words are combined into progressively more comprehensive or inclusive constituents.

Phrase-Structure Grammar: a collection of lexical and phrase-structure rules such as those illustrated above comprise one of the four components of a conventional phrase-structure grammar. A phrase-structure grammar, G,

is defined by the entries in an ordered quadruple of the form $G = \langle V_N, V_T, S, P \rangle$ where the entries are identified as follows:

- V_N is a non-terminal vocabulary consisting of the lexical and syntactic category leabels (such as the N, V, NP and VP symbols described above).
- V_T denotes as set of words, called the terminal vocabulary of G.
- S is a special member of V_N that, in addition to being the label of the sentence category, identifies the starting symbol of G.
- P identifies the collection of rules, which is sometimes described as the production set of the grammar.

The sets V_N and V_T are not empty, but they are normally disjoint so that their intersection is empty.

Derivation of sentences: sentences can be produced or derived on the basis of G by beginning with the starting symbol and applying re-writing rules from the production set P. the grammar G is thus treated as are writing system whereby the elements of its non-terminal vocabulary $V_{\rm N}$ are treated as symbols that can be re-written or replaced by applying rules from P. this re-writing process continues by applying rules with left-hand sides that match the non-terminal symbols produced by previous replacements. The derivation stops when no further rules can be applied because all of the non-terminal symbols have been replaced by words from the terminal vocabulary $V_{\rm T}$.

Note, however, that it is not considered that the cognitive system produces sentences by a rewriting or symbol replacement process. It is assumed only that the cognitive processes of sentence production are in some sense equivalent to the rewriting procedures. The cognitive system performs operations that can be described in their effect by the rewriting or replacement operations represented above.

Context-Free Language: the resulting sequence or string of words from V_T is said to comprise a sentence that is derived or generated by the grammar. The set of all

sentences that can be generated by a grammar G is called the language of G and is identified as L(G). Because all the rules in the grammar illustrated here have a single left-hand side symbol, and the rules can be applied whenever there is a match for this symbol, without regard for whatever other symbols might be adjacent to it, a grammar such as G is described as context-free.

VI. TREE DIAGRAM

A phrase-structure grammar G can also produce a structure for a sentence that it generates. If G is context-free, this structure can have the form of a tree with its root node corresponding to the starting symbol of the grammar.

Other nodes correspond to, and are labeled by symbols from the non-terminal vocabulary $V_{\rm N}$. The leaves of the tree correspond to the words of the sentence and are therefore identified by elements of the terminal vocabulary $V_{\rm T}$. A tree can be constructed for a sentence a process

analogous to the rewriting procedure just described; but rather than replacing a non-terminal symbol, a rule attaches branches to it, with these branches ending in nodes that are labeled by the right-hand side symbols of the rule. The operation of applying rules then consists of tree substitution, rather than replacement. The substitution procedure continues as long as there are branches that end in nodes labeled with symbols from V_N, and it stops when all the branches end in leaves consisting of those words from V_T that comprise the sentence.

The resulting tree is usually represented upside down relative to the normal orientation of a tree, with its root

node at the top and the leaves, corresponding to the words of the sentence, at the bottom. A tree constructed in the course of producing the sentence 'nice dogs like cats' is down in the figure. A tree such as this represents an analysis of the sentence and may be described variously as an analysis tree, a phrase marker, or a parse tree, or is called simply an analysis or a parse for the sentence.

VII. CONCLUSION

This paper presents a formula relationship that allows us to use the concept of quantum theory in the domain of IR. We have seen the retrieval of objects such as text, data, image etc as an aspect centre to the user with such centrality

measured by relevance. The paper shows that the lexical content of an object and the relationship between the terms can affect the meaning representation. It was established that an improvement of the underlying concept of algebra as used in QT can form a good basis for natural language test retrieval without losing the richness contained in the constituents of such grammar. Finally, the context of syntactic analysis of an English grammatical sentence of the IR framework using retrieval based on relevance defined by quantum probability is still achievable. This is essential as opens a new research opportunity which will be aimed at solving the generalized IR problem.

REFERENCES

- E. Merzbacher, (1998): Quantum Mechanics, John Wiley & Sons, ISBN 0-471-88702-1
- C. J. van Rijsbergen, (1979): Information Retrieval. Butterworths, 2nd edition,
- A.Sitaram, (2001): Uncertainty Principle, mathematical", in Hazewinkel, Michiel, Encyclopedia of Mathematics, Springer, ISBN 978-1556080104).
- P.A.M. Dirac, (1930): The Principles of Quantum Mechanics, Clarendon Press, Oxford, 1930
- 5. V. B. Berestetskii, E. M. Lifshitz, L. P. Pitaevskii (1971).
- J. B. Sykes, J. S. Bell (1973): Translators, Relativistic Quantum Theory 4, part I. Course of Theoretical Physics (Landau and Lifshitz) ISBN 0-08-016025-5).
- B. Hjorland, (2010): The foundation of the concept of relevance, Journal of the American Society for information science and technology, 61(2), 217-237).
- F.I.,Dretske, (1981): Knowledge and the Flow of Information. Blackwell Publications

- D. Blair, M. Maron, (1990): Full-text information retrieval: further analysis and clarification. Information processing & management 26(3) 437-447)
- E.G Beltrametti, G. Cassinelli, (1981): The logic of quantum mechanics. Addison Wesley (
- A. Wilce, (2006): Quantum Logic and Probability Theory. In The Stanford Encyclopedia of Philosophy (Spring 2006 Edition), Edward N. Zalta (ed.)
- R. Cooke, K. Michael and M. William ,(1985): An elementary proof of Gleason's theorem, Math. Proc. Camb. Phil. Soc. 98, (117-128).
- G. Patricio, K. Ralph, and F. Bernd, (2009): CoRR, Vol. abs/0910.1938
- H. Billinge, (1997): A constructive formulation of Gleason's theorem, J. Philosophical Logic.
- B. Hjorland (2001). Towards a theory of aboutness, subject, topicality, theme, domain, field, content and relevance".
 Journal of the American society for information science and technology, 52(9), 774-778)
- P.D. Bruza, (1993). Stratified information disclosure: a synthesis between hypermedia and information retrieval. Unpublished doctoral dissertation, University of Nijmegen, Nijmegen, The Netherlands)
- 17 IT Commo (2002). Promometric Analysis Desertion II.II 5th
- H. Greene, (2003): Econometric Analysis, Prentice-Hall, 5edition,: pp. 808-809
- B. Simon, (2005): Trace ideals and their applications, second edition, Amer. Math. Soc
- C.R. Putnam, (1951): On normal operators in Hilbert space, American journal of mathematics Vol. 73, No. 2 pp. 357-362
- E. Zafiris, (2001). Probing quantum structure through Boolean localization systems for measurement of observables, international journal of theoretical physics 39, (12)
- C.H. Bennett, E. Bernstein, G. Brassard & U. Vazirani (2003): Strenghts and weaknesses of quantum computing, to be published in the SIAM journal on computing
- 22 P.W. Shor(2003), Algorithms for quantum computation: discrete logarithms and factoring, proceedings, 35th Annual symposium on fundamentals of comp
- R.C. Miller, V.H. Chou, M. Bernstein, G. Little, M. Van Kleek, and D. Kargern (2008). Inky: a sloppy command line for the web with rich visual feedback. In proceedings of the 21st annual ACM symposium on user interface software and technology (UIST'08), pages 131-140. ACM New York, NY, USA.
- F. Coecke, (2003): Quantum logic meets quantum information, special session of quantum theory: reconsideration of the foundations 2, Vaxjo, Sweden
- E. Grefenstette, (2011): A compositional distributional semantics, two concrete constructions, and some experimental evaluations in lecture notes in computer science. Vol. 7052. Pages 35-47.
- Lu et al., 2008; Kwiatkowski et al., 2010])
- D. Holdcroft, (1991). Saussure: Signs, System, and Arbitrariness. Cambridge University Press